Transmit Strategies for MIMO Multi-User Channels with AF-Relay

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Contents

Abstract

The use of multiple input multiple output systems as well as the use of relay nodes are without any doubt key elements of future wireless networks. Both techniques ensure high data rates and wide-range coverage. Due to the variety of possibilities to precode the signals before transmission and to process them at the relay, the search for the optimal transmit strategy is an enormous challenge. This holds especially if the channel is shared by multiple users and the interference among them becomes the main limiting factor for the performance. Hence, for scenarios with multiple antennas, multiple users, and a relay, the transmit strategies that achieve optimal rates are mostly unknown.

In this thesis, we aim for finding transmit strategies that achieve optimal or at least near-optimal rates in multi-user channels with an amplify-and-forward relay, where all nodes except the receiver(s) have multiple antennas. As amplify-and-forward relays simply amplify the incoming signals without any decoding or re-encoding, a low computational complexity and small delays are ensured. The channel models that we consider are the multiple-access relay channel as well as the broadcast relay channel. Moreover, we compare a time division multiple-access (TDMA)-based transmit scheme to a strategy where all stations use the channel at the same time (referred to as joint relaying).

In a first step, we neglect the direct links between the transmitter(s) and the receiver(s) and derive the optimal sum-rates of TDMA and joint relaying for the multiple-access relay channel. Moreover, we show that TDMA achieves higher sum-rates than joint relaying. For the broadcast relay channel without direct links we derive a heuristic scheme. This scheme has a lower complexity than prevailing schemes and is shown to achieve approximately the same sum-rates.

Subsequently, we generalize the model of the multiple-access relay channel by considering also the direct links between the transmitters and the receiver. For this generalization, we find upper and lower bounds on the sum-rate both for TDMA and joint relaying. As the respective upper and lower bounds are sufficiently close, TDMA and joint relaying can be compared by means of simulation results. It turns out that the superiority of TDMA persists for weak direct links but gets lost quickly if the direct links become stronger.

Finally, we consider the use of finite alphabets in the broadcast channel at low signal-to-noise ratio. Using the minimum required energy per bit and the wideband slope as quality criteria, we show that the optimal performance of Gaussian alphabets is also achieved by quadrature phase shift keying. As a consequence of this, the suboptimality of TDMA at low signal-to-noise ratio, which was already shown for Gaussian alphabets, persists if finite alphabets are used.

List of Symbols and Acronyms

$\lvert a \rvert$	Absolute value of a
R	Achievable rate
R_Σ	Achievable sum-rate
AWGN	Additive white Gaussian noise
AF	Amplify-and-forward
$\mathbf{F}$	Amplifying matrix at relay
$\angle(a)$	Angle of $a \in \mathbb{C}$, i.e., $\angle\left(e^{i\varphi}\right) = \varphi$
P	Available average transmit power
BPSK	Binary phase shift keying
BRC	Broadcast relay channel
$\mathcal{C}$	Capacity region
C	Channel capacity
h	Channel gain
CSI	Channel state information
DF	Decode-and-forward
$\lvert\mathbf{A}\rvert$	Determinant of matrix $\mathbf{A}$
$\operatorname{diag}(a_1, \ldots, a_n)$	Diagonal matrix, where $a_1, \ldots, a_n$ are the values on the diagonal
DPC	Dirty paper coding
τ	Duration of a time slot when using TDMA
EVD	Eigenvalue decomposition
$\mathbf{v}_{\max}(\mathbf{A})$	Eigenvector corresponding to maximum eigenvalue of a matrix $\mathbf{A}$
E_b	Energy per information bit
$\lvert\lvert\mathbf{h}\rvert\rvert$	Euclidean norm of a vector $\mathbf{h}$
FDMA	Frequency division multiple access
$\mathcal{CN}(\mu, \sigma^2)$	Gaussian distributed random variable with mean μ and variance σ^2
$\mathcal{CN}(\mu, \mathbf{A})$	Gaussian distributed random vector with mean μ and covariance matrix $\mathbf{A}$
∇f	Gradient of a scalar field f
$\mathbf{1}[\Upsilon]$	Indicator function, which is 1 if condition Υ is true, otherwise zero
IC	Interference channel
IWF	Iterative water-filling
JR	Joint relaying (only used partly in simulation results)

KKT (conditions)	Karush-Kuhn-Tucker (conditions)
$\lambda_{\max}(\mathbf{A})$	Maximum eigenvalue of a matrix $\mathbf{A}$
$\max(.)$	Maximum of an expression
$\lambda_{\min}(\mathbf{A})$	Minimum eigenvalue of a matrix $\mathbf{A}$
$\frac{E_b}{N_0}_{\min}$	Minimum energy per bit to noise ratio
$\min(.)$	Minimum of an expression
$\frac{E_b}{N_0}^r_{\min}$	Minimum received energy per bit to noise ratio
MIMO	Multiple input multiple output
MISO	Multiple input single output
MAC	Multiple-access channel
MARC	Multiple-access relay channel
$I(X;Y)$	Mutual information of random variables X and Y
N_r	Number of antennas at receiver
N_f	Number of antennas at relay
N_t	Number of antennas at transmitter
K	Number of users
$\mathbf{A} \succeq 0$	Positive semidefiniteness of a matrix $\mathbf{A}$
QPSK	Quadrature phase shift keying
y	Received signal
z	Receiver noise
$\mathcal{C}^n$	Set of n-dimensional complex vector
$\mathcal{C}^{m\times n}$	Set of complex $m \times n$ matrices
SNR	Signal-to-noise ratio
SIMO	Single input multiple output
SISO	Single input single output
SVD	Singular value decomposition
C_Σ	Sum channel capacity
TDMA	Time division multiple access
$\mathrm{tr}(\mathbf{A})$	Trace of a matrix $\mathbf{A}$
$\mathbf{Q}$	Transmit covariance matrix
x	Transmitted signal
UB	Upper bound (only used partly in simulation results)
$\mathcal{S}_0$	Wideband slope
ZF-DPC	Zero-forcing dirty paper coding

1

Introduction

In today's wireless communication systems, the demand for high data rates and wide-range coverage is steadily growing. To meet these requirements, a high density of base stations is necessary, which entails high costs for installation and maintenance. Two other possibilities to increase throughput and coverage are the use of relay nodes as well as the use of multiple input multiple output (MIMO) technology. The goal of this thesis is to combine these two paradigms and optimize the transmit strategies such that the highest possible data rate is achieved in the up- and downlink of wireless networks.

Relay channels were considered in [CG79] first and have been extensively studied by researchers in the last decades. As there are several possibilities of processing the signals at the relay, different types of relays are distinguished. One very powerful possibility, referred to as decode-and-forward (DF) or regenerative relaying, is to decode the signals at the relay, re-encode them, and subsequently transmit them to the destination(s). However, although very efficient decoding algorithms exist [Bos99], this requires high computational power at the relay and entails large delays. Another widely-used possibility, referred to as amplify-and-forward (AF) or non-regenerative relaying, is to simply amplify the received signal. The drawback of this scheme is that the noise, which occurs at the relay, is also amplified and thus decreases the performance.

Also the use of relays in multi-user channels has been considered in numerous works. A comparison of different relaying techniques as well as upper bounds on the achievable data rates for the multiple-access relay channel (MARC) are given in [SKM04]. This work is extended in [SKM07], where an improved scheme for the MARC based on DF is presented. Moreover, [MDG13] introduces two joint source-channel coding schemes for a MARC with correlated sources and DF-relay. Contrary to those works, we will focus on AF-relaying techniques in this thesis, because AF has much lower delays and ensures that the relay nodes remain cheap.

Similar to relay channels, MIMO channels have been the focus of many works in the last years. During that time the capacity region was found for the most important channels occurring in wireless communication like the point-to-point channel [Tel99], the multiple-access channel (MAC) [CV93, YRBC04], and the broadcast channel [CS03, YC01, WSS06].

However, if a MIMO AF-relay is added to those channels, both the distribution of the transmit power to the antennas (identified by the transmit covariance matrix) as well as the amplifying matrix at the relay have to be optimized. This turned out to be an enormous challenge and therefore the capacity region is unknown for most scenarios. Especially in case the direct links between the transmitter(s) and the receiver(s) are not neglected, few results exist. Nevertheless, considerable progress has been made in the field of MIMO channels with AF-relay during the last years:

The structure of the relay matrix and the transmit covariance matrices that leads to the optimal sum-rate has been found both for the point-to-point channel without direct links as well as for the MARC without direct links in [FHK06, YH10]. However, this structure still contains parameters that are subject to optimization and the optimal sum-rate of this channel remains unknown in general.

In [TH07], a single-user system was considered, where the transmit covariance matrix was fixed to a scaled identity matrix. With this restriction, an algorithm was found that optimizes the relay matrix. However, for the case of nonzero direct links, only upper and lower bounds on the sum-rate could be provided. Different from all previously mentioned works, a half-duplex relay was assumed in [VSBNS06]. Using this relay in single-user systems both with and without direct links, suboptimal transmit strategies based on iterative algorithms were derived.

The broadcast relay channel (BRC) is considered in [CTHC08, YH10]. For this channel, only suboptimal algorithms are presented, which partly suffer from a high computational complexity. A duality between the BRC and the MARC, as existent for the corresponding channels without relay [VJG03], is found in [JGH07]. Unfortunately, this duality only holds if the receiver(s) and transmitter(s) have only a single antenna.

In this thesis, our goal is to find the optimal sum-rates that are achievable with AF-relays in the multiple input single output (MISO) BRC without direct links and in the MISO MARC with and without direct links, i.e., we assume that the receiver(s) has/have a single antenna. This assumption allows us to find either closed-form solutions for the optimal sum-rate or close upper and lower bounds. Thus, it provides a first step in finding the optimal sum-rate of general MIMO relay channels, where also the receiver(s) has/have multiple antennas.

For this purpose, we generally assume Gaussian channels and Gaussian transmit alphabets. An exception to this is Chapter 6, where we assume Gaussian channels but finite alphabets. The reason for this is that Gaussian alphabets can not be used but only approximated in practical applications.

In detail, the contributions of this thesis and the corresponding publications are the following:

- **MISO MARC without direct links (published in [KMH12b]):** We adopt the approach from [YH10] and derive a solution that reaches the optimal sum-rate for the MISO MARC without direct links in closed-form. Furthermore, we propose a different transmission scheme based on time division multiple access (TDMA). In contrast to the prevailing scheme (like in [YH10]), where all users access the channel jointly (referred to as joint relaying), our scheme decomposes the K-user MARC in K orthogonal single-user relay channels. After deriving the optimal duration of the time slots, we obtain a closed-form solution for the sum-rate achievable with TDMA. Subsequently, we show that this rate is always higher than the sum-rate of joint relaying in practical channels.

- **MISO BRC without direct links (unpublished):** We present a heuristic algorithm which exploits the duality of MAC and broadcast channels without relays from [VJG03]. Compared to existing algorithms, our algorithm has a much lower complexity and it is shown by means of simulation results that the achieved rates are approximately the same.

- **MISO MARC with direct links (unpublished):** Contrary to the MISO MARC without direct links, a closed-form solution of the achievable rates with TDMA or joint relaying does not seem feasible for nonzero direct links. Instead, we derive upper and lower bounds, which are shown to be close to each other by means of simulation results. The lower bounds are achieved by an efficient iterative algorithm, which is shown to converge within a few iterations by means of simulation results. Further simulations reveal that the superiority of TDMA, which was shown for absent direct links, only persists if the direct links are weak. For strong direct links, it is observed that joint relaying delivers better results.

- **SISO MARC with direct links (published in [KMH12a]):** As a special case of the MISO MARC with direct links, we consider the single input single output (SISO) MARC with direct links. The single transmit antennas allow to derive an algorithm which computes the optimal sum-rate achievable with TDMA. Moreover, an analytic criterion for the asymptotic optimality of TDMA is derived.

- **Finite alphabets in broadcast channels (published in [KWHK09]):** As practical systems can not use the unbounded Gaussian alphabets, we also consider the performance of finite alphabets at low signal-to-noise ratio (SNR) in broadcast channels. As the low SNR entails low data rates, these rates are not an adequate quality criterion for this case. Instead, Verdú defined the minimum energy per bit and the wideband slope [Ver02] as quality criteria for low-SNR channels. Using these criteria, we show that the optimal performance of Gaussian alphabets is also achieved by quadrature phase shift keying (QPSK) in low-SNR broadcast channels. Moreover, we show that the suboptimality of TDMA, which was found for Gaussian alphabets in these channels [CTV04], persists if finite alphabets are used.

Outline of this Thesis

In Chapter 2, we introduce the two channel models considered in this work in their most general form. Hence, we describe the MIMO MARC with direct links as well as the MIMO BRC without direct links together with the constraints on the transmitted signals that reflect the limited transmit power at the transmitter(s) and the relay. The simplifications that result from assumptions like zero direct links or single receiver antennas are mentioned in the chapters where they are assumed.

The fundamentals of multi-user systems with and without relay as well as the current state of research are discussed in Chapter 3. First, we describe the MAC and broadcast channel in Section 3.1 and 3.2, respectively. In both sections, we first consider the respective SISO channel and describe the capacity achieving transmit strategy as well as the capacity region. Subsequently, the MIMO case is considered and the solutions to the new optimization problems that multiple antennas pose as well as the resulting capacity regions are discussed. Moreover, for both channels and both SISO and MIMO, we derive the rates achievable by TDMA.

As it turns out that the optimization problems for the MIMO broadcast channel are, due to their nonconvexity, much harder to solve, we discuss the duality theory from [VJG03] in Subsection 3.2.3. This theory builds a relation between the capacity regions of the broadcast channel and its dual MAC, such that the problem of finding the broadcast channel capacity region can be simplified.

Section 3.3 describes the current state of research in multi-user channels with AF-relay. This includes a description of the results for the MARC without direct links in Subsection 3.3.1 as well as for the broadcast channel without direct links in Subsection 3.3.2. For channels with direct links, results are only available for single-user channels. These are described in Subsection 3.3.3.

Our work on multi-user channels with AF-relay and without direct links is described in Chapter 4. This chapter starts with a description of our proposed TDMA-based relaying scheme, which contains the consideration of single-user relay channels. Subsequently, we consider the MISO MARC in Section 4.2, where we find the optimal sum-rates of joint relaying in 4.2.1 and of our proposed TDMA scheme in 4.2.2. The achieved rates are compared in Subsection 4.2.3, where it is also shown that the rates achieved by TDMA are always superior in practical systems.

The BRC without direct links is considered in Section 4.3. In this section, we derive an efficient algorithm to optimize the transmit strategy in Subsection 4.3.1 and compare this strategy to prevailing schemes and upper bounds in Subsection 4.3.2.

Chapter 5 considers the MARC with direct links. We derive three upper bounds on the achievable sum-rate in Section 5.1. The first upper bound is for single-user channels and is used to provide an upper bound for TDMA. It is determined by an algorithm with brute-force methods, where the structure of the corresponding optimization problem is exploited to reduce the complexity of the algorithm. The two other upper bounds are intended for multi-user channels with joint relaying. One of these bounds is also computed by a brute-force algorithm, while the other one is found by bounding the expression for the sum-rate directly.

In Section 5.2, we describe achievable schemes for the MISO MARC with direct links. First, we explain why, contrary to the upper bounds, the complexity of brute-force methods seems to be too high to calculate solutions with an adequate precision. Subsequently, in Subsection 5.2.2, we derive a heuristic and iterative algorithm with low complexity. This algorithm computes covariance matrices and a relaying matrix that achieve a sum-rate which is close to the optimum. The special case of a SISO MARC is discussed in Section 5.3, where an algorithm is given to calculate the optimal sum-rate achievable with TDMA. This allows the derivation of an analytic criterion for the asymptotic optimality of TDMA.

Finally, Section 5.4 presents simulation results for the MISO and SISO MARC with direct links. These results show how close the derived upper and lower bounds are and allow a comparison of TDMA and joint relaying. Moreover, the convergence behavior of the algorithm derived in Subsection 5.2.2 is analyzed.

The use of the more practical finite alphabets in broadcast channels at low SNR is discussed in Chapter 6. For this purpose, the minimum energy per bit and the wideband slope, which were defined as quality criteria for low-SNR channels in previous works, are introduced in Section 6.1. This definition is followed by a review of the results for low-SNR broadcast channels with Gaussian alphabets in Section 6.2. Our derivation of the wideband slope and minimum energy per bit for BPSK and QPSK alphabets is conducted in Section 6.3. In Section 6.4, the obtained slope regions are evaluated graphically for one example.

Finally, the thesis is closed by Chapter 7, which gives a summary of the results, some concluding remarks and an outlook on possible future research in the considered fields.

Notation

Our notation follows the list of symbols and acronyms, which precedes this chapter. However, due to the variety of the considered problems, the variables that are listed there are only a basis, which is extended where necessary. Generally, we use boldface lowercase letters to denote column vectors and boldface uppercase letters to denote matrices.

Moreover, if multiple instances or transformations of one variable are required, we use subscripts or decorations like tildes, bars, etc.. As we consider multi-user systems, superscripts with numbers in braces are exclusively used to denote variables that can be associated with one user.

To give an example, h denotes the channel gain if a channel is scalar, while for vector channels we would use $\mathbf{h}$ and for channel matrices we use $\mathbf{H}$. In a MIMO MARC, the channel gain from transmitter k to the relay is denoted by $\mathbf{H}_r^{(k)}$, while the channel gain for the direct link between transmitter k and the receiver is called $\mathbf{H}_d^{(k)}$.

As explained above, the goal of this thesis is to maximize the achievable (sum-)rates. Therefore, if not otherwise identified, the attribute optimal means optimal with respect to the maximum (sum-)rate throughout the whole thesis. Note that, as we restrict ourselves to AF relaying schemes in this thesis, a scheme that is called optimal in this context might be outperformed by a DF-based relaying scheme. The same holds for the upper bounds derived here. A scheme that is not optimal (or can not shown to be optimal) but could be realized under the given constraints, is referred to as "achievable scheme".

2

Considered Channel Models

This section describes the two channel models that are discussed in Chapters 4 and 5 as well as in Section 3.3. For these descriptions, we use the most general form of the channel models that is required in the following chapters. Thus, we describe the multiple-access relay channel (MARC) with multiple antennas at all nodes, with direct links between transmitters and receiver, and with an arbitrary number of users K. For the broadcast relay channel (BRC), we use the same assumptions except the direct links, because they are not considered for the BRC in this work. In the following chapters, we will then describe the simplifications, e.g., the restriction to a single receive antenna, where they are assumed.

In this chapter, we restrict ourselves to a description of the channel model without any analysis or transmit strategies. The analysis of existing transmit strategies is done step by step in Chapter 3.

The common channel properties that we assume throughout this whole thesis are the following: We consider additive white Gaussian noise (AWGN) channels with independent block Rayleigh fading. This means, the channel gains between two antennas are Rayleigh distributed, independent of each other, and remain constant during the transmission of one codeword. The number of antennas at the receiver and transmitter are denoted by N_r and N_t, respectively. Due to the independence of the channel gains, the channel matrices have rank $\min\{N_r, N_t\}$ with probability 1. Finally, we assume perfect channel state information (CSI) at both the transmitter(s) and the receiver(s) as well as, if existent, the relay. Hence, the received signal can be modeled as the sum of the transmitted signal(s), multiplied by the channel matrix, and the additive Gaussian noise.

As already stated in the introduction, we consider full-duplex and amplify-and-forward (AF) relays. Moreover, we assume that the relay transmits in a different frequency band than the transmitter(s). For the sake of conformity with the underlying references [YH10, TH07], we do not account for the doubled bandwidth demand by multiplying the rates (given in Bit/s/Hz) with a factor of 1/2. However, the additional bandwidth requirement has to be considered when comparing the rates with those of a channel without relay.

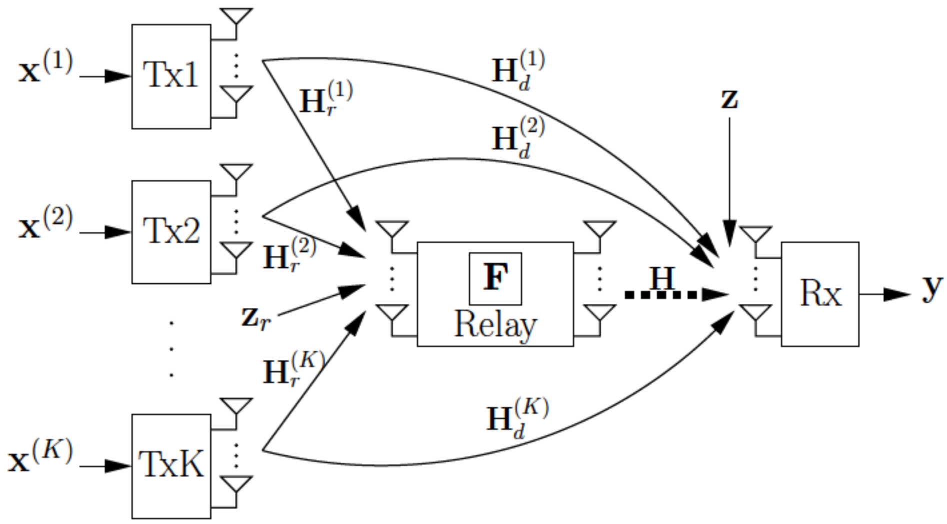

Figure 2.1: K-user multiple-access relay channel (MARC)

2.1 Multiple-Access Relay Channel (MARC)

The considered MARC is depicted in Figure 2.1 and consists of K transmitting nodes with $N_t^{(1)}, \dots, N_t^{(K)}$ antennas, one relay with N_f antennas, and one receiver with N_r antennas. Each user $k \in \{1, \dots, K\}$ transmits the signal $\mathbf{x}^{(k)} \in \mathbb{C}^{N_t^{(k)}}$, which reaches both the relay and the receiver. The channel matrix for the transmission to the relay is given by the matrix $\mathbf{H}_r^{(k)} \in \mathbb{C}^{N_f \times N_t^{(k)}}$, while the direct channel to the receiver is described by the matrix $\mathbf{H}_d^{(k)} \in \mathbb{C}^{N_r \times N_t^{(k)}}$. Thus, the received signal $\mathbf{y}_r \in \mathbb{C}^{N_f}$ of the relay can be written as

$$\mathbf{y}_r = \sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{x}^{(k)} + \mathbf{z}_r,$$

where $\mathbf{z}_r \in \mathbb{C}^{N_f}$ is the noise vector at the relay. We assume $\mathbf{z}_r \sim \mathcal{CN}(0, \mathbf{I})$, i.e., the components of $\mathbf{z}_r$ are Gaussian distributed with zero mean and unit covariance matrix. The relay amplifies the signals by the matrix $\mathbf{F} \in \mathbb{C}^{N_f \times N_f}$ and transmits the signal $\mathbf{x}_r = \mathbf{F}\mathbf{y}_r$ over the channel $\mathbf{H} \in \mathbb{C}^{N_r \times N_f}$ to the receiver.

It is assumed that the transmission from the relay to the receiver takes place in a different frequency band, i.e., the signals transmitted by the relay are orthogonal to the signals transmitted by the users. To overcome the problem that the signals from the relay arrive with the delay of one symbol, we assume that the direct signal can be buffered and processed together with the delayed signal from the relay. Hence, the out-of-band reception can be modeled by doubling the number of antennas at the receiver, where one half receives the signal from the relay and the other half receives the signals from the transmitters. This results in the received signal

$$\mathbf{y} = \begin{bmatrix} \mathbf{HF} \sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{x}^{(k)} + \mathbf{HF}\mathbf{z}_r + \mathbf{z}_1 \\ \sum_{k=1}^{K} \mathbf{H}_d^{(k)} \mathbf{x}^{(k)} + \mathbf{z}_2 \end{bmatrix},$$

where $\mathbf{z}_i \sim \mathcal{CN}(0, \mathbf{I})$ $(i = 1, 2)$ denote the Gaussian noise vectors at the receiver.

As it can be seen, the first N_r components of $\mathbf{y}$ contain noise both from the relay and from the receiver. This leads to a noise covariance matrix of

$$\mathbf{z} \triangleq \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix} \sim \mathcal{CN}\left(0, \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}\right),$$

where $\mathbf{S} = \mathbf{I} + \mathbf{HFF}^H\mathbf{H}^H$. As the formulation of the achievable rates is much easier with unit noise covariance matrices, we will normalize the first N_r components of $\mathbf{y}$ by multiplying the channel outputs with $\mathbf{S}^{-1/2}$. This does not change the achievable rates and results in an equivalent output

$$\widetilde{\mathbf{y}} = \sum_{k=1}^{K} \mathbf{H}_{\text{eff}}^{(k)} \mathbf{x}^{(k)} + \widetilde{\mathbf{z}}$$

at the receiver, where $\widetilde{\mathbf{z}} \sim \mathcal{CN}(0, \mathbf{I})$, and $\mathbf{H}_{\text{eff}}^{(k)} \in \mathbb{C}^{2N_r \times N_t^{(k)}}$ is defined as

$$\mathbf{H}_{\text{eff}}^{(k)} \triangleq \begin{bmatrix} \mathbf{S}^{-1/2}\mathbf{HF}\mathbf{H}_r^{(k)} \\ \mathbf{H}_d^{(k)} \end{bmatrix}.$$

Of course, the available transmit power is limited both at the relay and at the transmitters. If the transmitters' covariance matrices are defined as $E\left(\mathbf{x}^{(k)}\mathbf{x}^{(k)H}\right) \triangleq \mathbf{Q}^{(k)}$, then the average transmit power constraints are given by

$$\begin{aligned} \mathbf{Q}^{(k)} &\succeq 0 \quad \forall k \in \{1, \ldots, K\} \\ \operatorname{tr}\left(\mathbf{Q}^{(k)}\right) &\leq P^{(k)} \quad \forall k \in \{1, \ldots, K\} \\ E\left(\operatorname{tr}\left(\mathbf{x}_r\mathbf{x}_r^H\right)\right) = \operatorname{tr}\left(\mathbf{F}\left(\mathbf{I} + \sum_{k=1}^{K} \mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)H}\right)\mathbf{F}^H\right) &\leq P_r, \end{aligned} \tag{2.1}$$

where $\operatorname{tr}(\mathbf{A})$ denotes the trace of a matrix $\mathbf{A}$.

The parameters that are subject to optimization are the distribution of the available transmit powers $P^{(k)}$ to the antennas of user k, which are determined by the covariance matrix $\mathbf{Q}^{(k)}$ as well as the construction of the relay's output from its input, which is determined by the relaying matrix $\mathbf{F}$. Because of the complexity of finding a global optimal solution for this problem, we will first explain the derivation of the optimal covariance matrices $\mathbf{Q}^{(k)}$ in the multiple-access channel (MAC) without relay. In a second step, the relay is added to the channel, which raises the additional challenge of finding the optimal relaying matrix $\mathbf{F}$.

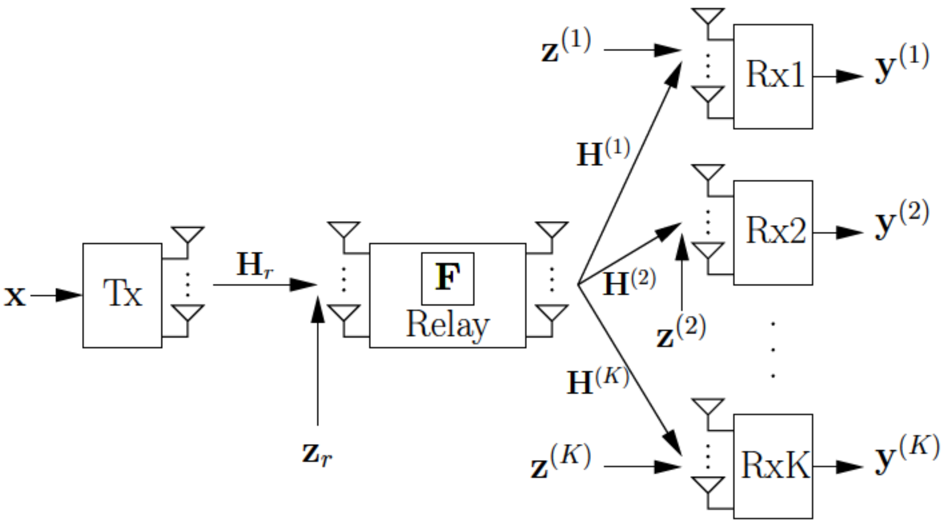

Figure 2.2: K-user broadcast relay channel (BRC)

2.2 Broadcast Relay Channel (BRC)

The BRC is depicted in Figure 2.2, where it can be seen that it is a mirrored copy of the MARC without direct links constructed by reversing the direction of transmission. Thus, we have one transmitter with N_t antennas, one relay with N_f antennas and K receivers with $N_r^{(1)}, \ldots, N_r^{(K)}$ antennas. In this thesis, we will focus on the achievable rates with superposition coding, which is the optimal coding strategy for broadcast channels. This means, the transmitted signal $\mathbf{x} \in \mathbb{C}^{N_t}$ is constructed as the sum of the signals $\mathbf{x}^{(k)} \in \mathbb{C}^{N_t}$, i.e.,

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{x}^{(k)},$$

where $\mathbf{x}^{(k)}$ is the signal containing the information for user k. Hence, the received signal at the relay can be written as

$$\mathbf{y}_r = \mathbf{H}_r \mathbf{x} + \mathbf{z}_r,$$

where $\mathbf{z}_r \sim \mathcal{CN}(0, \mathbf{I})$ is the noise at the relay and $\mathbf{H}_r \in \mathbb{C}^{N_f \times N_t}$ denotes the channel between transmitter and relay.

As in the MARC, the relay amplifies the signal with the matrix $\mathbf{F} \in \mathbb{C}^{N_f \times N_f}$ and constructs the output signal $\mathbf{x}_r = \mathbf{F}\mathbf{y}_r$. However, in contrast to the MARC, we do not consider the direct links between transmitter and receivers such that the receivers obtain their signal from the relay only. Therefore, the received signal at receiver k can be written as

$$\mathbf{y}^{(k)} = \mathbf{H}^{(k)}\mathbf{F}\mathbf{H}_r\mathbf{x} + \mathbf{H}^{(k)}\mathbf{F}\mathbf{z}_r + \mathbf{z}^{(k)} = \mathbf{H}^{(k)}\mathbf{F}\mathbf{H}_r \sum_{k=1}^{K} \mathbf{x}^{(k)} + \mathbf{H}^{(k)}\mathbf{F}\mathbf{z}_r + \mathbf{z}^{(k)}, \tag{2.2}$$

where $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{I})$ is the additional noise at the receiver and $\mathbf{H}^{(k)} \in \mathbb{C}^{N_r^{(k)} \times N_f}$ denotes the channel between the relay and receiver k.

Also in the BRC there is noise both from the relay and the receiver such that the total noise covariance matrix at user k is $\mathbf{I} + \mathbf{H}^{(k)}\mathbf{F}\mathbf{F}^H{\mathbf{H}^{(k)}}^H$. However, the absence of the direct links allows a compact formulation of the achievable rates such that a normalization of the noise is superfluous.

As the BRC has only one transmitting station, there is in principle only one transmit covariance matrix $\mathbf{Q} \triangleq E\left(\mathbf{x}\mathbf{x}^H\right)$, which is subject to optimization. However, it makes sense to consider the covariance matrices $\mathbf{Q}^{(k)} \triangleq E\left(\mathbf{x}^{(k)}{\mathbf{x}^{(k)}}^H\right)$ instead, which, due to the independence of the signals $\mathbf{x}^{(k)}$, sum up to the total covariance matrix $\mathbf{Q}$. Together with the power constraint at the relay, the average power constraints can be formulated as

$$\begin{aligned} \mathbf{Q}^{(k)} &\succeq 0 \quad \forall k \in \{1, \ldots, K\} \\ \operatorname{tr}(\mathbf{Q}) = \operatorname{tr}&\left(\sum_{k=1}^{K} \mathbf{Q}^{(k)}\right) \leq P \\ E\left(\operatorname{tr}\left(\mathbf{x}_r\mathbf{x}_r^H\right)\right) = \operatorname{tr}&\left(\mathbf{F}\left(\mathbf{I} + \mathbf{H}_r\mathbf{Q}{\mathbf{H}_r}^H\right)\mathbf{F}^H\right) \leq P_r. \end{aligned} \tag{2.3}$$

The most important differences to the MARC are the joint power constraint for the covariance matrices and the fact that, from the receiver perspective, the interference and the desired signal go through the same channel. Although the optimization parameters are still $\mathbf{F}$ and the covariance matrices $\mathbf{Q}^{(k)}$, we will see in the following chapters that the latter point makes the optimization in the BRC much more complicated than the one for the MARC. Concerning the explanation of the BRC, we will proceed in the same way as for the MARC, i.e., give an explanation of the broadcast channel without relay first.

3

Fundamentals

This chapter discusses the fundamentals of MIMO multi-user systems. The MAC and the broadcast channel are considered in Section 3.1 and 3.2, respectively. For both channels the capacity achieving transmit technique is described and the capacity region is formulated for both single- and multi-antenna systems. As, especially in the MIMO BC, the optimization of the matrices in order to achieve the capacity is very challenging, the duality between MAC and broadcast channels is discussed in Subsection 3.2.3, which allows a simplification of this problem. Moreover, for both channels the use of TDMA as alternative transmit strategy is considered.

In Section 3.3, we describe the current state of research on these two channels with AF-relay. The MARC without direct links is discussed in Subsection 3.3.1, while the BRC without direct links is considered in Subsection 3.3.2. Subsection 3.3.3 considers a result on relay channels with direct links. However, this result is only for single-user channels with fixed transmit covariance matrices.

All the results presented is this chapter are from existing works that serve as a basis for this thesis. These results shall be extended by the contributions of this thesis in Chapters 4, 5, and 6.

3.1 Gaussian MAC

One of the standard scenarios in multi-user wireless communications is the transmission from $K > 1$ users to one receiving node. This scenario occurs for example in the uplink of cellular networks. The corresponding channel is referred to as K-user MAC. In this section, the single input single output (SISO) Gaussian MAC will be introduced first. Hereafter, the more involved case of a multiple input multiple output (MIMO) Gaussian MAC is described and the differences to the SISO MAC are explained.

3.1.1 Single-Antenna Gaussian MAC

The SISO Gaussian MAC is depicted in Figure 3.1. In this channel, the received signal y is obtained as the sum of the user's inputs $x^{(k)}$ multiplied by their corresponding

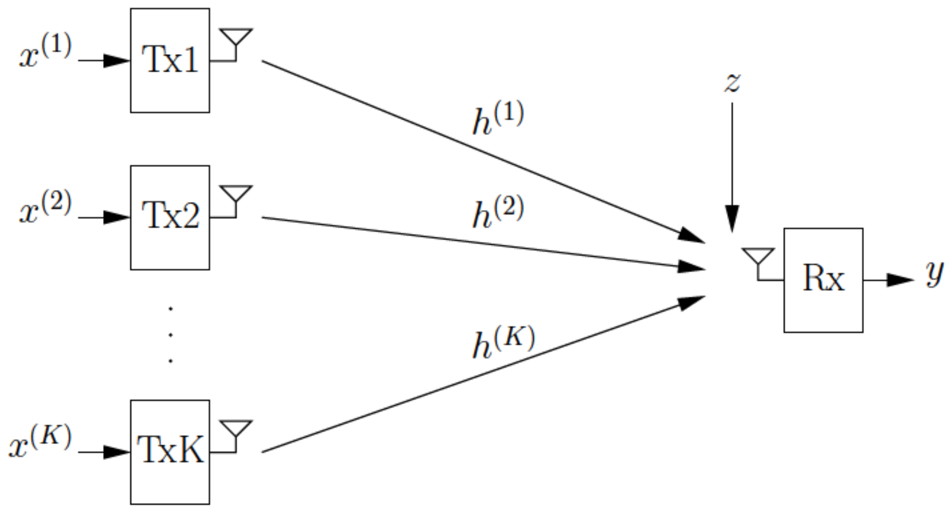

Figure 3.1: K-user SISO MAC

channel gains $h^{(k)}$ and the Gaussian receiver noise $z \sim \mathcal{CN}(0,1)$, i.e.,

$$y = \sum_{k=1}^{K} h^{(k)} x^{(k)} + z.$$

The capacity region of this channel was found in [Ahl71, Lia72]. For the Gaussian case considered here, with power constraints given by $E\left(|x^{(k)}|^2\right) \leq P^{(k)}$, it can be written as (cf. [Gol05])

$$\mathcal{C}_{\text{MAC}} = \Bigg\{ (R^{(1)}, \ldots, R^{(K)}) : \\ \sum_{k \in S} R^{(k)} \leq \log_2 \left(1 + \sum_{k \in S} |h^{(k)}|^2 P^{(k)}\right) \forall S \subseteq \{1, \ldots, K\} \Bigg\}.$$

For the simplest case of $K = 2$, the capacity region of the MAC has two dimensions and is given by the constraints

$$\begin{aligned} R^{(1)} &\leq C^{(1)} = \log_2\left(1 + |h^{(1)}|^2 P^{(1)}\right) \\ R^{(2)} &\leq C^{(2)} = \log_2\left(1 + |h^{(2)}|^2 P^{(2)}\right) \\ R^{(1)} + R^{(2)} &\leq C_\Sigma = \log_2\left(1 + |h^{(1)}|^2 P^{(1)} + |h^{(2)}|^2 P^{(2)}\right). \end{aligned} \tag{3.1}$$

This capacity region is visualized in Figure 3.2. It has the form of a pentagon with corner points 0, A, B, C, and D (cf. [CT06]). Obviously, point A can be achieved if user 2 is silent ($x^{(2)} = 0$), such that user 1 transmits as in a single-user channel and

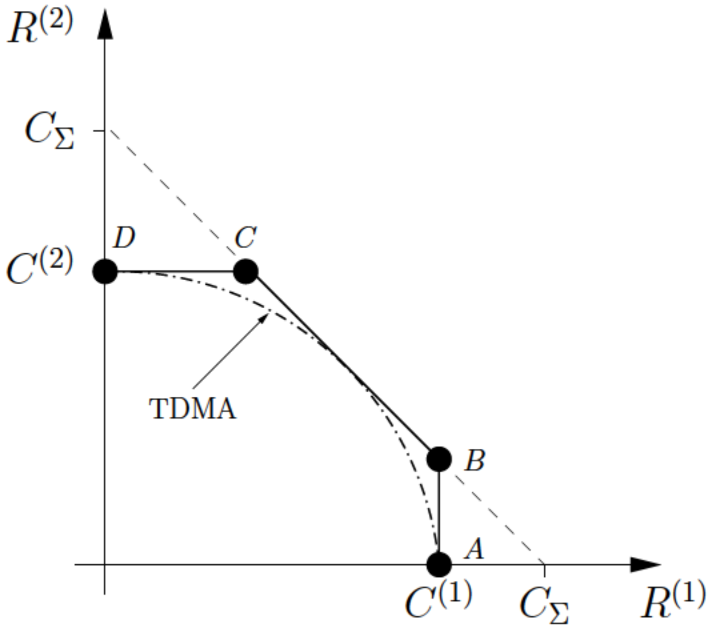

Figure 3.2: Capacity region of two-user MAC

achieves the rate $C^{(1)}$ given in the first equation of (3.1). In order to achieve point B, also user 2 transmits information. However, for decoding $x^{(2)}$, the receiver treats $x^{(1)}$ as additional noise, i.e., $R^{(2)} = \log_2\left(1 + \frac{|h^{(2)}|^2 P^{(2)}}{1+|h^{(1)}|^2 P^{(1)}}\right)$. In a second decoding step $x^{(2)}$ is known and its influence can be subtracted from the received signal, i.e., $x^{(1)}$ is again only interfered by the Gaussian noise and $R^{(1)} = C^{(1)}$. The points C and D can be achieved by swapping the roles of user 1 and 2.

In a MAC with $K > 2$, the same principle (also called successive decoding or onion peeling [CT06]) can be applied to achieve the capacity region. In each decoding step, the influence from already decoded users can be subtracted and all not yet decoded signals have to be considered as additional noise.

Another possibility to share the multiple-access channel among the transmitting users is the use of time division multiple access (TDMA)[1]. This means, the channel is divided into K time slots and each user occupies one time slot exclusively. This principle is in general not capacity achieving, but it is simple to use and allows simple receivers, because successive decoding is no longer necessary.

Let $\tau^{(k)}$ be the length of the time slot that user k occupies, where $\sum_{k=1}^{K} \tau^{(k)} = 1$.

[1] The use of frequency division multiple access (FDMA) would also be possible, but as it leads to the same rates as TDMA, it is not further mentioned here.

Then, the achievable rate of user k is

$$R_{\text{TDMA}}^{(k)} = \tau^{(k)} \log_2 \left(1 + \frac{|h^{(k)}|^2 P^{(k)}}{\tau^{(k)}}\right), \tag{3.2}$$

where the $\tau^{(k)}$ in the denominator inside the logarithm is due to an increased transmit power. As user k occupies the channel only for $\tau^{(k)}$ fraction of time and only the average power is constrained, it can increase its transmit power to $\frac{P^{(k)}}{\tau^{(k)}}$. Averaging over time, this results in a transmit power of $P^{(k)}$ again.

The overall rate region achievable by TDMA is obtained by evaluating the user rates (3.2) for all possible values of $\tau^{(1)}, \ldots, \tau^{(K)}$ with $\sum_{k=1}^{K} \tau^{(k)} = 1$. A point of this region, which is of special interest, is the point that achieves the optimal sum-rate. In contrast to the general capacity region, where the sum capacity

$$C_\Sigma \triangleq \max \sum_{k=1}^{K} R^{(k)} = \log_2 \left(1 + \sum_{k=1}^{K} |h^{(k)}|^2 P^{(k)}\right)$$

follows directly from the constraints independent of the user order, the optimal sum-rate of TDMA has to be found by optimizing the duration of the time slots, such that

$$R_{\Sigma,\text{TDMA}} \triangleq \max_{\sum_{k=1}^{K} \tau^{(k)}=1} \sum_{k=1}^{K} \tau^{(k)} \log_2 \left(1 + \frac{|h^{(k)}|^2 P^{(k)}}{\tau^{(k)}}\right). \tag{3.3}$$

The optimal duration of the time slots is given by (cf. [CT06])

$$\tau^{(k)} = \frac{|h^{(k)}|^2 P^{(k)}}{\sum_{l=1}^{K} |h^{(l)}|^2 P^{(l)}}, \tag{3.4}$$

i.e., each user's time slot duration is proportional to the product of its channel gain and its available transmit power. Combining (3.3) and (3.4) we obtain

$$R_{\Sigma,\text{TDMA}} = \log_2 \left(1 + \sum_{k=1}^{K} |h^{(k)}|^2 P^{(k)}\right),$$

which is obviously the same as C_Σ. This both verifies the optimality of (3.4) and shows the sum-rate optimality of TDMA, although the achievable rate region of TDMA is smaller, as can be seen in the following example.

For the case of $K = 2$, the achievable rate region of TDMA is also visualized as dash-dotted line in Figure 3.2 (cf. [CT06]). It can be seen that the TDMA curve touches the capacity region in 3 points, where points A and D are achievable by allocating the complete time to user 1 or 2 and the third point is the optimal sum-rate point described above. The equality of the sum-rates of TDMA and successive decoding will be one of the major topics in the following chapters. In the following section, it will be shown that TDMA does usually not achieve the sum capacity, if the receiver has multiple antennas. On the other hand, the following chapter will show that TDMA can also achieve higher rates than successive decoding if a relay is added to the channel.

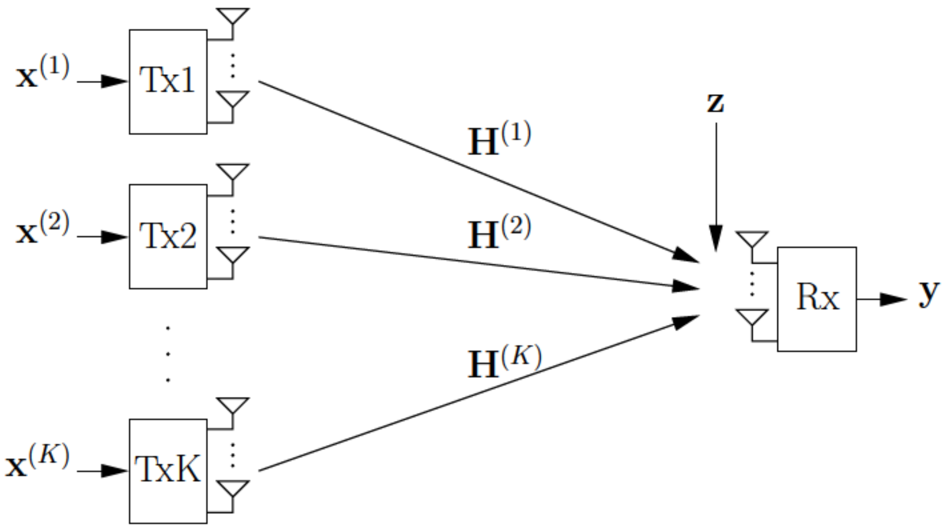

Figure 3.3: K-user MIMO MAC

3.1.2 Multi-Antenna Gaussian MAC

If the number of antennas is increased to $N_r > 1$ at the receiver and $N_t^{(1)}, \ldots, N_t^{(K)}$ ($N_t^{(k)} > 1 \quad \forall k$) at the respective transmitters, we obtain a K-user MIMO MAC as shown in Figure 3.3. Formally, the differences are that the input signals $\mathbf{x}^{(k)} \in \mathbb{C}^{N_t^{(k)}}$, the noise $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{I})$, and the output signal

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}^{(k)} \mathbf{x}^{(k)} + \mathbf{z}$$

are now vectors representing the signals of the different antennas. Moreover, the channel gains are identified by matrices $\mathbf{H}^{(k)} \in \mathbb{C}^{N_r \times N_t^{(k)}}$, where the entries represent the channel gains between all antennas of transmitter k and the receiver.

However, finding the capacity region is much more involved for this channel. The main reason for this is that each user can distribute its power arbitrarily among its antennas, which provides more degrees of freedom. This power distribution is reflected by the transmit covariance matrices $\mathbf{Q}^{(k)} \in \mathbb{C}^{N_t^{(k)} \times N_t^{(k)}}$ defined as

$$\mathbf{Q}^{(k)} \triangleq E\left(\mathbf{x}^{(k)} \mathbf{x}^{(k)H}\right),$$

where $\operatorname{tr}\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)}$ has to be ensured to fulfill the transmit power constraint.

The capacity region of the K-user Gaussian MIMO MAC is described in [CV93, YRBC04] and can be written as (cf. [Gol05])

$$\mathcal{C}_{MAC} = \bigcup_{\mathbf{Q}^{(k)} \succeq 0,\ \mathrm{tr}\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)} \forall k} \left\{ (R^{(1)}, \ldots, R^{(K)}) : \right.$$
$$\left. \sum_{k \in S} R^{(k)} \leq \log_2 \left| \mathbf{I} + \sum_{k \in S} \mathbf{H}^{(k)} \mathbf{Q}^{(k)} \mathbf{H}^{(k)^H} \right| \forall S \subseteq \{1, \ldots, K\} \right\}, \tag{3.5}$$

where $|\mathbf{A}|$ denotes the determinant of a matrix $\mathbf{A}$. The union over all covariance matrices $\mathbf{Q}^{(k)}$ with $\mathbf{Q}^{(k)} \succeq 0$ and $\mathrm{tr}\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)}$ is due to the fact that there is not a single set of covariance matrices, which is optimal for all points of the capacity region.

One possibility to compute the set of covariance matrices that contribute to the capacity region is to sequentially optimize a weighted sum-rate $\sum_{k=1}^{K} \mu^{(k)} R^{(k)}$, where $\mu^{(k)} \geq 0$ are weights which reflect the importance of the corresponding rate $R^{(k)}$ [YRBC04]. These weights are varied until all "edges" of the capacity region are found. Although the weighted sum-rate optimization is a convex problem, finding the exact capacity region entails large computational complexity. Algorithms that compute the capacity region can be found, e.g., in [CV93, VTA01].

If the goal is to find only the sum capacity instead of the whole capacity region, the problem simplifies significantly, since it can be formulated as a single convex optimization problem (cf . [YRBC04])

$$\begin{aligned} \max_{\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}} \quad & \log_2 \left| \mathbf{I} + \sum_{k=1}^{K} \mathbf{H}^{(k)} \mathbf{Q}^{(k)} \mathbf{H}^{(k)^H} \right| \\ \text{s.t.} \quad & \mathrm{tr}\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)} \qquad \forall k, \\ & \mathbf{Q}^{(k)} \succeq 0 \qquad \forall k. \end{aligned} \tag{3.6}$$

For this problem, a very simple and efficient algorithm was presented in [YRBC04]. This algorithm solves the problem of finding K covariance matrices by iteratively optimizing one transmit matrix at a time until the sum-rate converges. For this purpose, it uses the fact that changing the decoding order of the users does not influence the sum-rate. Hence, the covariance matrix which is currently optimized can always be assumed to belong to the user which is decoded first and suffers from interference from all other users. This optimization is than equivalent to finding the capacity of a single-user MIMO channel, which has a simple water-filling solution [Tel99] (see also Appendix A.2). The algorithm is listed in Algorithm 1 in terms of pseudo code.

As in the preceding subsection, the capacity region (3.5) is visualized for the case of $K = 2$ in Figure 3.4. In contrast to the single-antenna case, the capacity region is not quite a pentagon but rather a pentagon with rounded corners, which is due to the union operation mentioned above. However, the pentagon given by $R^{(1)} \leq C^{(1)}$, $R^{(2)} \leq C^{(2)}$, and $R^{(1)} + R^{(2)} \leq C_{\Sigma}$ often gives a very good approximation of the capacity region. This

Algorithm 1 Iterative water-filling for solving (3.6) [YRBC04]

Set $\mathbf{Q}^{(k)} = 0 \qquad \forall k$
repeat
    for $k = 1$ to K **do**
        $\mathbf{N} = \mathbf{I} + \sum_{l=1, l \neq k}^{K} \mathbf{H}^{(l)} \mathbf{Q}^{(l)} {\mathbf{H}^{(l)}}^H$
        $\mathbf{Q}^{(k)} = \arg\max_{\mathbf{Q}} \log_2 \left| \mathbf{H}^{(k)} \mathbf{Q} {\mathbf{H}^{(k)}}^H + \mathbf{N} \right|$
    end for
until sum-rate convergence

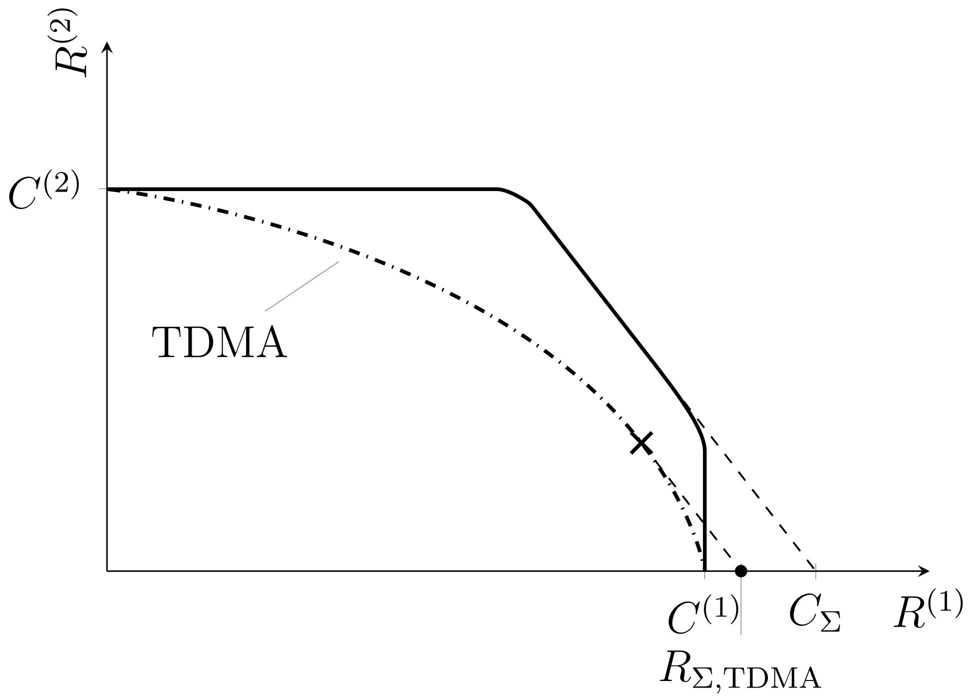

Figure 3.4: Capacity region of two-user MIMO MAC

approximation can be calculated with much less complexity, because $C^{(1)}$ and $C^{(2)}$ are the capacities of the single-user MIMO channels with only user 1 or user 2, respectively, which are simple to compute. The sum capacity C_Σ can be calculated with Algorithm 1.

Also in the MIMO MAC, it is possible to use TDMA, which makes it much simpler to find the achievable rate region. Similar to (3.2), the achievable rates can be calculated as

$$R^{(k)}_{\mathrm{TDMA}} = \tau^{(k)} \log_2 \left| \mathbf{I} + \mathbf{H}^{(k)} \mathbf{Q}^{(k)} {\mathbf{H}^{(k)}}^H \right|,$$

where the covariance matrices are limited by $\mathrm{tr}\left(\mathbf{Q}^{(k)}\right) \leq \frac{P^{(k)}}{\tau^{(k)}}$. Hence, the optimal covariance matrices can be found separately by simple water-filling as described in Appendix A.2. The achievable region is then obtained by varying the duration of the time slots $\tau^{(k)}$.

Also here, the optimal sum-rate $R_{\Sigma,\mathrm{TDMA}}$ is often of interest. However, for MIMO systems there is in general no closed-form solution for the optimal $\tau^{(k)}$ as in (3.4). Moreover, the optimal sum-rate point of TDMA does in general not lie on the capacity region of the MIMO MAC.

This can be observed in Figure 3.4, where the achievable TDMA region is also plotted (using the same channel matrices as for the capacity region) and the optimal sum-rate point is marked with an X. Obviously, there is now a gap between the TDMA region and the capacity region and $R_{\Sigma,\mathrm{TDMA}} < C_\Sigma$. The reason for this gap is that TDMA can not benefit much from the spatial diversity gained by the multiple antennas.

A good example to illustrate this is a single input multiple output (SIMO) MAC with $K = 2$, where the channel vectors of the users are orthogonal to each other. Here, the capacity region is a rectangle because the orthogonality allows the users to transmit without disturbing each other. On the other hand, TDMA can not benefit from this orthogonality, because the users transmit in separate time slots. Thus, the capacity region is only touched at the points, where one user owns the channel exclusively.

One special case, which is of particular interest for the following chapters, is the multiple input single output (MISO) MAC. Here, the channel matrices $\mathbf{H}^{(k)}$ are row vectors ${\mathbf{h}^{(k)}}^H \in \mathbb{C}^{1\times N_t^{(k)}}$. Looking at the capacity region, the MISO MAC behaves like a SISO MAC. The reason for this is that the expressions ${\mathbf{h}^{(k)}}^H \mathbf{Q}^{(k)} \mathbf{h}^{(k)}$ appearing in the rate expressions can be upper bounded by

$${\mathbf{h}^{(k)}}^H \mathbf{Q}^{(k)} \mathbf{h}^{(k)} \leq ||\mathbf{h}^{(k)}||^2 \lambda_{\max}\left(\mathbf{Q}^{(k)}\right) \leq ||\mathbf{h}^{(k)}||^2 \mathrm{tr}\left(\mathbf{Q}^{(k)}\right) \leq ||\mathbf{h}^{(k)}||^2 P^{(k)},$$

where $||\mathbf{a}||$ denotes the euclidean norm of a vector $\mathbf{a}$ and $\lambda_{\max}(\mathbf{A})$ denotes the maximum eigenvalue of a matrix $\mathbf{A}$. As it can be seen, the choice

$$\mathbf{Q}^{(k)} = \frac{\mathbf{h}^{(k)} {\mathbf{h}^{(k)}}^H}{{\mathbf{h}^{(k)}}^H \mathbf{h}^{(k)}} P^{(k)}$$

achieves the upper bound and is therefore optimal for any $\mathbf{Q}^{(k)}$ in all rate expressions from 3.5. Therefore, the above choice of $\mathbf{Q}^{(k)}$ is optimal for the whole capacity region,

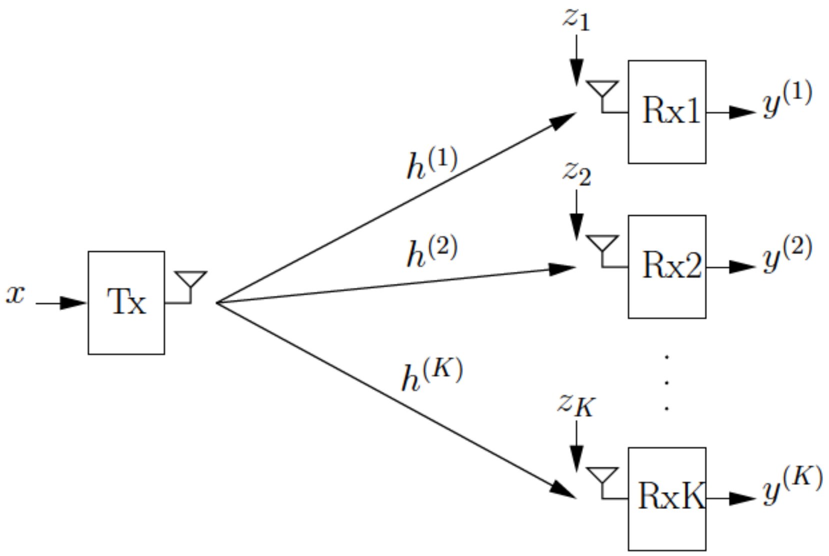

Figure 3.5: K-user SISO broadcast channel

which makes the union operation from (3.3) superfluous. Hence, the capacity region for $K = 2$ is a pentagon again and TDMA is sum-rate optimal. However, these properties hold only in the MISO MAC. If the single antenna is at the transmitter side, we obtain the SIMO MAC mentioned above, where these properties are not valid.

3.2 Gaussian Broadcast Channel

As a channel with one transmitter and multiple receivers, the broadcast channel is the natural counterpart to the MAC. Although the capacity region is different, this relation between MAC and broadcast channel can be used to construct a relation between the two capacity regions, which is commonly called duality. Before describing this duality, we will proceed the same ways as in the preceding section and explain the simpler SISO case first, followed by the MIMO case. Note that, in the capacity regions considered here, it is assumed that no common data (like in radio or TV broadcast channels) is transmitted. Instead, each user receives its own individual data. However, it is possible to extend the region to also include common data [Gol05].

3.2.1 Single-Antenna Gaussian Broadcast Channel

Figure 3.5 shows the SISO broadcast channel. The input signal $x \in \mathbb{C}$ can be written as

$$x = \sum_{k=1}^{K} x^{(k)},$$

where $x^{(k)} \in \mathbb{C}$ is the signal that contains the information for user k. The signal

$$y^{(k)} = \mathbf{h}^{(k)} x + \mathbf{z}^{(k)}$$

at receiver k consists of the sum of all signals multiplied by the respective channel gain $\mathbf{h}^{(k)}$ and the additive receiver noise $\mathbf{z}^{(k)} \sim \mathcal{CN}(0,1)$.

In the considered SISO case, the Gaussian broadcast channel is always degraded [Cov72], which means the receivers can be ordered according to their channel quality. The capacity region for degraded broadcast channels is well-known [Ber73, Gal74]. For the channel in Figure 3.5 with a transmit power constraint of $E\left(|x|^2\right) \leq P$, it can be written as (cf. [Gol05])

$$\begin{aligned} \mathcal{C}_{BC} = \bigcup_{\sum_{k=1}^{K} P^{(k)} = P} \Bigg\{ & (R^{(1)}, \ldots, R^{(K)}) : \\ & R^{(k)} = \log_2 \left(1 + \frac{|h^{(k)}|^2 P^{(k)}}{1 + \sum_{l=1}^{K} |h^{(l)}|^2 P^{(l)} \mathbf{1}\left[|h^{(k)}| \leq |h^{(l)}|\right]} \right) \Bigg\}, \end{aligned} \tag{3.7}$$

where $\mathbf{1}[.]$ denotes the indicator function, which is 1 if its argument is true and 0 otherwise. This capacity region can be achieved by superposition coding with successive interference cancellation [Cov72]. This means the signals $x^{(k)}$ are encoded as in a single-user channel with $E\left(|x^{(k)}|\right) \leq P^{(k)}$, $\sum_{k=1}^{K} P^{(k)} = P$ and then superposed at the transmitter. At receiver k, the degradedness of the channel allows to decode the signals of all users l with $|h^{(l)}| < |h^{(k)}|$ before decoding $x^{(k)}$.

Decoding the signals of these users is possible, because they have a worse channel and their code rate is chosen such that their respective receiver can decode this signal. As receiver k obtains a better version of the signal, it can also decode the signal. However, for the signals of users with higher channel gains, this does not hold. Hence, the signals of these users are treated as additional noise, which can be seen in the denominator in (3.7).

Considering the sum-rate of the SISO Gaussian broadcast channel, the situation is different to the MAC. Using (3.7), it can be shown that the sum capacity is achieved when all power is allocated to the user with the highest channel gain. Therefore, we can write the sum capacity as

$$C_\Sigma = \log_2 \left(1 + \max_k \mathbf{h}^{(k)} P \right).$$

Also in the broadcast channel, it is possible to use TDMA. If the channel is divided in time slots of lengths $\tau^{(1)}, \ldots, \tau^{(K)}$, user k can achieve a rate of

$$R^{(k)}_{\text{TDMA}} = \tau^{(k)} \log_2 \left(1 + \mathbf{h}^{(k)} P \right).$$

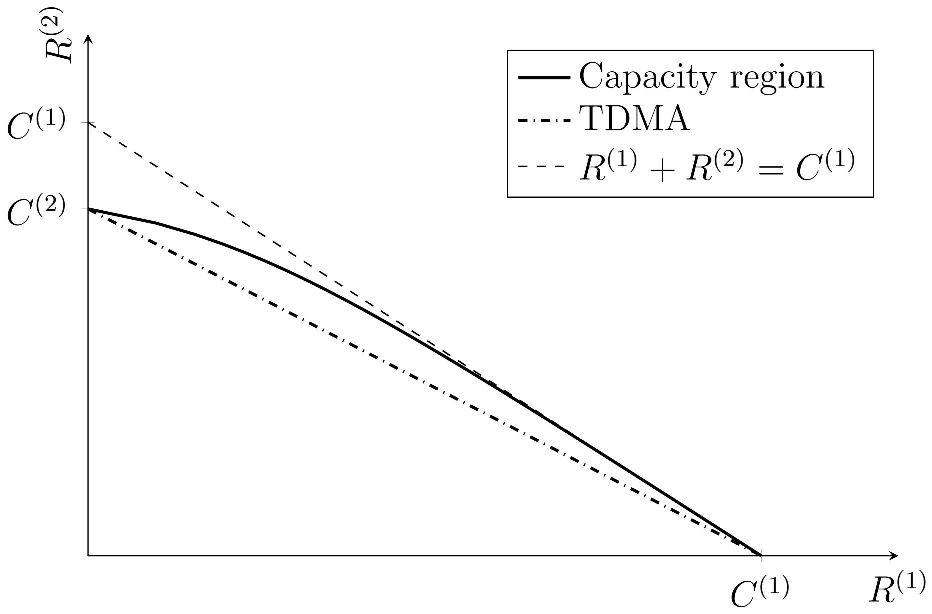

Figure 3.6: Capacity region of two-user SISO broadcast channel

In contrast to the MAC, the transmitter is active all the time and can not increase the transmit power due to partial inactivity and the duration of the time slots appears only outside the logarithm. Thus, in the broadcast channel the rate region achievable with TDMA is (only) the convex combination of the "corner points" of the capacity region, i.e., the points where all the power is allocated to one user. As the region described by (3.7) usually exceeds their convex combination, the TDMA region touches the capacity region in general only at the corner points. However, as one of those points yields the optimal sum-rate, TDMA achieves the sum capacity in the SISO broadcast channel as well.

An example of a capacity region with $K = 2$ is plotted in Figure 3.6. As previously mentioned, the corner points of the capacity region, where user 1 and 2 use the channel exclusively and achieve their single-user capacities $C^{(1)}$ and $C^{(2)}$, respectively, are achievable both by TDMA and superposition coding. While the TDMA region is only the convex combinations of those points, which is a straight line for $K = 2$, the capacity region achievable by superposition coding with successive interference cancellation is larger. It can also be seen, that the optimal sum-rate (indicated by the dashed line) is achieved at $R^{(1)} = C^{(1)}$ and $R^{(2)} = 0$, i.e., $C_\Sigma = C^{(1)}$.

3.2.2 Multi-Antenna Gaussian Broadcast Channel

As in the MAC, the computation of the capacity region becomes much more involved if the transmitter has $N_t > 1$ antennas and the receivers are equipped with $N_r^{(1)}, \ldots, N_r^{(K)}$

($N_r^{(k)} > 1 \;\; \forall k$) antennas. The output signal $\mathbf{y}^{(k)} \in \mathbb{C}^{N_r^{(k)}}$ at receiver k now consists of the channel matrix $\mathbf{H}^{(k)} \in \mathbb{C}^{N_t^{(k)} \times N_t}$, multiplied by the input vector $\mathbf{x} \in \mathbb{C}^{N_t}$, and the additional noise $\mathbf{z}^{(k)} \sim \mathcal{CN}(0, \mathbf{I})$, i.e.,

$$\mathbf{y}^{(k)} = \mathbf{H}^{(k)}\mathbf{x} + \mathbf{z}^{(k)},$$

where $\mathbf{x} = \sum_{k=1}^{K} \mathbf{x}^{(k)}$. Since the only difference to Figure 3.5 is the change from scalars to vectors and matrices, the channel model is not plotted again for the MIMO broadcast channel. As in the MAC, it is possible to distribute the power between the multiple transmit antennas. The power distribution of the signals $\mathbf{x}^{(k)}$ is again given by the covariance matrices $\mathbf{Q}^{(k)} \in \mathbb{C}^{N_t \times N_t}$ defined by

$$\mathbf{Q}^{(k)} \triangleq E\left(\mathbf{x}^{(k)}\mathbf{x}^{(k)^H}\right),$$

where $\mathbf{Q} = \sum_{k=1}^{K} \mathbf{Q}^{(k)}$ denotes the covariance of the total transmitted signal $\mathbf{x}$, which has to fulfill the power constraint $\operatorname{tr}(\mathbf{Q}) \leq P$. The main challenge of the transition from scalar to matrix channel gains is that the degradedness of the channel does no longer hold in general.

But, although the capacity region of the general nondegraded broadcast channel is unknown, a transmission technique was proposed in [CS03, YC01], which was shown to achieve the capacity region in [WSS06]. This scheme is based on dirty paper coding (DPC) [Cos83], a technique which allows the cancellation of previously known interference. As the interference in broadcast channels is given by the signals for the (other) users and all signals are known at the transmitter, it is possible to sequentially encode the user's signals.

For this purpose let $\pi : \{1, \ldots, K\} \to \{1, \ldots, K\}$ be a permutation of the users specifying the encoding order, such that user $\pi(1)$ is encoded first, followed by user $\pi(2)$, etc. until user $\pi(K)$ is encoded last. Hence, when user $\pi(k)$ is encoded, the transmitter knows the signals $\mathbf{x}^{(\pi(1))}, \ldots, \mathbf{x}^{(\pi(k-1))}$ and can use DPC to encode $\mathbf{x}^{(\pi(k))}$ such that it is not interfered by these signals. The achievable rate vector $\mathbf{r}(\pi, \mathbf{Q})$ with user order π and covariance matrix $\mathbf{Q} = \sum_{k=1}^{K} \mathbf{Q}^{(k)}$ can be written as[2] (cf. [Gol05])

$$\begin{aligned} &\mathbf{r}(\pi, \mathbf{Q}) = \left[R^{(\pi(1))}, \ldots, R^{(\pi(K))}\right] : \\ &\qquad R^{(\pi(k))} = \log_2 \left| \mathbf{I} + \frac{\mathbf{H}^{(\pi(k))}\mathbf{Q}^{(\pi(k))}\mathbf{H}^{(\pi(k))^H}}{\mathbf{I} + \mathbf{H}^{(\pi(k))}\left[\sum_{j>k} \mathbf{Q}^{(\pi(j))}\right]\mathbf{H}^{(\pi(k))^H}} \right| \quad \forall k. \end{aligned} \tag{3.8}$$

Clearly, the encoding order influences the achievable rate of the users, such that a fix ordering is not sufficient to achieve the capacity region. Instead, the capacity region

[2]The matrix division is used to shorten the formula. Due to the determinant rule that $|\mathbf{I} + \mathbf{AB}| = |\mathbf{I} + \mathbf{BA}|$ it does not matter whether the inverse of the denominator is multiplied from the left or from the right

can be described as convex hull over all possible orders π and covariance matrices $\mathbf{Q}$ [Gol05], i.e.,

$$\mathcal{C}_{\text{BC}} = \text{Co}\left(\bigcup_{\pi, \mathbf{Q} \succeq 0, \text{tr}(\mathbf{Q}) \leq P} \mathbf{r}\left(\pi, \mathbf{Q}\right) \right). \tag{3.9}$$

Unfortunately, finding the achievable rate region, even for a fixed user order π, can not be formulated as convex optimization problem [Gol05]. The reason for this is that the functions in (3.8) are not concave with respect to the covariance matrices. This increases the complexity of solving this problem dramatically. Thus, the usual way of computing the capacity region of the MIMO broadcast channel is to exploit the duality with the MIMO MAC [VJG03], which will be discussed in the following subsection.

Another interesting suboptimal transmission technique is the so called zero-forcing dirty paper coding (ZF-DPC) [CS03], which is a combination of DPC and a precoding technique that orthogonalizes the channel. For this scheme, all channels are stacked in one matrix $\mathbf{H}_c \in \mathbb{C}^{N_\Sigma \times N_t}$

$$\mathbf{H}_c = \begin{bmatrix} \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{H}^{(K)} \end{bmatrix} = \mathbf{RG},$$

where $N_\Sigma = \sum_{k=1}^{K} N_r^{(k)}$ and $\mathbf{RG}$ is the QR decomposition [HJ90] obtained by applying the Gram-Schmidt orthogonalization to the rows of $\mathbf{H}_c$.

The resulting matrix $\mathbf{R} \in \mathbb{C}^{N_\Sigma \times N_t}$ is a lower triangular matrix, while $\mathbf{G} \in \mathbb{C}^{N_t \times N_t}$ is a unitary matrix. By using $\mathbf{G}^H$ to encode the transmitted signals, i.e., $\mathbf{x} = \mathbf{G}^H \widetilde{\mathbf{x}}$, the received vector $\mathbf{y}_c \in \mathbb{C}^{N_\Sigma}$ can be written as

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{x} + \mathbf{z}_c = \mathbf{RGG}^H \widetilde{\mathbf{x}} + \mathbf{z}_c = \mathbf{R}\widetilde{\mathbf{x}} + \mathbf{z}_c.$$

Note that $\mathbf{y}_c$ is a vector that contains the received signals of all antennas of all users.

The principle of ZF-DPC is to construct the transmitted signal, such that it can be decoded by the first N_t antennas[3] (corresponding to the first N_t rows of $\mathbf{H}_c$). This has two consequences: First, each antenna is treated as a separate entity. Second, especially if N_Σ is significantly larger than N_t, a lot of antennas (and thus also users) can not receive any data. For the i-th ($i = 1, \ldots, N_t$) receiving antenna, the received signal can be written as

$$y_{ci} = R_{i,i}\widetilde{x}_i + \sum_{j<i} R_{i,j}\widetilde{x}_j + z_{ci}.$$

As it can be seen, the precoding with $\mathbf{G}^H$ eliminates the interference from the signals $\widetilde{x}_j$ with $j > i$. With the additional use of DPC and encoding the signals in the order

[3]We assume here that $N_\Sigma \geq N_t$. If this is not the case the number of transmitted signals reduces to N_Σ, but the principle remains the same

$\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_{N_t}$, also the interference from the signals $\widetilde{x}_j$ with $j > i$ can be eliminated. Thus, the the corresponding rate can be written as (cf. [CS03])

$$R_i = \log_2 \left(1 + |R_{i,i}|^2 P_i\right),$$

where $P_i \triangleq E\left(|\widetilde{x}_i|^2\right)$ and $\sum_{i=1}^{N_t} P_i \leq P$.

Finally, besides the power allocation to the different signals, the diagonal entries of $\mathbf{R}$ are the main factor for the achievable rates. As the relevant diagonal entries of $\mathbf{R}$ depend only on the first N_t rows of $\mathbf{H}_c$, these entries can be optimized by permuting the rows of $\mathbf{H}_c$. This means, the antennas that are provided with data (and also their order) are switched. Although, there are $\binom{N_\Sigma}{N_t} N_t! = \frac{N_\Sigma}{(N_\Sigma - N_t)!}$ possibilities to select N_t antennas and permute them, the computation of the ZF-DPC region is usually much easier than computing the capacity region, whereas the rates, especially the achievable sum-rates are often very close to the capacity.

Last but not least, also for the MIMO broadcast channel, the TDMA protocol can be used. Contrary to the MAC, the differences of TDMA in SISO and MIMO broadcast channels are not too big. Thus, we will only briefly mention TDMA in this subsection.

Similar to the SISO case, the achievable rate of user k with TDMA is given by its single-user capacity multiplied by the time slot length $\tau^{(k)}$, i.e.,

$$R^{(k)}_{\mathrm{TDMA}} = \tau^{(k)} \log_2 \left| \mathbf{I} + \mathbf{H}^{(k)} \mathbf{Q}^{(k)} \mathbf{H}^{(k)H} \right|.$$

By varying the duration of the time slots $\tau^{(k)}$, it is possible to achieve the convex combination of the corner points of the capacity region as explained in the previous subsection. However, TDMA is not generally sum-rate optimal in the MIMO broadcast channel, because the optimal sum-rate point is usually not a corner point of the capacity region as in the SISO case. Moreover, as TDMA can not exploit the diversity as good as the capacity achieving DPC, the performance gap to the capacity region is rather big. The advantage of TDMA that remains, is that the achievable rate region is easy to compute, because it is sufficient to calculate the convex combinations of the single-user capacities.

Also for the broadcast channel, the MISO case will be of special interest in the following chapters. Again, the channel matrices $\mathbf{H}^{(k)}$ can be written as row vectors $\mathbf{h}^{(k)H} \in \mathbb{C}^{1 \times N_t}$. However, different from the MAC, the MISO broadcast channel has more in common with the MIMO broadcast channel than with the SISO broadcast channel: It is in general nondegraded and the performance of TDMA is rather poor.

This is illustrated in Figure 3.7, where an example of a MISO broadcast capacity region for $K = 2$ is visualized. Furthermore, the figure shows that the performance of ZF-DPC is very good. As there are only two single-antenna users, we have $N_\Sigma = 2$ and hence only two possible orders of the users. For order A, we assume $\mathbf{H}_c^H = [\mathbf{h}^{(1)}, \mathbf{h}^{(2)}]$, while for order B we take $\mathbf{H}_c^H = [\mathbf{h}^{(2)}, \mathbf{h}^{(1)}]$. At the corner points of the capacity

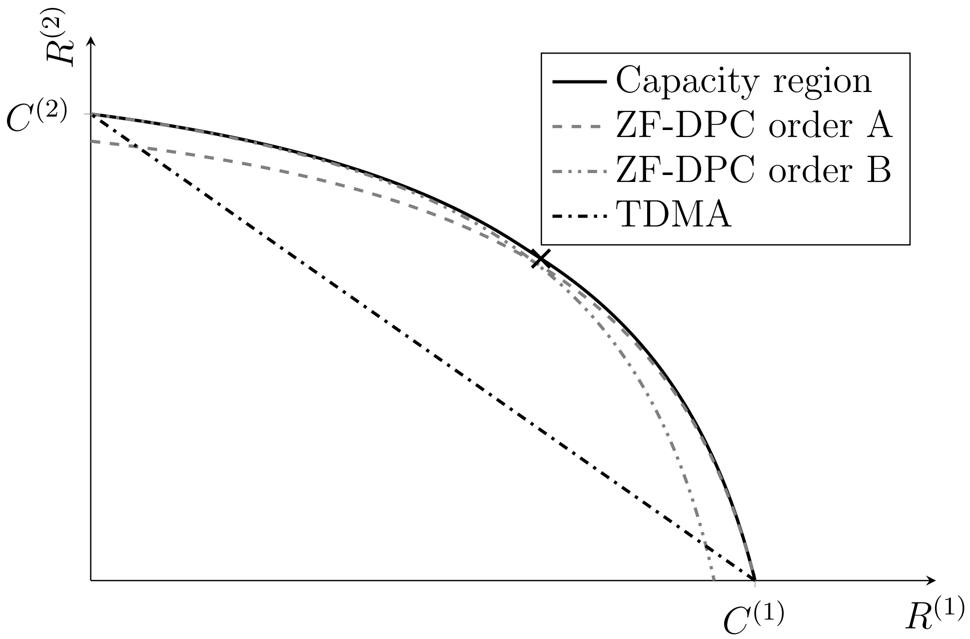

Figure 3.7: Capacity region of two-user MISO broadcast channel

region, ZF-DPC with the appropriate order touches the capacity region. Looking at the optimal sum-rate point (indicated by an X), the ZF-DPC region does not touch the capacity region although it is very close. Overall, the achievable region with ZF-DPC is almost as large as the capacity region, while the effort to compute the transmit strategies is much lower.

For the sake of completeness, the SIMO broadcast channel shall also be mentioned at this point. Unlike the MISO broadcast channel, the properties of the SIMO channel are more or less the same as in the SISO case. In detail, the SIMO broadcast channel is always degraded and hence achieves the optimum sum-rate if the best user occupies the channel exclusively. The same holds for TDMA, such that, as for SISO channels the TDMA region touches the capacity region in that point. As, for the BRC, we focus on optimizing the achievable sum-rate and this optimization degenerates to a single-user problem for SIMO and SISO broadcast channels, the SIMO and SISO BRC are not further discussed in this work.

3.2.3 Duality with Gaussian Multiple-Access Channel

As mentioned in the preceding section, the capacity region of the MIMO broadcast channel in (3.9) is very hard to compute as the corresponding optimization problem is nonconvex. However, a duality between MIMO BC and MIMO MAC has been shown in [VJG03], which allows a much simpler computation of the capacity region. The duality is established by defining a dual MIMO MAC, where the channel matrices are given by the Hermitian of the channel matrices of the broadcast channel. Thus, the output signal $\bar{\mathbf{y}}$ of the dual MIMO MAC is obtained by

$$\bar{\mathbf{y}} = \sum_{k=1}^{K} \mathbf{H}^{(k)^H} \bar{\mathbf{x}}^{(k)} + \bar{\mathbf{z}}^{(k)},$$

where $\bar{\mathbf{z}}^{(k)} \sim \mathcal{CN}(0, \mathbf{I})$ and $\bar{\mathbf{Q}}^{(k)} \triangleq E\left(\bar{\mathbf{x}}^{(k)} \bar{\mathbf{x}}^{(k)^H}\right)$ define the covariance matrix of the input signals.

The result of [VJG03] is that the capacity region of the MIMO broadcast channel from (3.9) is equal to the capacity region of the dual MIMO MAC if the power constraints of the latter are replaced by the single constraint

$$\sum_{k=1}^{K} \operatorname{tr}\left(\bar{\mathbf{Q}}^{(k)}\right) \leq P.$$

Hence, the individual power constraints of the conventional MIMO MAC described in Subsection 3.1.2 are changed into one sum power constraint such that (as in the dual MIMO BC) the power can be arbitrarily divided among the users. This new constraint changes neither the convexity of the corresponding optimization problem nor the optimal transmit strategy, which is still superpositioning with successive decoding at the receiver. Thus, the capacity region of the MIMO BC and its dual MAC are given by (cf. [VJG03])

$$\mathcal{C}_{\mathrm{BC}} = \bar{\mathcal{C}}_{\mathrm{MAC}} = \bigcup_{\bar{\mathbf{Q}}^{(k)} \succeq 0,\ \sum_{k=1}^{K} \operatorname{tr}\left(\bar{\mathbf{Q}}^{(k)}\right) \leq P} \Bigg\{ (R^{(1)}, \ldots, R^{(K)}) : \\ \sum_{k \in S} R^{(k)} \leq \log_2 \left| \mathbf{I} + \sum_{k \in S} \mathbf{H}^{(k)^H} \bar{\mathbf{Q}}^{(k)} \mathbf{H}^{(k)} \right| \forall S \subseteq \{1, \ldots, K\} \Bigg\}.$$

Besides the proof of duality, [VJG03] also provides formulas to calculate the covariance matrices $\mathbf{Q}^{(k)}$ of the broadcast channel that achieve the same point of the region as the corresponding dual matrices $\bar{\mathbf{Q}}^{(k)}$ in the dual MAC. As in the MIMO MAC with individual power constraints, the capacity region can be computed, e.g., by weighted sum-rate optimization. However, also with one sum power constraint and although the problem is convex, the computational complexity is high.

If, instead, only the sum capacity is of interest, the complexity can be reduced. As for the conventional MAC (cf. (3.6)) the convex problem of finding the optimal sum capacity can be written as

$$\begin{aligned}
\max_{\bar{\mathbf{Q}}^{(1)},\ldots,\bar{\mathbf{Q}}^{(K)}} \quad & \log_2 \left| \mathbf{I} + \sum_{k=1}^{K} \mathbf{H}^{(k)^H} \bar{\mathbf{Q}}^{(k)} \mathbf{H}^{(k)} \right| \\
\text{s.t.} \quad & \sum_{k=1}^{K} \operatorname{tr}\left(\bar{\mathbf{Q}}^{(k)}\right) \leq P \\
& \bar{\mathbf{Q}}^{(k)} \succeq 0 \qquad \forall k.
\end{aligned} \tag{3.10}$$

Motivated by the results of [YRBC04] (Algorithm 1 from Subsection 3.1.2), another algorithm was proposed in [JRV+05] which maximizes (3.10). This algorithm also proceeds in an iterative way, where the main difference is that, due to the joint power constraint, water-filling has to be done jointly for all covariance matrices. The pseudo code of this algorithm is given in Algorithm 2.

Algorithm 2 Iterative block water-filling for solving (3.6) [JRV+05]

1: Set $\bar{\mathbf{Q}}^{(k)} = 0 \qquad \forall k$
2: **repeat**
3:     **for** $k = 1$ to K **do**
4:         $\mathbf{G}^{(k)} = \mathbf{H}^{(k)} \left(\mathbf{I} + \sum_{l=1, l \neq k}^{K} \mathbf{H}^{(l)} \bar{\mathbf{Q}}^{(l)} \mathbf{H}^{(l)^H} \right)^{-1/2}$
5:     **end for**
6:     $\left\{ \bar{\mathbf{Q}}^{(k)} \right\}_{k=1}^{K} = \arg\max_{\mathbf{Q}^{(l)}} \sum_{l=1}^{K} \log_2 \left| \mathbf{I} + \mathbf{G}^{(l)^H} \mathbf{Q}^{(l)} \mathbf{G}^{(l)} \right|$
7: **until** sum-rate convergence

It can be seen that the algorithm is closely related to Algorithm 1. The differences are the computation of $\mathbf{G}^{(k)}$ and the water-filling in line 6. Due to the joint power constraint, the water-filling has to be done jointly for all covariance matrices. This water-filling is equivalent to a single-user water-filling with a block diagonal channel $\mathbf{H} = \operatorname{diag}\left(\mathbf{G}^{(1)}, \ldots, \mathbf{G}^{(K)}\right)$ [JRV+05].

It is shown in [JRV+05] that Algorithm 2 always converges for $K = 2$. However, for $K > 2$, some modifications are necessary in order to ensure convergence. These modifications shall not be further mentioned here, because their description is lengthy and they do not change the principle of the algorithm. They are precisely explained in [JRV+05], together with the proof of convergence for any K.

3.3 Gaussian Channels with AF-Relay

As already mentioned in the introduction, the use of relays can increase throughput and coverage of wireless networks, where the restriction to AF-relays ensures that complexity and costs of the relays remain reasonable. If relays are used in multi-user channels like the MAC or broadcast channel, especially with multiple antennas, the computation of the capacity region becomes an enormous challenge.

The main reason for the high complexity of optimizing MIMO relay channels is that both the relay amplifying matrix and the transmit covariance matrices have to be optimized. To make things worse, these two optimization problems can not be considered separately but are highly dependent on each other. Therefore, this thesis and most of the existing publications on this subject focus on maximizing the achievable sum-rate only. As it could be seen in subsection 3.1.2, the sum-rate (together with the single-user capacities) already gives a good approximation for the capacity region of the MIMO MAC.

Another parameter with significant influence on the complexity of the optimization problems is the consideration of the direct links between transmitter and receiver. If they are considered, the complexity of the optimization problems grows even further. Thus, there are few publications that consider direct links and, to our knowledge, the problem of considering direct links in combination with MAC or broadcast channels has not been addressed yet in a comparable setup.

In this section, we will describe the existing results that can be viewed as a basis for the results of this thesis: First, we describe the results for MAC and broadcast channels without direct links and then proceed with the results for single-user channels with direct links.

3.3.1 Gaussian MAC with AF-Relay and No Direct Links

If the direct links of the MARC are neglected, i.e., set to zero, the model introduced in Section 2.1 simplifies significantly. As the receiver obtains the signal from the relay only, the last N_r rows of the equivalent channel matrices $\mathbf{H}_{\text{eff}}^{(k)}$ can be ignored, such that

$$\mathbf{H}_{\text{eff}}^{(k)} = \mathbf{S}^{-1/2}\mathbf{H}\mathbf{F}\mathbf{H}_r^{(k)}$$

can be used instead, whereas the power constraints described in (2.1) remain the same.

Assuming that the relay matrix $\mathbf{F}$ is fixed, the channel can be considered as a k-user MIMO MAC with channel matrices $\mathbf{H}_{\text{eff}}^{(1)}, \ldots, \mathbf{H}_{\text{eff}}^{(K)}$. As discussed in Subsection 3.1.2, the achievable sum-rate can be written as

$$R_\Sigma = \log_2 \left|\mathbf{I} + \mathbf{S}^{-1/2}\mathbf{H}\mathbf{F}\mathbf{R}\mathbf{F}^H\mathbf{H}^H\mathbf{S}^{-H/2}\right|,$$

where $\mathbf{R} \triangleq \sum_{k=1}^K \mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)^H}$. Together with the transmit power constraints the problem of optimizing the achievable sum-rate in the K-user MIMO MARC without direct links can be summarized and reformulated as (cf. [YH10])

$$\begin{aligned}
\max_{\mathbf{F},\mathbf{Q}^{(1)},\ldots,\mathbf{Q}^{(K)}} \quad & R_\Sigma = \log_2 \frac{\left|\mathbf{H}\mathbf{F}\mathbf{R}\mathbf{F}^H\mathbf{H}^H + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H + \mathbf{I}\right|}{\left|\mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H + \mathbf{I}\right|} \\
\text{s.t.} \quad & \mathbf{Q}^{(k)} \succeq 0 \quad \forall k \\
& \operatorname{tr}\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)} \ \ \forall k \\
& \operatorname{tr}\left(\mathbf{F}\left(\mathbf{I}+\mathbf{R}\right)\mathbf{F}^H\right) \leq P_r.
\end{aligned} \tag{3.11}$$

Compared to the classical MAC without relay, the additional challenge in the MARC is not only to find the optimal relaying matrix $\mathbf{F}$, but also that there are now three instead of two constraints for $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$. The consequence of this is that, even if the optimal $\mathbf{F}$ would be known, the optimal covariance matrices can in general not be found by water-filling, because the additional constraint inhibits a diagonalization of all constraints. Thus, although the problem of finding the optimal covariance matrices is still convex for fixed $\mathbf{F}$, the optimization has to be done with matrices which entails a high computational complexity.

To make things worse, the optimal relay matrix $\mathbf{F}$ and the set of covariance matrices $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ depend on each other. Hence, if the relay matrix is changed, also the channels $\mathbf{H}_{\text{eff}}^{(k)}$ change and the optimal covariance matrices have to be calculated again. This is especially crucial, because finding the optimal relaying matrix is very difficult. To put it in a nutshell, a general optimal solution for this problem with acceptable complexity or even in closed-form seems infeasible.

Therefore, two suboptimal algorithms for optimizing the sum-rate are presented in [YH10]. One of those algorithms is based on a joint gradient search over $\mathbf{F}$ and $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$. The other one optimizes these matrices separately and in a cyclic fashion. For the scenarios simulated in [YH10], both algorithms achieve the same performance. But, as the joint gradient search is computationally intensive and a good choice of the search gradient parameters is difficult and crucial, we will focus on the latter algorithm here.

This algorithm, referred to as Algorithm 6 in [YH10], can be divided into two steps, where the first step optimizes the covariance matrices and the second step optimizes the relay matrix. These two steps are iteratively executed until the matrices remain constant.

For the covariance matrix optimization, there is another (inner) iteration over the users. For each user k, all matrices except $\mathbf{Q}^{(k)}$ are fixed and the definitions $c = \log_2 \left|\mathbf{I} + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H\right|$ and

$$\mathbf{G}^{(k)} = \mathbf{H}\mathbf{F}\left(\mathbf{R} - \mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)^H}\right)\mathbf{F}^H\mathbf{H}^H + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H + \mathbf{I}$$

are used to rewrite the sum-rate as

$$R_\Sigma = \log_2\left|\mathbf{I} + \mathbf{G}^{(k)^{-\frac{1}{2}}}\mathbf{H}\mathbf{F}\mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)^H}\mathbf{F}^H\mathbf{H}^H\mathbf{G}^{(k)^{-\frac{H}{2}}}\right| + \log_2\left|\mathbf{G}^{(k)}\right| - c.$$

The relevant power constraints for maximizing R_Σ with respect to $\mathbf{Q}^{(k)}$ are formulated as

$$\begin{aligned} \operatorname{tr}\left(\mathbf{Q}^{(k)}\right) &\leq P^{(k)} \\ \operatorname{tr}\left(\mathbf{F}\mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)^H}\mathbf{F}^H\right) &\leq P_r - \operatorname{tr}\left(\mathbf{F}\left(\mathbf{I} + \mathbf{R} - \mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)^H}\right)\mathbf{F}^H\right). \end{aligned}$$

The resulting optimization problem for $\mathbf{Q}^{(k)}$ is convex and solved for all $k = 1, \ldots, K$.

For the optimization of the relay matrix, all covariance matrices are fixed and the singular value decomposition (SVD) of $\mathbf{H}$ is given by $\mathbf{H} = \mathbf{U}_h \boldsymbol{\Sigma}_h \mathbf{V}_h^H$, where $\boldsymbol{\Sigma}_h = \operatorname{diag}(\sigma_1, \ldots, \sigma_n)$ with descending diagonal order and $n = \min\{N_r, N_f\}$. Furthermore, $\mathbf{R} = \mathbf{U}_r \boldsymbol{\Sigma}_r \mathbf{U}_r^H$ represents the eigenvalue decomposition (EVD) of $\mathbf{R}$, where $\boldsymbol{\Sigma}_r = \operatorname{diag}\left(\lambda_1, \ldots, \lambda_{N_f}\right)$ with descending diagonal order. As shown in [FHK06], the optimal structure of the relaying matrix is given by

$$\mathbf{F} = \mathbf{V}_h \boldsymbol{\Sigma}_f \mathbf{U}_r^H, \tag{3.12}$$

where $\boldsymbol{\Sigma}_f = \operatorname{diag}\left(f_1, \ldots, f_{N_f}\right)^{1/2}$ is a diagonal matrix whose entries have to be found by optimization. Using these decompositions, the optimization with respect to $\mathbf{F}$ can be reformulated as (cf. [YH10])

$$\begin{aligned} \max_{f_1,\ldots,f_{N_f}} \quad & R_\Sigma = \sum_{i=1}^{n} \log_2\left(1 + \frac{\sigma_i^2 \lambda_i f_i}{1 + \sigma_i^2 f_i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{N_f} (1 + \lambda_i) f_i \leq P_r \\ & f_i \geq 0 \ \ \forall i. \end{aligned} \tag{3.13}$$

As derived in [YH10] by exploiting the Karush-Kuhn-Tucker (KKT) conditions, the solution of this problem can be written as

$$f_i = \frac{1}{2\sigma_i^2(1+\lambda_i)} \cdot \max\left\{0, \sqrt{\lambda_i^2 + 4\lambda_i \sigma_i^2 \mu} - \lambda_i - 2\right\},$$

where the parameter μ is chosen such that the power constraint is fulfilled, i.e.,

$$\sum_{i=1}^{N_f} \frac{1}{2\sigma_i^2} \cdot \max\left\{0, \sqrt{\lambda_i^2 + 4\lambda_i \sigma_i^2 \mu} - \lambda_i - 2\right\} = P_r.$$

Although an optimal solution is obtained in both steps, it can not be ensured that the algorithm reaches the global optimum for two reasons. First, there is no proof of convergence. Second, even if the algorithm would converge it can not be ensured that it will find the global optimum, because the overall problem is not convex and both steps of the algorithm find the optimal solution only for the parameters calculated in the other step, for which global optimality can not be ensured.

3.3.2 Gaussian BC with AF-Relay and No Direct Links

For broadcast channels with an additional relay, as introduced in Section 2.2, the problems concerning the computation of the capacity region or the sum capacity are the same as in the broadcast channel without relay. The main issue is again the nonconvexity of the problems, which makes the computation very complex. As in the MARC, the optimization of the relay can not be done separately, which increases the complexity even further.

This subsection will give an overview of existing approaches for the BRC. First, we briefly describe a result on the establishment of a duality between BRC and MARC [JGH07]. Unfortunately, this duality is only shown for single-antenna transmitters and receivers, such that it is not usable for most of the cases considered in this work. Subsequently, we describe approaches for the MISO BRC based on ZF-DPC [CTHC08]. Like the duality approach, we do not explain these results in detail. The reason for this is that [YH10], which describes results both for DPC and ZF-DPC in MIMO channels, has a more general channel model. Hence, we use these results for a detailed explanation.

Looking at the results for the broadcast channel without relay, the most obvious approach for simplifying the search for optimal covariance matrices in the BRC is the use of some kind of duality with the MARC. However, the duality of MAC and BC from [VJG03] can not be directly transferred to channels with relay. Even if the relay matrix $\mathbf{F}$ would be fixed and only the covariance matrices are considered, the creation of a duality relationship as in [VJG03] collapses with the relay power constraint, because the matrices between transmitter and relay are different in the dual channels.

A novel approach to establish a duality between BRC and MARC has been made in [JGH07]. It is shown, that a MARC with AF-relay and sum power constraint P and relay power P_r is dual to the BRC with opposite transmit direction, transmit power P_r, and relay power P. Thus, the relay power of the dual MARC is the transmit power of the BRC and the total transmit power of the dual MARC is the relay power of the BRC. Although the dual MARC is different from the one discussed in the preceding section (because of the sum transmit power constraint instead of individual per user constraints), this duality is an enormous simplification for finding the capacity region of a class of BRCs.

As the model in [JGH07] also encompasses multiple hops and relays, it is in some parts more general than the models considered here. However, there is one restriction, which makes the model unusable for most of the cases considered in this work: It is assumed that all nodes except the relay have a single antenna. Furthermore, the model is restricted to real signals and the necessary phase optimization for the complex case is not discussed. The authors of [JGH07] only conjecture that their results also hold for channels with MIMO nodes, but state that the duality with MIMO nodes would be especially challenging as the structure of the transmitted signal covariance matrix on the dual MAC and BC will be different.

Another approach, which is also applicable with MIMO nodes, is the use of ZF-DPC. As in the broadcast channel without relay, this simplifies the optimization of the transmit strategy significantly. However, also here the joint search for optimal relaying and transmit covariance matrices poses an enormous challenge.

This challenge is faced in [CTHC08], where several algorithms are described to optimize the sum-rate in MISO BRCs:

- As first simple idea, the authors of [CTHC08] use a so-called all-pass relay with relaying matrix $\mathbf{F} = \gamma \mathbf{I}$, where γ is chosen such that the relay power constraint is met. Although this approach is far away from being optimal, it has the same complexity as finding the optimal sum-rate of a broadcast channel using ZF-DPC.

- A more involved approach optimizes also the relay by using a relaying matrix with a structure similar to the one in (3.12). By using a high signal-to-noise ratio (SNR) approximation of the sum-rate the problem is then turned into a convex problem, which allows for an easier solution, although the complexity is still considerable. Furthermore, the optimum selection of the active users in ZF-DPC (cf. Subsection 3.2.2) is done by exhaustive search, which makes the complexity even higher.

- In order to reduce the complexity of the preceding approach, a third algorithm is presented in [CTHC08]. This algorithm uses a reduced-complexity algorithm for selecting the active users, which is based on a lower bound. This lower bound includes an equal power distribution on the active users in the system, which yields a further reduction of the complexity. The remaining optimization parameter, the power allocation at the relay can then be found by water-filling.

A generalization to MIMO BRCs with both DPC and ZF-DPC is given in [YH10]. Using DPC with encoding order π as described in Subsection 3.2.2 for the model described in Section 2.2, the rate of user $\pi(k)$ can be written as (cf. [YH10])

$$R^{\pi(k)} = \log_2 \frac{\left| \mathbf{H}^{(k)} \mathbf{F} \mathbf{H}_r \sum_{i \geq k} \mathbf{Q}^{\pi(i)} \mathbf{H}_r^H \mathbf{F}^H {\mathbf{H}^{(k)}}^H + \mathbf{H}^{(k)} \mathbf{F} \mathbf{F}^H {\mathbf{H}^{(k)}}^H + \mathbf{I} \right|}{\left| \mathbf{H}^{(k)} \mathbf{F} \mathbf{H}_r \sum_{i > k} \mathbf{Q}^{\pi(i-1)} \mathbf{H}_r^H \mathbf{F}^H {\mathbf{H}^{(k)}}^H + \mathbf{H}^{(k)} \mathbf{F} \mathbf{F}^H {\mathbf{H}^{(k)}}^H + \mathbf{I} \right|}.$$

Together with the power constraints for the transmitter and the relay, the problem of finding the optimal sum-rate can be written as

$$\begin{aligned} \max_{\mathbf{F}, \mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}} \quad & R_\Sigma = \sum_{k=1}^{K} R^{\pi(k)} \\ \text{s.t.} \quad & \operatorname{tr}\left(\sum_{k=1}^{K} \mathbf{Q}^{(k)} \right) \leq P \\ & \operatorname{tr}\left(\mathbf{F} \left(\mathbf{I} + \mathbf{H}_r \sum_{k=1}^{K} \mathbf{Q}^{(k)} \mathbf{H}_r^H) \right) \mathbf{F}^H \right) \leq P_r \\ & \mathbf{Q}^{(k)} \succeq 0 \ \ \forall k. \end{aligned}$$

Unfortunately, this problem is nonconvex and hard to solve. In [YH10] a joint gradient algorithm is used to solve the problem. However, as in the MARC, the joint gradient search is computationally intensive and a good choice of the search gradient parameters is difficult and crucial. Therefore, this solution is not further considered in this thesis. Instead, we propose a solution of the problem based on duality in the following chapter and give a more detailed explanation of the ZF-DPC based approach of [YH10] here.

As explained in Subsection 3.2.2, ZF-DPC can simplify the problem of finding good transmit strategies for the MIMO broadcast channel. The same holds in the BRC, but as in the MARC, the additional optimization of the relay matrix makes the problem more involved and requires some modifications. Instead of considering the full channel from transmitter to receiver, the stacked channel matrix $\mathbf{H}_c \in \mathbb{C}^{N_\Sigma \times N_f}$ is built with the channel matrices from the relay to the receivers in [YH10], i.e.,

$$\mathbf{H}_c = \begin{bmatrix} \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{H}^{(K)} \end{bmatrix} = \mathbf{RG}, \tag{3.14}$$

where $N_\Sigma = \sum_{k=1}^{K} N_r^{(k)}$ and $\mathbf{RG}$ is the QR-decomposition of $\mathbf{H}_c$, where $\mathbf{R} \in \mathbb{C}^{N_\Sigma \times N_f}$ is lower triangular and $\mathbf{G} \in \mathbb{C}^{N_f \times N_f}$ is unitary.

Defining the SVD of $\mathbf{H}_r$ as $\mathbf{H}_r = \mathbf{U}_h \mathbf{\Sigma}_h \mathbf{V}_h^H$, where $\mathbf{\Sigma}_h = \operatorname{diag}(\sigma_1, \ldots, \sigma_n)$ with descending diagonal elements and $n = \min\{N_f, N_t\}$, the authors of [YH10] (cf. [CTHC08]) construct the relay matrix as

$$\begin{aligned}\mathbf{F} &= \mathbf{G}^H \mathbf{\Sigma}_f \mathbf{U}_h^H \\ \mathbf{\Sigma}_f &= \operatorname{diag}\left(f_1, \ldots, f_{N_f}\right)^{1/2}.\end{aligned}$$

This construction is a heuristic choice without any proof of optimality, but the motivation of this choice becomes clear when looking at the received vector $\mathbf{y}_c \in \mathbb{C}^{N_\Sigma}$. As the input signals are precoded as $\mathbf{x} = \mathbf{V}_h \widetilde{\mathbf{x}}$, the received vector can be written as

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{F} \mathbf{H}_r \mathbf{x} + \mathbf{H}_c \mathbf{F} \mathbf{z}_r + \mathbf{z}_c = \mathbf{R} \mathbf{\Sigma}_f \mathbf{\Sigma}_h \widetilde{\mathbf{x}} + \mathbf{R} \mathbf{\Sigma}_f \widetilde{\mathbf{z}}_r + \mathbf{z}_c,$$

where $\mathbf{z}_r \sim \mathcal{CN}(0, \mathbf{I})$ is the noise from the relay and $\widetilde{\mathbf{z}}_r = \mathbf{U}_h^H \mathbf{z}_r$. It can be seen that the above choice of the matrix $\mathbf{F}$ ensures that the overall matrix between the transmitted vector $\widetilde{\mathbf{x}}$ and the received vector $\mathbf{y}_c$ (without noise) remains lower triangular.

As in the broadcast channel without relay, $\mathbf{y}_c$ contains the signals of all receive antennas, where at most the first N_t antennas are in use, such that the signal of the i-th receiving antenna can be written as

$$y_{ci} = R_{i,i} f_i^{1/2} \sigma_i \widetilde{x}_i + \sum_{j<i} R_{i,j} f_j^{1/2} \sigma_j \widetilde{x}_j + \sum_{j \leq i} R_{i,j} f_j^{1/2} \widetilde{z}_j + z_{ci}.$$

The second term, which contains the interference from the signals $\widetilde{x}_j$ with $j > i$ can again be eliminated by the use of DPC as in Subsection 3.2.2. On the contrary, the additional noise terms from the relay $\widetilde{z}_j$ ($j \geq i$) can not be mitigated by DPC, also not for $j > i$. Hence, signals that are encoded early in DPC suffer from more relay noise.

Finally, using $P_i \triangleq E\left(|\widetilde{x}_i|^2\right)$ to denote the power given to the i-th data stream, the problem of finding the optimal sum-rate is given by (cf. [YH10])

$$\begin{aligned}
\max_{\substack{f_1,\ldots,f_{N_f},\\ P_1,\ldots,P_{N_t}}} \quad & R_\Sigma = \sum_{i=1}^{N_t} \log_2\left(1 + \frac{|R_{i,i}|^2|\sigma_i|^2 f_i P_i}{1+\sum_{j=1}^{i}|R_{i,j}|^2 f_j}\right) \\
\text{s.t.} \quad & \sum_{i=1}^{N_t} P_i \leq P \\
& \sum_{i=1}^{N_t} f_i\left(1+|\sigma_i|^2 P_i\right) \leq P_r \\
& P_i \geq 0 \ \ \forall i.
\end{aligned} \tag{3.15}$$

Note that this optimization problem assumes a fix order of the channel matrices from the relay to the receivers in $\mathbf{H}_c$, where the order given in (3.14) is only one example. As mentioned in Subsection 3.2.2, further gains can be achieved by permuting the rows of $\mathbf{H}_c$.

In [YH10], two approaches are used to solve the optimization problem (3.15). The first approach is adopted from [CTHC08], where the sum-rate is lower bounded by

$$R_\Sigma \geq \sum_{i=1}^{N_t} \log_2\left(\frac{|R_{i,i}|^2|\sigma_i|^2 f_i P_i}{1+\sum_{j=1}^{i}|R_{i,j}|^2 f_j}\right) = -\log_2 \prod_{i=1}^{N_t}\left(\frac{1+\sum_{j=1}^{i}|R_{i,j}|^2 f_j}{|R_{i,i}|^2|\sigma_i|^2 f_i P_i}\right),$$

which is a good approximation at high SNR. Optimizing this lower bound is a geometric program, which can be transformed into a convex problem and solved with well-known standard algorithms [BV04].

The second approach separates the optimization of $f_1,\ldots,f_{N_f}$ and $P_1,\ldots,P_t$ and optimizes both parameter sets alternately, similar as in the MARC. While the problem of finding $P_1,\ldots,P_t$ is convex and can again be solved by standard algorithms for convex problems, the relay matrix parameters $f_1,\ldots,f_{N_f}$ are found by gradient search.

3.3.3 Gaussian Single-User Channel with Direct Links

A more general model for relay channels is obtained if also the direct links between transmitter(s) and receiver(s) are taken into consideration. Especially if a direct link is strong, which might be the case if the receiver is closer to the transmitter than the relay, a model that neglects the direct link does not reflect the situation and the achievable performance correctly. However, integrating direct links in the relay channel is mathematically challenging. Therefore, most of the work on relay channels with direct links make use of some other simplifications.

In the following, we will review the most interesting result for the models considered in this work, which is given in [TH07]. This paper considers a single-user MIMO channel with MIMO AF-relay, where the transmitter does not know the channel. Thus, the transmitter can not distribute its power on the antennas according to a precoding matrix that fits the channel conditions. Nevertheless, this assumptions lie in the scope of our channel model introduced in Section 2.1 if we set[4] $K = 1$ and $\mathbf{Q} = \frac{P}{N_t}\mathbf{I}$. Hence, the remaining variable for optimization is only the relay matrix $\mathbf{F}$.

Using the well-known formula, the achievable rate of this single-user channel can be written as

$$R = \log_2 \left| \mathbf{I} + \mathbf{H}_{\text{eff}} \frac{P}{N_t} \mathbf{I}\, \mathbf{H}_{\text{eff}}^H \right| = \log_2 \left| \mathbf{I} + \frac{P}{N_t} \mathbf{H}_{\text{eff}}^H \mathbf{H}_{\text{eff}} \right|,$$

where $\mathbf{H}_{\text{eff}}$ is the effective channel defined in Section 2.1. A form of the rate that is better suited for optimizing over $\mathbf{F}$ is obtained by expanding $\mathbf{H}_{\text{eff}}$, such that

$$\begin{aligned} R &= \log_2 \left| \mathbf{I} + \frac{P}{N_t} \left[\mathbf{H}_r^H \mathbf{F}^H \mathbf{H}^H \mathbf{S}^{-H/2} \;\; \mathbf{H}_d^H \right] \begin{bmatrix} \mathbf{S}^{-1/2} \mathbf{H} \mathbf{F} \mathbf{H}_r \\ \mathbf{H}_d \end{bmatrix} \right| \\ &= \log_2 \left| \mathbf{I} + \frac{P}{N_t} \left(\mathbf{H}_r^H \mathbf{F}^H \mathbf{H}^H \mathbf{S}^{-1} \mathbf{H} \mathbf{F} \mathbf{H}_r + \mathbf{H}_d^H \mathbf{H}_d \right) \right| \\ &= \log_2 \left| \mathbf{I} + \frac{P}{N_t} \left(\mathbf{H}_r^H \mathbf{H}_r - \mathbf{H}_r^H \left(\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{H} \mathbf{F} \right)^{-1} \mathbf{H}_r + \mathbf{H}_d^H \mathbf{H}_d \right) \right|. \end{aligned}$$

For further simplification, [TH07] introduces $\widetilde{\mathbf{H}} \triangleq \mathbf{H}_r \left(\mathbf{I} + \frac{P}{N_t} \mathbf{H}_d^H \mathbf{H}_d \right)^{-1/2}$, which can be interpreted as the "projection" of $\mathbf{H}_r$ onto $\mathbf{H}_d$, and write

$$\begin{aligned} R &= \log_2 \left| \left(\mathbf{I} + \frac{P}{N_t} \mathbf{H}_d^H \mathbf{H}_d \right) \left(\mathbf{I} + \frac{P}{N_t} \left(\widetilde{\mathbf{H}}^H \widetilde{\mathbf{H}} - \widetilde{\mathbf{H}}^H \left(\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{H} \mathbf{F} \right)^{-1} \widetilde{\mathbf{H}} \right) \right) \right| \\ &= \underbrace{\log_2 \left| \mathbf{I} + \frac{P}{N_t} \mathbf{H}_d^H \mathbf{H}_d \right|}_{\triangleq c} + \log_2 \left| \mathbf{I} + \frac{P}{N_t} \left(\widetilde{\mathbf{H}} \widetilde{\mathbf{H}}^H - \widetilde{\mathbf{H}} \widetilde{\mathbf{H}}^H \left(\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{H} \mathbf{F} \right)^{-1} \right) \right|. \end{aligned}$$

In this form of R, the first term is a constant denoted as c. For optimizing the second term, we introduce the SVDs $\widetilde{\mathbf{H}} = \widetilde{\mathbf{U}} \widetilde{\mathbf{\Sigma}} \widetilde{\mathbf{V}}^H$, $\mathbf{H} = \mathbf{U}_h \mathbf{\Sigma}_h \mathbf{V}_h^H$ and fix the structure of $\mathbf{F}$ as

$$\mathbf{F} = \mathbf{V}_h \mathbf{\Sigma}_f \mathbf{V}_f^H,$$

where $\mathbf{V}_f$ is an arbitrary unitary matrix and $\mathbf{\Sigma}_f$ is arbitrary and not necessarily diagonal. Hence, this choice of $\mathbf{F}$ does not restrict the possible solutions for $\mathbf{F}$. Using this notations, the rate can be further rewritten as

$$\begin{aligned} R_\Sigma &= c + \log_2 \left| \mathbf{I} + \frac{P}{N_t} \left(\widetilde{\mathbf{\Sigma}} \widetilde{\mathbf{\Sigma}}^H - \widetilde{\mathbf{\Sigma}} \widetilde{\mathbf{\Sigma}}^H \widetilde{\mathbf{U}}^H \left(\mathbf{I} + \mathbf{V}_f \mathbf{\Sigma}_f^H \mathbf{\Sigma}_h^H \mathbf{\Sigma}_h \mathbf{\Sigma}_f \mathbf{V}_f^H \right)^{-1} \widetilde{\mathbf{U}} \right) \right| \\ &= c + \log_2 \left| \mathbf{I} + \frac{P}{N_t} \left(\widetilde{\mathbf{\Sigma}} \widetilde{\mathbf{\Sigma}}^H - \widetilde{\mathbf{\Sigma}} \widetilde{\mathbf{\Sigma}}^H \widetilde{\mathbf{U}}^H \mathbf{V}_f \left(\mathbf{I} + \mathbf{\Sigma}_f^H \mathbf{\Sigma}_h^H \mathbf{\Sigma}_h \mathbf{\Sigma}_f \right)^{-1} \mathbf{V}_f^H \widetilde{\mathbf{U}} \right) \right|. \end{aligned}$$

[4]As there is only one user, we omit the superscript

Clearly, it can be seen that choosing $\mathbf{V}_f = \widetilde{\mathbf{U}}$ and using a diagonal $\boldsymbol{\Sigma}_f$ would diagonalize the determinant and allow a simplified optimization with respect to the diagonal elements. However, looking at the relay power constraint, we obtain

$$\operatorname{tr}\left(\mathbf{F}\left(\mathbf{I}+\frac{P}{N_t}\mathbf{H}_r\mathbf{H}_r^H\right)\mathbf{F}^H\right) = \operatorname{tr}\left(\boldsymbol{\Sigma}_f\left(\mathbf{I}+\frac{P}{N_t}\mathbf{V}_f^H\mathbf{H}_r\mathbf{H}_r^H\mathbf{V}_f\right)\boldsymbol{\Sigma}_f^H\right) \leq P_r,$$

i.e., choosing $V_f = \widetilde{\mathbf{U}}$ with a diagonal $\boldsymbol{\Sigma}_f$ would not diagonalize the power constraint. For this purpose $\mathbf{V}_f$ should be the left singular vectors of $\mathbf{H}_r$. The reason for this problem is that the direct links influence the sum-rate and hence also the choice of $\mathbf{F}$ that maximizes the sum-rate, while for the power constraint the direct links do not play a role. Consequently, the optimal structure of $\mathbf{F}$ is not known.

In order to approach the optimal achievable rates, two lower bounds and one upper bound are given in [TH07]. The upper bound is obtained by relaxing the relay power constraint as

$$\begin{aligned}
\operatorname{tr}\left(\mathbf{F}\left(\mathbf{I}+\frac{P}{N_t}\mathbf{H}_r\mathbf{H}_r^H\right)\mathbf{F}^H\right) &= \operatorname{tr}\left(\mathbf{F}\left(\mathbf{I}+\frac{P}{N_t}\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^H\right)\mathbf{F}^H\right) \\
&\quad + \frac{P^2}{N_t^2}\operatorname{tr}\left(\mathbf{F}\widetilde{\mathbf{H}}\mathbf{H}_d^H\mathbf{H}_d\widetilde{\mathbf{H}}^H\mathbf{F}^H\right) \\
&\geq \operatorname{tr}\left(\mathbf{F}\left(\mathbf{I}+\frac{P}{N_t}\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^H\right)\mathbf{F}^H\right),
\end{aligned}$$

i.e., the relationship between $\mathbf{H}_r$ and $\widetilde{\mathbf{H}}$ is used to split up the power constraint in two terms, where the second one is neglected. Obviously, this constraint is now also diagonalized by choosing $V_f = \widetilde{\mathbf{U}}$ and $\boldsymbol{\Sigma}_f$ diagonal. This leads to the optimal solution for the upper bound, which is explained in detail in [YH10].

The first lower bound is directly computed from this solution, the optimal $\mathbf{F}$ for the upper bound is multiplied by a scalar $\gamma \in \mathbb{R}$, such that the original (and not the relaxed) power constraint is fulfilled with equality. For the second lower bound, the direct link is simply neglected, i.e., the relay matrix that would be optimal if $\mathbf{H}_d = 0$ is used which leads to $\mathbf{V}_f$ being the left singular values of $\mathbf{H}_r$ and a diagonal $\boldsymbol{\Sigma}_f$ (details about the channel without direct links can be found in [TH07]). This solution is then simply used in the channel with nonzero direct links, although this is clearly not optimal. The simulation results in [TH07] show that usually the first lower bound performs better.

4

Relaying for MISO Multi-User Channels without Direct Links

In this section, the results for multi-user channels with AF-relay and without direct links between the transmitter(s) and receiver(s) are presented, where we restrict ourselves to MISO channels. First, we introduce a novel transmit strategy for relay channels based on TDMA, which is especially useful in the MARC. In Section 4.2, we will compare this technique which divides the channel by using time slots to the prevailing strategy of transmitting all signals jointly. We will show that the TDMA-based strategy achieves strictly higher sum-rates in the MISO MARC. Subsequently, we will consider the MISO BRC in Section 4.3. As the use of ZF-DPC for this channel is well studied [CTHC08, YH10] but the issues concerning the complexity remain, we will introduce an efficient algorithm which makes use of the duality between MAC and BC (without relay). This algorithm has a reduced complexity and achieves approximately the same rates as the best possible ZF-DPC algorithms.

4.1 TDMA-based Relaying and Single-User Relay Channels

As described in the preceding section, finding the optimal transmit strategy for channels with relay can be very challenging in multi-user channels. As in channels without relay, the use of TDMA can simplify this problem. The main reason for this is that only one user is active at a time, i.e., there is no interference and the optimization of the transmit covariance matrices becomes simpler. On the other hand, the duration of the time slots has to be optimized, which increases the complexity again. However, this optimization is usually much easier to solve such that the overall complexity is smaller.

But complexity is not the only advantage of TDMA. With the TDMA strategy proposed here, the relay also incorporates the time slot structure, i.e., a different relay matrix can be chosen in each time slot. This allows to choose the relay matrix such that it exactly fits to the channel of the active user only, which is an advantage. If

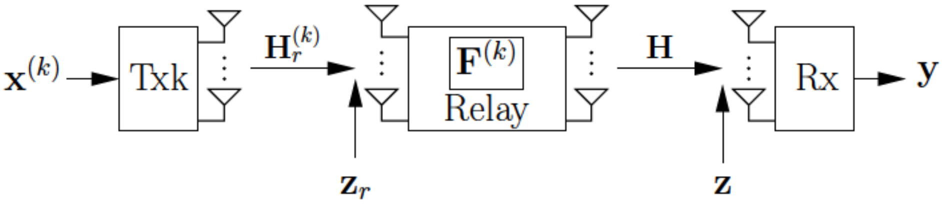

Figure 4.1: Channel of user k in MARC if TDMA is used

TDMA is not used and all signals are transmitted jointly, which will be referred to as "joint relaying" in the remainder of this thesis, there is only one matrix $\mathbf{F}$, which amplifies the signals of all users. Especially if the number of users is large, choosing such a matrix is always a compromise because it will not be perfect for all users.

To put it in a nutshell, TDMA divides a K-user MARC or BRC in K orthogonal single-user channels, which are described by the channel model in Figure 4.1. In the following, we will therefore describe transmit strategies for single-user relay channels. In order to combine them with TDMA in a K-user system, these strategies can be used in the K time slots. Choosing the duration of the time slots is a further optimization problem, which is different for the MARC and BRC. Therefore, we explain this problem later. Note that in Figure 4.1 and in the following description, we use the same notation and channel matrices as in the MARC. However, everything explained in this section also holds for the use of TDMA in the BRC.

As the MIMO single-user relay channel can be interpreted as a special case of a MIMO MARC, the results on the optimal structure of the relay and covariance matrices found in [FHK06] and explained in Subsection 3.3.1 hold here as well. Hence, using the SVDs $\mathbf{H}_r^{(k)} = \mathbf{U}_r^{(k)}\mathbf{\Sigma}_r^{(k)}\mathbf{V}_r^{(k)H}$ with $\mathbf{\Sigma}_r^{(k)} = \operatorname{diag}\left(\lambda_1^{(k)}, \dots, \lambda_{\min\left(N_t^{(k)}, N_f\right)}^{(k)}\right)^{1/2}$ and $\mathbf{H} = \mathbf{U}_h\mathbf{\Sigma}_h\mathbf{V}_h^H$ with $\mathbf{\Sigma}_h = \operatorname{diag}\left(\sigma_1, \dots, \sigma_{\min\left(N_f, N_r\right)}\right)$, it is known from [FHK06, YH10] that

$$
\begin{aligned}
\mathbf{Q}^{(k)} &= \mathbf{V}_r^{(k)}\mathbf{\Sigma}_q^{(k)}\mathbf{V}_r^{(k)H} \\
\mathbf{F}^{(k)} &= \mathbf{U}_h^H\mathbf{\Sigma}_f^{(k)}\mathbf{U}_r^{(k)H},
\end{aligned}
$$

where $\mathbf{\Sigma}_q = \operatorname{diag}\left(q_1^{(k)}, \dots, q_{N_t^{(k)}}^{(k)}\right)$ and $\mathbf{\Sigma}_f = \operatorname{diag}\left(f_1^{(k)}, \dots, f_{N_f}^{(k)}\right)^{1/2}$ define the optimal structure of $\mathbf{F}^{(k)}$ and $\mathbf{Q}^{(k)}$.

With this structure and recalling that the noise covariance at the receiver is given by $\mathbf{S} = \mathbf{I} + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H$, the achievable rate can be written as

$$
\begin{aligned}
R &= \log_2\left|\mathbf{I} + \mathbf{S}^{-1/2}\mathbf{H}\mathbf{F}\mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)H}\mathbf{F}^H\mathbf{H}^H\mathbf{S}^{-H/2}\right| \\
&= \log_2\left|\mathbf{I} + \mathbf{\Sigma}_h\mathbf{\Sigma}_f^{(k)}\mathbf{\Sigma}_r^{(k)}\mathbf{\Sigma}_q^{(k)}\mathbf{\Sigma}_r^{(k)H}\mathbf{\Sigma}_f^{(k)H}\mathbf{\Sigma}_h^H\left(\mathbf{I} + \mathbf{\Sigma}_h\mathbf{\Sigma}_f^{(k)}\mathbf{\Sigma}_f^{(k)H}\mathbf{\Sigma}_h^H\right)^{-1}\right|
\end{aligned}
$$

By simplifying the sum-rate and considering the power constraints at the transmitter and the relay, we can formulate the optimization problem of the single-user relay channel as (cf. [FHK06, KMH12b])

$$\begin{aligned}
\max_{\substack{f_1^{(k)},\dots,f_{N_f}^{(k)}\\ q_1^{(k)},\dots,q_{N_t^{(k)}}^{(k)}}} \quad & R = \sum_{i=1}^{n} \log_2\left(1 + \frac{\sigma_i^2 \lambda_i^{(k)} q_i^{(k)} f_i^{(k)}}{1+\sigma_i^2 f_i^{(k)}}\right) \\
\text{s.t.} \quad & \sum_{i=1}^{N_r} (\lambda_i^{(k)} q_i^{(k)} + 1) f_i^{(k)} \leq P_r, \quad f_i^{(k)} \geq 0 \;\; \forall i \\
& \sum_{i=1}^{N_t^{(k)}} q_i^{(k)} \leq P^{(k)}, \;\; q_i^{(k)} \geq 0 \;\; \forall i,
\end{aligned} \tag{4.1}$$

where $n = \min\left(N_t^{(k)}, N_f, N_r\right)$. Unfortunately, although there is only a single user, the problem above is nonconvex and therefore hard to solve. However, in the following we will see that by considering MISO channels this problem has a simple solution, which allows several interesting insights about the performance of TDMA in MISO channels.

4.2 Gaussian MISO MAC with AF-Relay

If we consider a MISO MARC instead of a MIMO MARC, the basic problem remains the same: both the transmit covariance matrices and the relay matrix have to be optimized. However, there is one advantage which finally gives us the opportunity to solve the problem. The former channel matrix $\mathbf{H}$ between relay and receiver now becomes a row vector denoted by $\mathbf{h}^H$.

We will show that for this case a relaying matrix of rank 1 is optimal both with joint relaying and with TDMA, for which we can give a closed-form solution. Using this solution, we will also identify the optimal covariance matrices and obtain a closed-form solution for the sum-rate, again both for joint relaying and TDMA. This allows us to finally compare the achievable sum-rates of the two schemes in Subsection 4.2.3, where we will see that TDMA achieves always at least the same performance as joint relaying. Moreover, in practical scenarios, TDMA is strictly better.

4.2.1 Optimal Sum-Rate with Joint Relaying

If the signals are all transmitted at the same time, the optimization of the sum-rate is basically the same as in (3.11). However, considering the single-antenna receiver and recalling that $\mathbf{R} = \sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{Q}^{(k)} \mathbf{H}_r^{(k)^H}$ the problem reduces to

$$\begin{aligned}
\max_{\mathbf{F},\mathbf{Q}^{(1)},\ldots,\mathbf{Q}^{(K)}} \quad & R_\Sigma = \log_2 \left(1 + \frac{\mathbf{h}^H \mathbf{F} \mathbf{R} \mathbf{F}^H \mathbf{h}}{\mathbf{h}^H \mathbf{F} \mathbf{F}^H \mathbf{h} + 1} \right) \\
\text{s.t.} \quad & \mathbf{Q}^{(k)} \succeq 0 \quad \forall k \\
& \operatorname{tr}\left(\mathbf{Q}^{(k)} \right) \leq P^{(k)} \ \ \forall k \\
& \operatorname{tr}\left(\mathbf{F} \left(\mathbf{I} + \mathbf{R} \right) \mathbf{F}^H \right) \leq P_r,
\end{aligned} \tag{4.2}$$

i.e., the determinant vanishes because the respective left and right matrices in the expression are now row and column vectors, respectively.

Now, let $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ be fixed and consider the optimization with respect to $\mathbf{F}$. Recall from [FHK06] and Subsection 3.3.1 that the optimal structure for the relaying matrix is given by (3.12). Thus, we can give the same scalar reformulation of the optimization with respect to $\mathbf{F}$ as in (3.13). The difference is only that here $n = 1$ (as $N_r = 1$), such that there is only one logarithmic term. Thus, we obtain

$$\begin{aligned}
\max_{f_1,\ldots,f_{N_f}} \quad & R_\Sigma = \log_2 \left(1 + \frac{\mathbf{h}^H \mathbf{h}\, \lambda_1 f_1}{1 + \mathbf{h}^H \mathbf{h}\, f_1} \right) \\
\text{s.t.} \quad & \sum_{i=1}^{N_f} (1 + \lambda_i) f_i \leq P_r \\
& f_i \geq 0 \ \ \forall i.
\end{aligned}$$

Clearly, the squared norm of $\mathbf{h}$ replaces the largest squared singular value σ_1 of $\mathbf{H}$, because $\mathbf{h}^H$ has only one singular value, which is its norm.

Obviously, the solution of this optimization problem is to assign all possible relay power to f_1, because it is the only value from $f_1, \ldots, f_{N_f}$ that contributes to the rate. This results in $f_1 = P_r \left(1 + \lambda_1\right)^{-1}, f_2 = \ldots = f_{N_f} = 0$ being the optimal solution of the scalarized problem, whereas the optimal relaying matrix can be written as

$$\mathbf{F} = \frac{\mathbf{h}\, \mathbf{v}_{\max}^H \left(\mathbf{R} \right)}{||\mathbf{h}||} \sqrt{\frac{P_r}{1 + \lambda_1}}, \tag{4.3}$$

where $\mathbf{v}_{\max} \left(\mathbf{R} \right)$ denotes the eigenvector corresponding to the largest eigenvalue $\lambda_{\max} \left(\mathbf{R} \right) = \lambda_1$ of the matrix $\mathbf{R}$. Using this result in the sum-rate expression of (4.2), we get

$$
\begin{aligned}
R_\Sigma &= \log_2\left(1+\frac{P_r\mathbf{h}^H\mathbf{h}\mathbf{v}_{\max}^H(\mathbf{R})\,\mathbf{R}\mathbf{v}_{\max}(\mathbf{R})\,\mathbf{h}^H\mathbf{h}}{\mathbf{h}^H\mathbf{h}\mathbf{v}_{\max}^H(\mathbf{R})\,\mathbf{v}_{\max}(\mathbf{R})\,P_r\mathbf{h}^H\mathbf{h}+||\mathbf{h}||^2(1+\lambda_{\max}(\mathbf{R}))}\right)\\
&= \log_2\left(1+\frac{||\mathbf{h}||^2P_r\lambda_{\max}(\mathbf{R})}{1+||\mathbf{h}||^2P_r+\lambda_{\max}(\mathbf{R})}\right)\\
&= \log_2\left(\frac{(1+||\mathbf{h}||^2P_r)\,(1+\lambda_{\max}(\mathbf{R}))}{(1+||\mathbf{h}||^2P_r+\lambda_{\max}(\mathbf{R}))}\right)\\
&= \log_2\left(1+||\mathbf{h}||^2P_r\right)-\log_2\left(1+\frac{||\mathbf{h}||^2P_r}{1+\lambda_{\max}(\mathbf{R})}\right).
\end{aligned}
$$

Hence, the optimization with respect to $\mathbf{F}$ is complete and maximizing the achievable sum-rate is equivalent to maximizing $\lambda_{\max}(\mathbf{R})$, which can be done by optimizing the covariance matrices $\mathbf{Q}^{(1)},\ldots,\mathbf{Q}^{(K)}$. Before we state a theorem about the optimal solution of this problem, we need to introduce two lemmas.

Lemma 1

Let $\mathbf{A},\mathbf{B}\in\mathbb{C}^{n\times n}$ be positive semidefinite matrices. Then, if $\mathbf{A}-\mathbf{B}$ is positive semidefinite ($\mathbf{A}\succeq\mathbf{B}$), we have $\lambda_{max}(\mathbf{A})\geq\lambda_{max}(\mathbf{B})$.

Proof: *Let $\mathbf{v}_{max}(\mathbf{B})$ be the eigenvector corresponding to the eigenvalue $\lambda_{max}(\mathbf{B})$ with $||\mathbf{v}_{max}(\mathbf{B})||=1$. Then, we have*

$$
\begin{aligned}
\lambda_{max}(\mathbf{B}) &= \mathbf{v}_{max}(\mathbf{B})^H\,\mathbf{B}\mathbf{v}_{max}(\mathbf{B})\\
&\leq \mathbf{v}_{max}(\mathbf{B})^H\,\mathbf{B}\mathbf{v}_{max}(\mathbf{B})+\mathbf{v}_{max}(\mathbf{B})^H\cdot(\mathbf{A}-\mathbf{B})\cdot\mathbf{v}_{max}(\mathbf{B})\\
&= \frac{\mathbf{v}_{max}(\mathbf{B})^H\,\mathbf{A}\mathbf{v}_{max}(\mathbf{B})}{\mathbf{v}_{max}(\mathbf{B})^H\,\mathbf{v}_{max}(\mathbf{B})}\\
&\leq \lambda_{max}(\mathbf{A}),
\end{aligned}
$$

where the first inequality is due to the positive semidefiniteness of $\mathbf{A}-\mathbf{B}$ and the last inequality can be derived from the Rayleigh quotient. ■

Lemma 2

Let $\mathbf{\Pi}\in\mathbb{C}^{n\times n}$ be a positive semidefinite matrix with $tr(\mathbf{\Pi})\leq P$ and $\mathbf{A}\in\mathbb{C}^{m\times n}$. Then,

$$\mathbf{A}\mathbf{A}^HP\succeq\mathbf{A}\mathbf{\Pi}\mathbf{A}^H.$$

Proof: *The lemma can be equivalently formulated as*

$$\mathbf{x}^H\mathbf{A}(P\mathbf{I}-\mathbf{\Pi})\mathbf{A}^H\mathbf{x}\geq 0\quad\forall\mathbf{x}\in\mathbb{R}^m.$$

Hence, it is sufficient to show that $P\mathbf{I}-\mathbf{\Pi}$ is a positive semidefinite matrix. Let $\lambda_1,\ldots,\lambda_n$ be the eigenvalues of $\mathbf{\Pi}$ with $\lambda_1\geq\lambda_2\geq\ldots\geq\lambda_n$. Then, we have

$$P\geq tr(\mathbf{\Pi})=\sum_{i=1}^n\lambda_i\geq\lambda_1,$$

where the second inequality holds because we have $\mathbf{\Pi} \succeq 0$ and thus $\lambda_i \geq 0 \ \forall i$. Using $\lambda_{min}(P\mathbf{I} - \mathbf{\Pi})$ to denote the minimum eigenvalue of $P\mathbf{I} - \mathbf{\Pi}$, we can write

$$\lambda_{min}(P\mathbf{I} - \mathbf{\Pi}) = P - \lambda_{max}(\mathbf{\Pi}) = P - \lambda_1 \geq 0,$$

i.e., $P\mathbf{I} - \mathbf{\Pi}$ is positive semidefinite. ■

With these two lemmas, we can now formulate the following theorem.

Theorem 3

The optimal value of $\lambda_{max}(\mathbf{R})$ given by the nonconvex optimization problem

$$\begin{aligned} \max_{\mathbf{Q}^{(k)}} \quad & \lambda_{max}(\mathbf{R}) \\ s.t. \quad & tr\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)} \ \forall k \end{aligned}$$

is $\lambda_{max}\left(\sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{H}_r^{(k)^H} P^{(k)}\right)$.

Proof: *Define $\widetilde{\mathbf{R}} = \sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{H}_r^{(k)^H} P^{(k)}$. Using Lemmas 1 and 2, it is directly seen that $\lambda_{max}(\mathbf{R}) \leq \lambda_{max}\left(\widetilde{\mathbf{R}}\right)$. For showing the achievability of this solution, let*

$$\mathbf{Q}^{(k)} = \frac{\mathbf{H}_r^{(k)^H} \mathbf{v}\mathbf{v}^H \mathbf{H}_r^{(k)}}{\mathbf{v}^H \mathbf{H}_r^{(k)} \mathbf{H}_r^{(k)^H} \mathbf{v}} P^{(k)} \tag{4.4}$$

with $\mathbf{v} = \mathbf{v}_{max}(\widetilde{\mathbf{R}})$. Apparently, $tr\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)}$ is satisfied and we can write

$$\lambda_{max}(\mathbf{R}) = \mathbf{v}_{max}(\mathbf{R})^H \left(\sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{Q}^{(k)} \mathbf{H}_r^{(k)^H}\right) \mathbf{v}_{max}(\mathbf{R}) \tag{4.5}$$

$$\geq \mathbf{v}^H \left(\sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{Q}^{(k)} \mathbf{H}_r^{(k)^H}\right) \mathbf{v} \tag{4.6}$$

$$= \sum_{k=1}^{K} \mathbf{v}^H \mathbf{H}_r^{(k)} \mathbf{H}_r^{(k)^H} P^{(k)} \mathbf{v} = \lambda_{max}\left(\widetilde{\mathbf{R}}\right), \tag{4.7}$$

where (4.6) follows from the Rayleigh quotient and (4.7) is obtained by using (4.4) and writing $\mathbf{v}$ and $\mathbf{v}^H$ inside the sum. Thus, choosing $\mathbf{Q}^{(k)}$ as in (4.4) leads to the optimal value $\lambda_{max}\left(\widetilde{\mathbf{R}}\right)$. ■

Using this optimal choice of the covariance matrices together with the optimal relaying matrix the achievable sum-rate for joint relaying can be finally expressed as

$$R_\Sigma = \log_2\left(1 + ||\mathbf{h}||^2 P_r\right) - \log_2\left(1 + \frac{||\mathbf{h}||^2 P_r}{1 + \lambda_{\max}\left(\widetilde{\mathbf{R}}\right)}\right). \tag{4.8}$$

4.2.2 Optimal Sum-Rate with TDMA

If the proposed TDMA scheme is used in the MISO MARC, the channel is, as described in Section 4.1, divided in K single-user MISO channels. For these single-user channels, we face the same optimization problem as for MIMO single-user channels from (4.1). However, we have the simplification of a single receive antenna, which makes $n = 1$ and reduces the sum of logarithms to a single logarithm, i.e.,

$$\max_{\substack{f_1^{(k)},\ldots,f_{N_f}^{(k)}\\ q_1^{(k)},\ldots,q_{N_t^{(k)}}^{(k)}}} \quad R = \log_2\left(1 + \frac{\mathbf{h}^H\mathbf{h}\lambda_1^{(k)} q_1^{(k)} f_1^{(k)}}{1 + \mathbf{h}^H\mathbf{h} f_1^{(k)}}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{N_r} (\lambda_i^{(k)} q_i^{(k)} + 1) f_i^{(k)} \le P_r, \quad f_i^{(k)} \ge 0 \ \forall i$$

$$\sum_{i=1}^{N_t^{(k)}} q_i^{(k)} \le P^{(k)}, \ \ q_i^{(k)} \ge 0 \ \ \forall i.$$

As with joint relaying, the squared norm of $\mathbf{h}$ replaces the squared largest singular value σ_1 of the former matrix $\mathbf{H}$, because they are the same. And again, the solution of this problem is simple, because also here only $f_1^{(k)}$ contributes to the rate, which makes it optimal to assign all possible relay power to $f_1^{(k)}$. But, contrary to the case of joint relaying, exactly the same holds for $q_1^{(k)}$, i.e., $q_2^{(k)}, \ldots, q_{N_t^{(k)}}^{(k)}$ do not contribute to the rate and all transmit power should be assigned to $q_1^{(k)}$.

Hence, the optimal solution for the optimization problem given above is $q_1^{(k)} = P^{(k)}$, $q_2^{(k)} = \ldots = q_{N_t^{(k)}}^{(k)} = 0$, $f_1^{(k)} = P_r\left(1 + \lambda_1^{(k)} P^{(k)}\right)^{-1}$, $f_2^{(k)} = \ldots = f_{N_f}^{(k)} = 0$. Using this solution, the optimal rate in MISO single-user relay channels can be written as

$$\begin{aligned} R &= \log_2\left(1 + \frac{\mathbf{h}^H\mathbf{h}\lambda_1^{(k)} P^{(k)} P_r}{1 + \mathbf{h}^H\mathbf{h} P_r + \lambda_1^{(k)} P^{(k)}}\right) \\ &= \log_2\left(1 + ||\mathbf{h}||^2 P_r\right) - \log_2\left(1 + \frac{||\mathbf{h}||^2 P_r}{1 + \lambda_1^{(k)} P^{(k)}}\right). \end{aligned}$$

When this optimal transmit strategy shall be combined with the use of TDMA with time slots of duration $\tau^{(1)}, \ldots, \tau^{(K)}$ in a MARC, two things have to be considered when calculating the rate of a user. First, as user k is only active for $\tau^{(k)}$ fraction of time, its rate has to be multiplied by $\tau^{(k)}$. Second, for the same reason user k can increase its transmit power to $\frac{P^{(k)}}{\tau^{(k)}}$ and still have an average transmit power of $P^{(k)}$. Taking this into account the rate of user k in a MISO MARC with TDMA structure is given by

$$R^{(k)}_{\text{TDMA}} = \tau^{(k)} \left[\log_2\left(1 + ||\mathbf{h}||^2 P_r\right) - \log_2\left(1 + \frac{||\mathbf{h}||^2 P_r}{1 + \lambda_1^{(k)} \frac{P^{(k)}}{\tau^{(k)}}}\right)\right]$$

As it can be seen, the remaining optimization variables to maximize the sum-rate $R_{\Sigma,\text{TDMA}} = \sum_{k=1}^{K} R^{(k)}_{\text{TDMA}}$ are the duration of the time slots $\tau^{(1)}, \ldots, \tau^{(K)}$. Their optimal values are given by the following theorem.

Theorem 4

For the considered TDMA transmission scheme in the K-user MISO MARC without direct links, the duration of the time slots $\tau^{(1)}, \ldots, \tau^{(K)}$ that maximizes the sum-rate is given by

$$\tau^{(k)}_{opt} = \frac{\lambda_1^{(k)} P^{(k)}}{\sum\limits_{l=1}^{K} \lambda_1^{(l)} P^{(l)}}. \tag{4.9}$$

Proof: *Consider the optimization problem*

$$\begin{aligned} \max_{\tau^{(1)},\ldots,\tau^{(K)}} \; & R_{\Sigma,TDMA}(\boldsymbol{\tau}) \\ \text{s.t.} \;\; & h(\boldsymbol{\tau}) = 1 - \sum_{k=1}^{K} \tau^{(k)} = 0, \end{aligned}$$

where we introduced the vector $\boldsymbol{\tau} = \left[\tau^{(1)}, \ldots, \tau^{(K)}\right]$ and wrote $R_{\Sigma,TDMA}$ as a function of $\boldsymbol{\tau}$ to be more consistent with the notation used in optimization theory. As $R^{(k)}_{TDMA}$ depends only on $\tau^{(k)}$ but not on the other components of $\boldsymbol{\tau}$, the first derivative of $R_{\Sigma,TDMA}(\boldsymbol{\tau})$ can be calculated as

$$\begin{aligned} \frac{\partial R_{\Sigma,TDMA}}{\partial \tau^{(k)}} = \frac{\partial}{\partial \tau^{(k)}}\left(R^{(k)}_{TDMA}\right) = & \log_2\left(1 + ||\mathbf{h}||^2 P_r\right) - \log_2\left(1 + \frac{||\mathbf{h}||^2 P_r}{1 + \lambda_1^{(k)} \frac{P^{(k)}}{\tau^{(k)}}}\right) \\ & - \frac{\log_2(e) ||\mathbf{h}||^2 P_r \lambda_1^{(k)} P^{(k)}}{\tau^{(k)} \left(1 + \lambda_1^{(k)} \frac{P^{(k)}}{\tau^{(k)}}\right)\left(1 + \lambda_1^{(k)} \frac{P^{(k)}}{\tau^{(k)}} + ||\mathbf{h}||^2 P_r\right)}. \end{aligned}$$

Hence, this derivative depends only on $\tau^{(k)}$, such that the Hessian matrix of $R_{\Sigma,TDMA}$ is diagonal, where the diagonal elements are the second derivatives with *respect to* $\tau^{(k)}$ *given by*

$$\begin{aligned}\frac{\partial^2 R_{\Sigma,TDMA}}{\partial {\tau^{(k)}}^2} &= \frac{\partial^2}{\partial {\tau^{(k)}}^2} R^{(k)}_{TDMA} \\ &= -\frac{\log_2(e)\left(2\tau^{(k)} + 2\lambda_1^{(k)} P^{(k)} + ||\mathbf{h}||^2 P_r \tau^{(k)}\right) {P^{(k)}}^2 {\lambda_1^{(k)}}^2 ||\mathbf{h}||^2 P_r}{\left(\tau^{(k)} + \lambda_1^{(k)} P^{(k)} + ||\mathbf{h}||^2 P_r \tau^{(k)}\right)^2 \left(\tau^{(k)} + \lambda_1^{(k)} P^{(k)}\right)^2}.\end{aligned}$$

As $\frac{\partial^2 R_{\Sigma,TDMA}}{\partial {\tau^{(k)}}^2} < 0$, *the target function* $R_{\Sigma,TDMA}$ *is concave with respect to* $\boldsymbol{\tau}$, *which makes the optimization problem convex.*

Therefore, the Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality. For the underlying problem, the KKT conditions of a solution $\boldsymbol{\tau}^*$ *with dual variable* ν^* *to be optimal can be formulated as*

$$\begin{aligned} h(\boldsymbol{\tau}^*) &= 0 \\ \nabla R_{\Sigma,TDMA}(\boldsymbol{\tau}^*) + \nu^* \nabla h(\boldsymbol{\tau}^*) &= 0, \end{aligned}$$

where ∇ *denotes the gradient of a function. If we choose* $\boldsymbol{\tau}^*$ *as suggested in Theorem 4, the first condition is obviously fulfilled. For the second condition we can calculate the derivatives of* $R_{\Sigma,TDMA}$ *and* h *at the point* $\boldsymbol{\tau} = \boldsymbol{\tau}^*$ *as*

$$\begin{aligned} \left.\frac{\partial R_{\Sigma,TDMA}}{\partial \tau^{(k)}}\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^*} &= \log_2\left(1 + ||\mathbf{h}||^2 P_r\right) - \log_2\left(1 + \frac{||\mathbf{h}||^2 P_r}{1+S}\right) \\ &\quad - \frac{||\mathbf{h}||^2 P_r S}{(1+S)\left(1+S+||\mathbf{h}||^2 P_r\right)} \\ \left.\frac{\partial h}{\partial \tau^{(k)}}\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^*} &= -1, \end{aligned}$$

where $S = \left(\sum_{k=1}^{K} \lambda_1^{(k)} P^{(k)}\right)^{-1}$. *As the derivatives with respect to* $\tau^{(k)}$ *are the same in all components* $k = 1, \ldots, K$, *we can select* $\nu^* = \left.\frac{\partial R_{\Sigma,TDMA}}{\partial \tau^{(k)}}\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^*}$ *and also the second KKT condition is fulfilled.* ■

Employing these optimal duration of the time slots, the maximum achievable sum-rate for the TDMA-based transmission scheme can be written as

$$R_{\Sigma,\mathrm{TDMA}} = \log_2\left(1 + ||\mathbf{h}||^2 P_r\right) - \log_2\left(1 + \frac{||\mathbf{h}||^2 P_r}{1 + \sum_{k=1}^{K} \lambda_1^{(k)} P^{(k)}}\right). \tag{4.10}$$

4.2.3 Comparison of TDMA and Joint Relaying

The superiority of the proposed TDMA-based scheme compared to the previously considered joint relaying scheme is established by the following theorem.

Theorem 5

In the considered K-user MARC, the TDMA-based transmission scheme, where the duration of the time slots $\tau^{(k)}$ is given by (4.9), achieves at least the same sum-rate as the joint relaying scheme, i.e.,

$$R_{\Sigma,TDMA} \geq R_{\Sigma}. \tag{4.11}$$

Proof: *Comparing the rate expressions of $R_{\Sigma,TDMA}$ and R_{Σ} from (4.10) and (4.8), respectively, it can be seen that it is sufficient to show that $\sum_{k=1}^{K} \lambda_1^{(k)} P^{(k)} \geq \lambda_{max}\left(\widetilde{\mathbf{R}}\right)$. Recalling that $\widetilde{\mathbf{R}} \triangleq \sum_{k=1}^{K} \mathbf{H}_r^{(k)} {\mathbf{H}_r^{(k)}}^H P^{(k)}$, this is obtained by writing*

$$\sum_{k=1}^{K} \lambda_1^{(k)} P^{(k)} = \sum_{k=1}^{K} \lambda_{max}\left(\mathbf{H}_r^{(k)} {\mathbf{H}_r^{(k)}}^H P^{(k)}\right) \tag{4.12}$$

$$\geq \lambda_{max}\left(\widetilde{\mathbf{R}}\right), \tag{4.13}$$

where (4.13) follows from the convexity of the maximum eigenvalue operation [BV04]. ■

Note that (cf. [KT01]) in (4.13), and therefore also in Theorem 5, equality holds only if all eigenvalues $\lambda_{\max}\left(\mathbf{H}_r^{(k)} {\mathbf{H}_r^{(k)}}^H P^{(k)}\right)$ have the same eigenvector. However, if the matrices $\mathbf{H}_r^{(k)}$ are statistically independent, which is a valid assumption in practical systems, this happens with probability zero if $N_r > 1$. The consequence of this is that the TDMA-based approach is strictly better in practice.

A plausible explanation for this fact is obtained by assuming the relay matrix $\mathbf{F}$ is fixed to the optimal value for joint relaying given in (4.3). The remaining optimization problem of finding $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ is then equivalent to the one that has to be solved for a MISO MAC. As explained at the end of Subsection 3.1.2, the optimal sum-rates of TDMA and successive decoding (which corresponds to joint relaying) are equal in the MISO MAC. If $\mathbf{F}$ is chosen as in (4.3), this sum-rate equals the optimal sum-rate of joint relaying in the MISO MARC. Concerning TDMA in the MISO MARC, this rate is also achievable by setting all $\mathbf{F}^{(1)}, \ldots, \mathbf{F}^{(K)}$ as in (4.3). However, as in TDMA the relay matrices must not be equal in all time slots, this choice does not achieve the optimal sum-rate. Finally, the advantage of TDMA is that the relay matrices can be perfectly adjusted to the channel of the single active user. On the contrary, there is only one relay matrix in joint relaying, which has to find a compromise such that not only one but all incoming signals are amplified as good as possible.

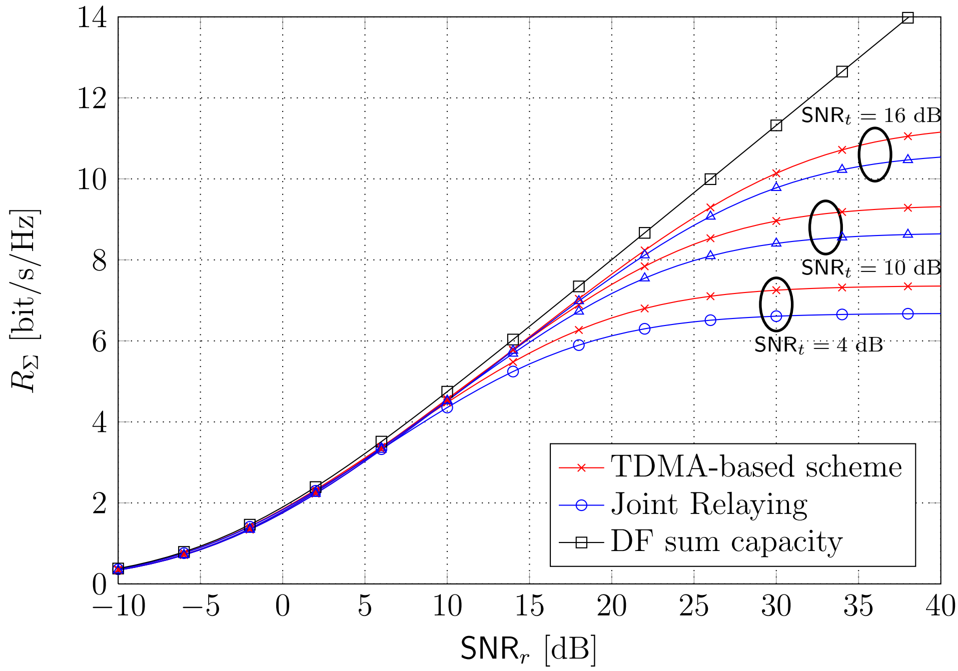

Figure 4.2: Sum-rates in the MISO MARC with $K = 10$ and $N_r = N_t = 3$

Although Theorem 5 clearly states the superiority of TDMA, the question for the quantitative gain in practical systems remains. In order to give a rough estimate, what gain can be expected, we will present a set of simulation results in the following. These simulations assume channels with independent Rayleigh fading, i.e., all entries of the channel matrices are $\sim \mathcal{CN}(0, 1)$ and independent from each other. Furthermore, the transmitters are assumed to have the same number of antennas $N_t^{(1)} = \ldots = N_t^{(K)} = N_t$, and the same transmit power $P^{(1)} = \ldots = P^{(K)} = P$. All results presented in this subsection are obtained by averaging over 1000 channel realizations

The achievable sum-rates with joint relaying and the TDMA-based scheme for a system with $K = 10$ users are visualized in Figure 4.2. We assumed that all transmitters and the relay have $N_t = N_f = 3$ antennas, while, as a prerequisite of this scenario, the receiver is assumed to have a single antenna. As we assume a unit noise variance the available power at transmitters and relay equals the associated SNRs, which are identified by SNR_t and SNR_r, respectively. While for SNR_t, we consider the three values 4 dB, 10 dB, and 16 dB, SNR_r is varied between -10 and 40 dB in steps of 1 dB. It can be observed that, especially for high values of SNR_r, the performance of the TDMA-based scheme is up to 10% better than that of the joint relaying scheme.

For a broader evaluation of the two investigated AF-based schemes, we also plotted the sum-rate achieved by decode-and-forward (DF). It can be shown that DF, with

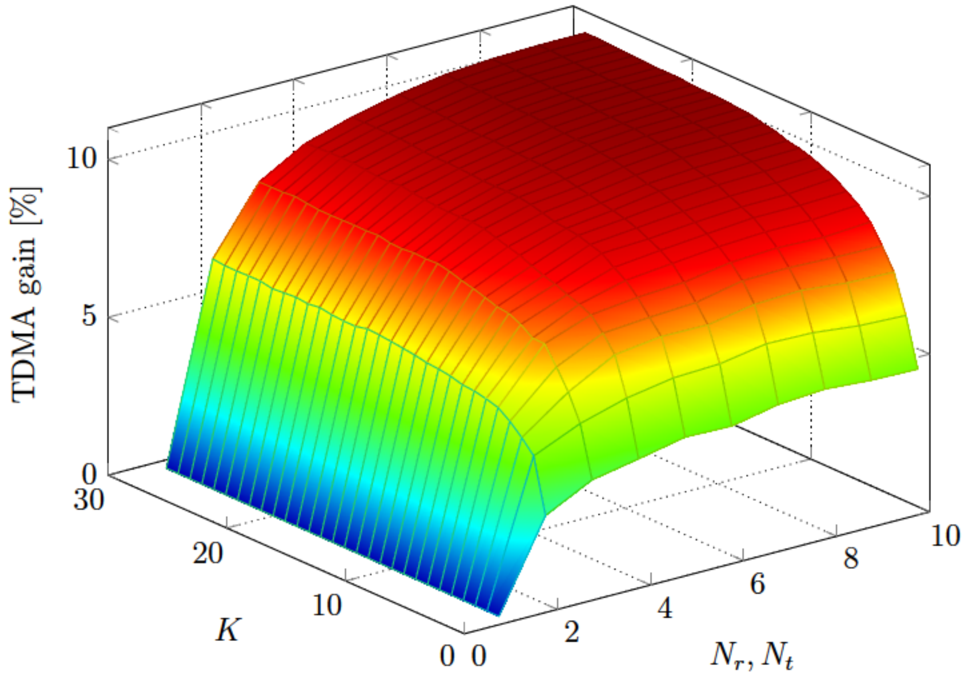

Figure 4.3: TDMA gain for $P_r \to \infty$, $N_t = N_r$, and $\mathsf{SNR}_t = 10$ dB

complete decoding and re-encoding of the signals at the relay, achieves the cut-set upper bound [CT06] and is therefore capacity achieving in the MARC without direct links. In short, this means the sum-rate of DF can be expressed as

$$R_{\Sigma,\mathrm{DF}} = \min\left(C_{\Sigma,\mathrm{Tx}\to\mathrm{Rl}}, C_{\mathrm{Rl}\to\mathrm{Rx}}\right), \tag{4.14}$$

where $C_{\Sigma,\mathrm{Tx}\to\mathrm{Rl}}$ and $C_{\mathrm{Rl}\to\mathrm{Rx}}$ are the (sum) capacities of the MAC from the transmitters to the relay and the single-user channel from the relay to the receiver, respectively.

As the MAC from transmitters to relay has multiple antennas at all nodes, the bottleneck in almost all cases considered here, is the channel from relay to receiver, which has a single antenna. Therefore, the achievable sum-rate with DF is well estimated by $C_{\mathrm{Rl}\to\mathrm{Rx}} = \log_2\left(1 + ||\mathbf{h}||^2 P_r\right)$ and does not depend on SNR_t. Comparing this rate to the ones achieved by AF, it can be seen that for reasonable relay power values, the AF-based schemes perform quite close to the sum-capacity although the relay does not decode and re-encode the signals. Only for high values of SNR_r, the DF scheme benefits from the decoding of the signals at the relay (which avoids the amplification of the relay noise) and outperforms the AF schemes clearly.

Returning to the comparison of joint relaying and our TDMA-based scheme, Figure 4.3 gives further insights into the behavior of the maximum gain of TDMA, which is achieved for $P_r \to \infty$. For this simulation, we assumed $\mathsf{SNR}_t = 10$ dB and that the number of relay and transmitter antennas are the same, i.e., $N_r = N_t$, where they

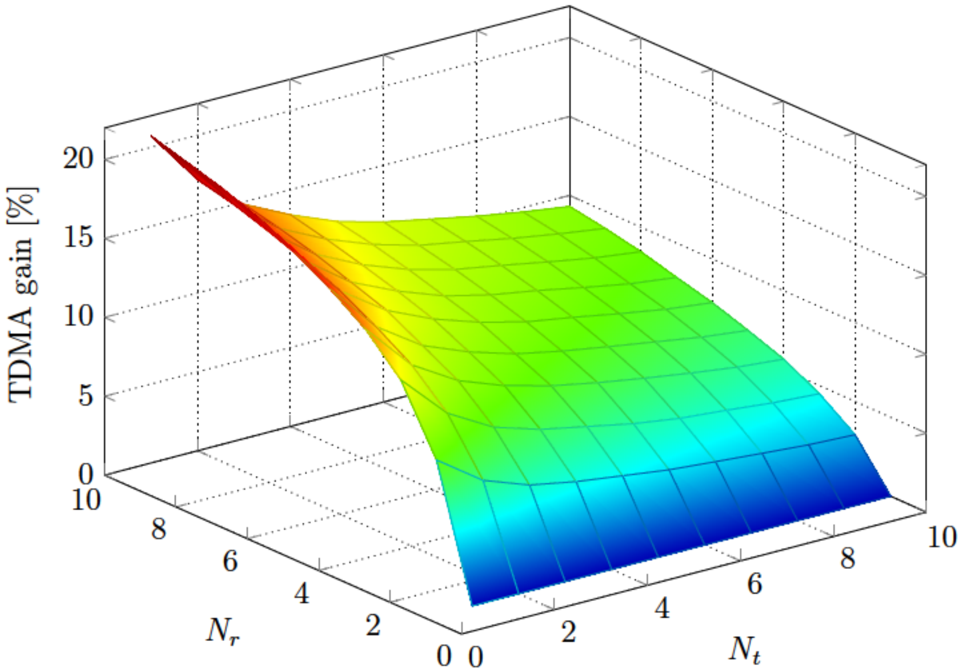

Figure 4.4: TDMA gain for $P_r \to \infty$, $K = 10$, and $P = 10$ dB

are varied as well as the number of users K. Obviously, we obtain no rate gain if $N_r = N_t = 1$, which follows from (4.12): For $N_r = 1$ the matrices $\mathbf{H}_r^{(k)}$ become row vectors and the maximum eigenvalue operation is superfluous as $\mathbf{H}_r^{(k)}\mathbf{H}_r^{(k)H}$ is a scalar. However, for $N_r, N_t > 1$, it can be seen that the gain of TDMA grows both with the number of users and with the number of relay and transmitter antennas. Concerning the number of users, this is easily explained because the difficulty in finding one relay matrix for joint relaying that fits to all users grows with K.

A more detailed view on the role of the number of antennas is given in Figure 4.4, where the number of users is fixed to $K = 10$, but the number of relay and transmitter antennas are varied separately. While the gain of TDMA increases with the number of relay antennas, it surprisingly decreases with the number of transmitter antennas. An explanation for this could be that joint relaying has a higher benefit from more transmit antennas than TDMA because more antennas increase number and magnitude of the eigenvalues of $\mathbf{H}_r^{(k)}$. Therefore, it becomes easier for joint relaying to find one matrix $\mathbf{F}$ which provides a good compromise between the "good" eigenvectors of the users. Looking at the application in cellular systems, this fact is rather an advantage of TDMA: As the mobile transmitting stations in the uplink usually have few space, they have mostly a single antenna while the relay can be made big enough to host a large antenna array. As Figure 4.4 shows this combination can lead to a rate gain of over 20% and even more if the number of users is higher.

4.3 Gaussian MISO Broadcast Channel with AF-Relay

As already mentioned in the previous chapter, the problem of finding optimal transmit strategies for broadcast channels is usually much harder to solve than for MACs. While for channels without relay, this problem can be simplified by utilizing the duality with the MAC, there is no general duality between the BRC and the MARC. So far, the most promising approach for the BRC is the use of ZF-DPC as explained in Subsection 3.3.2, which uses a heuristic choice of the relaying matrix to scalarize the problem. However, also this scalar problem is nonconvex and hard to solve such that the complexity remains high.

In the following subsection, we are going to introduce an algorithmic structure, which attempts to find a good transmit strategy for the MISO BRC with DPC instead of ZF-DPC. This algorithmic structure is heuristic and solves only parts of the problem. However, already this structure provides some significant simplifications such that the remaining optimization problems are much easier to solve than those of ZF-DPC. Using brute-force approaches for the remaining optimization problems of both ZF-DPC and our DPC algorithm, we will show in Subsection 4.3.2 that the performance of both schemes is approximately the same.

A key technique in our algorithmic structure is the use of the duality theory, which relates the capacity region of MAC and broadcast channels. Although this duality can not be directly extended to channels with a relay, we will use the MAC domain to solve a part of the problem.

4.3.1 Achievable Scheme Based on MAC-Broadcast Channel Duality

For the considered MISO BRC, the channel model described in Section 2.2 holds. Similar to the MARC, the restriction to a single receive antenna allows us to write the channel matrices from the relay to the destinations as row vectors ${\mathbf{h}^{(1)}}^H, \ldots, {\mathbf{h}^{(K)}}^H$ instead of matrices $\mathbf{H}^{(1)}, \ldots, \mathbf{H}^{(K)}$. Hence, the signal at receiver k can be written as (cf. (2.2))

$$y^{(k)} = {\mathbf{h}^{(k)}}^H \mathbf{F}\mathbf{H}_r \mathbf{x} + {\mathbf{h}^{(k)}}^H \mathbf{F}\mathbf{z}_r + z^{(k)} = {\mathbf{h}^{(k)}}^H \mathbf{F}\mathbf{H}_r \sum_{k=1}^{K} \mathbf{x}^{(k)} + {\mathbf{h}^{(k)}}^H \mathbf{F}\mathbf{z}_r + z^{(k)},$$

where the receiver noise $z^{(k)} \sim \mathcal{CN}(0,1)$ is now scalar. In order to simplify the notation, we will normalize the receiver output such that the overall noise has unit variance. Hence, we consider the equivalent output

$$\widetilde{y}^{(k)} = {s^{(k)}}^{-1/2} {\mathbf{h}^{(k)}}^H \mathbf{F}\mathbf{H}_r \mathbf{x} + \widetilde{z}^{(k)}$$

with $s^{(k)} = 1 + \mathbf{h}^{(k)^H}\mathbf{F}\mathbf{F}^H\mathbf{h}^{(k)}$ and $\widetilde{z}^{(k)} \sim \mathcal{CN}(0,1)$, such that the underlying MISO BRC can also be considered as MISO broadcast channel with channel vectors $\mathbf{h}_{\text{eff}}^{(k)^H} = s^{(k)^{-1/2}}\mathbf{h}^{(k)^H}\mathbf{F}\mathbf{H}_r$.

Assuming a fix relay matrix $\mathbf{F}$, it is known from the duality approach [VJG03] for channels without relay that this channel with power constraint P has the same sum capacity as the SIMO MAC with channel gains $\mathbf{h}_{\text{eff}}^{(k)}$ and sum power constraint P. The sum capacity of this channel is found by solving the optimization problem (3.10), which can be written as

$$\begin{aligned} \max_{P^{(1)},\dots,P^{(K)}} \quad & \log_2 \left| \mathbf{I} + \mathbf{H}_r^H\mathbf{F}^H \sum_{k=1}^{K} \frac{\mathbf{h}^{(k)}P^{(k)}\mathbf{h}^{(k)^H}}{1+\mathbf{h}^{(k)^H}\mathbf{F}\mathbf{F}^H\mathbf{h}^{(k)}} \mathbf{F}\mathbf{H}_r \right| \\ \text{s.t.} \quad & \sum_{k=1}^{K} P^{(k)} \leq P \\ & P^{(k)} \geq 0 \qquad \forall k \end{aligned} \tag{4.15}$$

in this case. Although the optimization variables $P^{(k)}$ are scalar, the solution of this problem is not trivial such that Algorithm 2 from Subsection 3.2.3 has to be used to compute the optimal power distribution.

Furthermore, the relay matrix $\mathbf{F}$ has to be found, which is much harder. Unfortunately, the structure of $\mathbf{F}$ that achieves the optimal sum-rate in the MARC does not generally achieve the optimal sum-rate in the BRC and an analytic solution seems infeasible. Therefore, we use a heuristic choice of $\mathbf{F}$ that is based on a similar structure as in the MARC. For this purpose, we define

$$\mathbf{D} \triangleq \sum_{k=1}^{K} \mathbf{h}^{(k)}P^{(k)}\mathbf{h}^{(k)^H} \tag{4.16}$$

and use the SVDs $\mathbf{D} = \mathbf{U}_d\mathbf{\Sigma}_d\mathbf{V}_d^H$ and $\mathbf{H}_r = \mathbf{U}_h\mathbf{\Sigma}_h\mathbf{V}_h^H$.

Looking at the sum-rate expression of (4.15), it can be seen that $\mathbf{F}$ is multiplied with $\mathbf{H}_r$ from the right side and with the sum expression from the left side. Our heuristic approach is to approximate the sum expression by $\mathbf{D}$, i.e., the denominator is ignored. If we use this approximation, choosing

$$\mathbf{F} = \mathbf{V}_d\mathbf{\Sigma}_f\mathbf{U}_h^H, \tag{4.17}$$

with $\mathbf{\Sigma}_f = \operatorname{diag}\left(f_1,\dots,f_{N_f}\right)^{1/2}$ diagonalizes $\mathbf{H}_r^H\mathbf{F}^H\mathbf{D}\mathbf{F}\mathbf{H}_r$ to $\mathbf{\Sigma}_h^H\mathbf{\Sigma}_f^H\mathbf{\Sigma}_d\mathbf{\Sigma}_f\mathbf{\Sigma}_h$.

From the Hadamard inequality [HJ90], it follows that this choice of $\mathbf{F}$ is optimal for the approximated version. However, we rather consider (4.17) as a heuristic choice for the exact problem instead of an optimal choice for the approximated problem. The advantage of choosing $\mathbf{F}$ as in (4.17) is that the number of scalar variables reduces from N_f^2 to N_f, which makes it easier to solve the problem by standard tools of optimization, e.g., a gradient search algorithm.

If the matrix $\mathbf{F}$ is determined and Algorithm 2 is applied to find the optimum power distribution $P^{(1)}, \ldots, P^{(K)}$, the transformation back to the broadcast channel domain has to be done. Using the transformation from [VJG03], the corresponding transmit covariance matrices $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$, which fulfill the transmit power constraint can be computed. However, as the duality transformation does not consider the relay power constraint, it has to be ensured that this is also fulfilled. In our heuristic algorithm, we achieve this by calculating the preliminary relay power

$$\widetilde{P}_r \triangleq \operatorname{tr}\left(\mathbf{F}\left(\mathbf{I} + \mathbf{H}_r \sum_{k=1}^{K} \mathbf{Q}^{(k)} \mathbf{H}_r^{\,H}\right)\mathbf{F}^H\right),$$

which is obtained by the current choices of $\mathbf{F}$ and $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$. In order to meet the relay power constraint with equality, the relay matrix is multiplied by $\gamma = \sqrt{\frac{P_r}{\widetilde{P}_r}}$.

The disadvantage of this approach is that the user order becomes important again. Although the user order does not influence the sum-rate in the dual SIMO MAC or in the broadcast channel directly, different user orders lead to different covariance matrices $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ in the transformation from MAC to broadcast channel domain. The consequence of this is that also $\widetilde{P}_r$ depends on the user order. As it is obtained from (4.15), the sum-rate is monotonically increasing with γ. Hence, the best user order is the one, which leads to the lowest $\widetilde{P}_r$. However, especially if P_r is large, the influence of the user order on $\widetilde{P}_r$ is marginal, such that the user order does not play such an important role as in the ZF-DPC approaches.

In order to evaluate the approach presented in this section, we will determine the remaining variables $f_1, \ldots, f_{N_f}$ by brute-force search, where the choice with the highest (preliminary) sum-rate is picked. Doing the same with the remaining optimization problem (3.15) of ZF-DPC, the capabilities of both schemes can be well estimated. In order to keep the computation time reasonable, we restrict ourselves to $K = 2$ users. The algorithmic structure explained above with the brute-force extensions for $K = 2$ is listed in Algorithm 3 in terms of pseudo code.

As there are only two users, it can be seen from (4.16) that $\mathbf{D}$ has only two nonzero eigenvalues. Although, our structure of $\mathbf{F}$ from (4.17) is only a heuristic choice and the matrix appearing in the real sum-rate is not exactly $\mathbf{D}$, making the rank of $\mathbf{F}$ larger than the number of eigenvalues of $\mathbf{D}$ is not reasonable. Therefore, we set all diagonal elements of $\mathbf{\Sigma}_f$ to zero except f_1 and f_2. Moreover, as $\mathbf{F}$ is scaled in line 27 to meet the power constraint, only the ratio between these two values has to be determined. This is done by setting $f_1 = \beta$ and $f_2 = 1 - \beta$, where $\beta \in [0, 1]$. The best possible β is determined by brute force search in lines 11-15 of Algorithm 3, where β is varied between 0 and 1 in steps of Δ_β.

Unfortunately, the interdependence of the best possible $\mathbf{F}$ and $P^{(1)}, P^{(2)}$ is the same as in the MARC. Hence, after changing β the power distribution $P^{(1)}, P^{(2)}$ has to be recalculated. This is done by Algorithm 2 from Subsection 3.2.3 (abbreviated by

Algorithm 3 Duality-based algorithm for the MISO BRC with $K = 2$ users

1: Initialize: $P^{(1)} = P^{(2)} = \frac{P}{2}$; $\beta = 0.5$; $\mathbf{\Sigma}_f = \mathrm{diag}\left(\sqrt{\beta}, \sqrt{1-\beta}, 0, \ldots, 0\right)$
2: $\mathbf{D} = \sum_{k=1}^{K} \mathbf{h}^{(k)} P^{(k)} \mathbf{h}^{(k)^H}$; $\mathbf{U}_d \mathbf{\Sigma}_d \mathbf{V}_d^H = \mathrm{svd}(\mathbf{D})$
3: $\mathbf{F} = \mathbf{V}_d \mathbf{\Sigma}_f \mathbf{U}_h^H$
4: **for** $k = 1 : 2$ **do**
5: $\quad s^{(k)} = 1 + \mathbf{h}^{(k)^H} \mathbf{F}\mathbf{F}^H \mathbf{h}^{(k)}$
6: $\quad \mathbf{h}_{\text{eff}}^{(k)} = s^{(k)^{-1/2}} \mathbf{h}^{(k)^H} \mathbf{F}\mathbf{H}_r$
7: **end for**
8: $\left[P^{(1)}, P^{(2)}\right] = \texttt{BlockIWF}\left(\left[\mathbf{h}_{\text{eff}}^{(1)}, \mathbf{h}_{\text{eff}}^{(2)}\right], P\right)$
9: **repeat**
10: $\quad \mathbf{D} = \sum_{k=1}^{K} \mathbf{h}^{(k)} P^{(k)} \mathbf{h}^{(k)^H}$; $\mathbf{U}_d \mathbf{\Sigma}_d \mathbf{V}_d^H = \mathrm{svd}(\mathbf{D})$
11: $\quad$ **for** $\beta = 0 : \Delta_\beta : 1$ **do**
12: $\quad\quad \mathbf{\Sigma}_f = \mathrm{diag}\left(\sqrt{\beta}, \sqrt{1-\beta}, 0, \ldots, 0\right)$
13: $\quad\quad$ Recalculate $\mathbf{F}$ and $s^{(k)}, \mathbf{h}_{\text{eff}}^{(k)}$ $\forall k$ as in lines 3-7
14: $\quad\quad \widetilde{R}_\Sigma(\beta) = \log_2 \left| \mathbf{I} + \sum_{k=1}^{2} \mathbf{h}_{\text{eff}}^{(k)} P^{(k)} \mathbf{h}_{\text{eff}}^{(k)^H} \right|$
15: $\quad$ **end for**
16: $\quad \beta^* = \arg\max_\beta \widetilde{R}_\Sigma(\beta)$
17: $\quad \mathbf{\Sigma}_f = \mathrm{diag}\left(\sqrt{\beta^*}, \sqrt{1-\beta^*}, 0, \ldots, 0\right)$
18: $\quad$ Recalculate $\mathbf{F}$ and $s^{(k)}, \mathbf{h}_{\text{eff}}^{(k)}$ $\forall k$ as in lines 3-7
19: $\quad \left[P^{(1)}, P^{(2)}\right] = \texttt{BlockIWF}\left(\left[\mathbf{h}_{\text{eff}}^{(1)}, \mathbf{h}_{\text{eff}}^{(2)}\right], P\right)$
20: $\quad \widetilde{R^*}_\Sigma = \log_2 \left| \mathbf{I} + \sum_{k=1}^{2} \mathbf{h}_{\text{eff}}^{(k)} P^{(k)} \mathbf{h}_{\text{eff}}^{(k)^H} \right|$
21: **until** $\mathbf{F}$ and $\left[P^{(1)}, P^{(2)}\right]$ converge
22: **for** all possible decoding orders π **do**
23: $\quad \left[\mathbf{Q}^{(1)}(\pi), \mathbf{Q}^{(2)}(\pi)\right] = \texttt{MAC2BC}\left(\left[\mathbf{h}_{\text{eff}}^{(1)}, \mathbf{h}_{\text{eff}}^{(2)}\right], \left[P^{(1)}, P^{(2)}\right]\right)$
24: $\quad \widetilde{P}_r(\pi) = \mathrm{tr}\left(\mathbf{F}\left(\mathbf{I} + \mathbf{H}_r\left(\mathbf{Q}^{(1)} + \mathbf{Q}^{(2)}\right)\mathbf{H}_r^H\right)\mathbf{F}^H\right)$
25: **end for**
26: $\pi^* = \arg\min_\pi \widetilde{P}_r(\pi)$
27: $\mathbf{F} = \sqrt{\frac{P_r}{\widetilde{P}_r(\pi^*)}}\mathbf{F}$
28: Recalculate $s^{(k)}, \mathbf{h}_{\text{eff}}^{(k)}$ $\forall k$ as in lines 4-7
29: $R_\Sigma = \sum_{k=1}^{2} \log_2 \left| \mathbf{I} + \frac{\mathbf{h}_{\text{eff}}^{(\pi^*(k))} \mathbf{Q}^{(\pi^*(k))}(\pi^*) \mathbf{h}_{\text{eff}}^{(\pi^*(k))^H}}{\mathbf{I} + \mathbf{h}_{\text{eff}}^{(\pi^*(k))}\left[\sum_{j>k} \mathbf{Q}^{(\pi^*(j))}(\pi^*)\right]\mathbf{h}_{\text{eff}}^{(\pi^*(k))^H}} \right|$

`BlockIWF` in Algorithm 3). As the new values of $P^{(1)}, P^{(2)}$ also require a recalculation of β, these two steps are repeatedly executed until the preliminary sum-rate $\widetilde{R^*}_\Sigma$ converges. Convergence is ensured by the facts that first, both the optimization with respect to β and with respect to $P^{(1)}, P^{(2)}$ can never decrease the sum-rate and second, the achievable sum-rate is limited.

After convergence, the final step is the transformation back to the BC according to [VJG03], which is indicated by `MAC2BC` in Algorithm 3. As already explained above, this transformation is done for every possible user order π in order to find the user order with the lowest preliminary relay power $\widetilde{P}_r(\pi)$. This order is used and the relay matrix is scaled, such that the achievable sum-rate can finally be calculated.

Extending the algorithm to $K > 2$ users is possible but the complexity of the brute force parts for finding $\mathbf{F}$ in lines 11-15 and for running `MAC2BC` for all possible decoding orders grows exponentially. However, the complexity of finding $\mathbf{F}$ is limited by the number of eigenvalues of $\mathbf{D}$, which is at most $\min(K, N_f)$, so if N_f is not too large the complexity remains reasonable. Moreover, the optimal decoding order only achieves small gains in the scaling factor γ, such that this step could be simplified by choosing a random decoding order.

4.3.2 Upper Bounds and Comparison

As a main benchmark for our algorithm, we consider the maximal achievable rate with ZF-DPC. This rate is approximated by a brute-force solution of the ZF-DPC optimization problem (3.15) with $K = 2$. For this purpose we transform our variables such that

$$\begin{aligned}
P_1 &= \alpha P \\
P_2 &= (1-\alpha)P \\
f_1 &= \frac{\beta P_r}{1 + |\sigma_1|^2 P_1} \\
f_2 &= \frac{(1-\beta) P_r}{1 + |\sigma_2|^2 P_2},
\end{aligned}$$

i.e., α and β are the new optimization variables. This simplifies the constraints significantly, as they can be equivalently written as $0 \leq \alpha, \beta \leq 1$. As writing the sum-rate as function of α and β delivers a bulky expression, it is not reformulated here but with the transformation given above it is possible to calculate the sum-rate with (3.15). Using the new variables, the optimal sum-rate with ZF-DPC can be approximated by varying α and β between 0 and 1 in steps of Δ_α and Δ_β, respectively.

Another simplified scheme, which will be investigated in the following simulation results is obtained by transmitting data to only one user. Of course, the user with the highest possible rate is chosen in that scheme and the optimal relay and covariance matrices can be derived as in Subsection 4.2.2. Recalling from Subsection 3.2.2 that the rate of serving only the best user equals the maximal sum-rate of TDMA in the

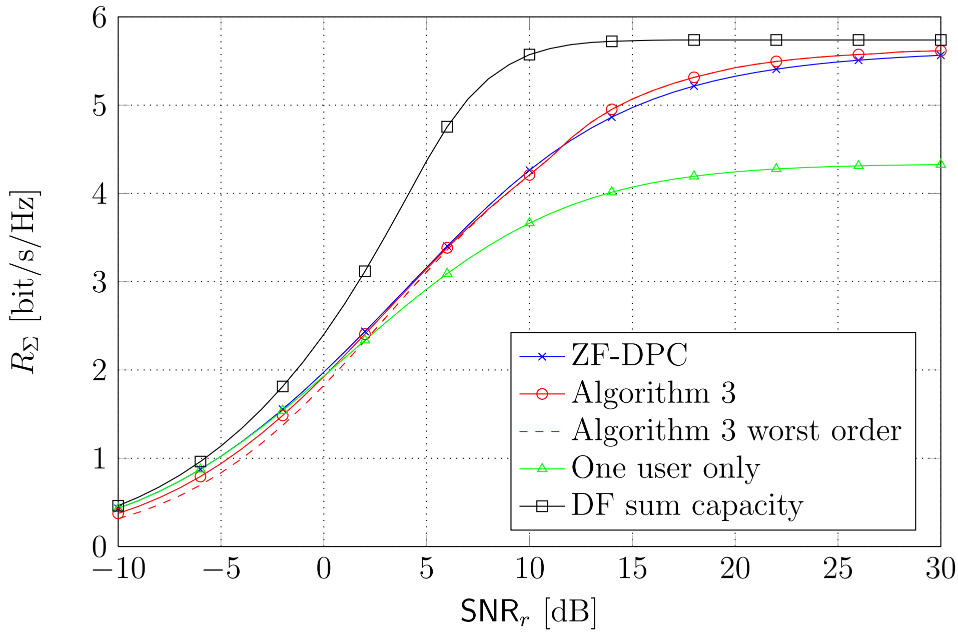

Figure 4.5: Achievable rates in MISO BRC for $\mathsf{SNR}_t = 4$ dB and $K = 2$

broadcast channel, this simple scheme gives an idea of the performance of TDMA in the MISO BRC.

An evaluation of the schemes mentioned above for $K = 2$ is given by simulation results. As in the MARC, we assume channels with independent Rayleigh fading. However, due to the increased complexity of the algorithms we take the average of 100 instead of 1000 channel realizations. The results are plotted in Figures 4.5, 4.6, and 4.7 for transmit power to noise ratios of $\mathsf{SNR}_t = 4$ dB, $\mathsf{SNR}_t = 10$ dB, and $\mathsf{SNR}_t = 16$ dB, respectively. The relay power to noise ratio SNR_r is varied between -10 and 30 dB in steps of 1 dB. Moreover, the precision of the brute-force searches is set to $\Delta_\alpha = \Delta_\beta = 0.01$ for ZF-DPC and $\Delta_\beta = 0.02$ for Algorithm 3. Again, we also compare the achievable rates with the sum capacity achieved by DF, which can be expressed similar to (4.14). The difference is that here the channel from the transmitter to the relay is a MIMO single-user channel, while the channel from the relay to the receivers is a MISO BC.

It can be seen that the achievable rates of Algorithm 3 are almost the same as those of ZF-DPC, although the complexity of this algorithm is much lower. In order to see that the decoding order, which is determined at the end of Algorithm 3, does not influence the rate too much, we also plotted the rate that is achieved by selecting the worst decoding order for each channel as dotted line. Obviously, this does not decrease the performance too much. For higher values of P_r, there is no difference at all. The scheme where only one user is served achieves a reasonable performance only for low

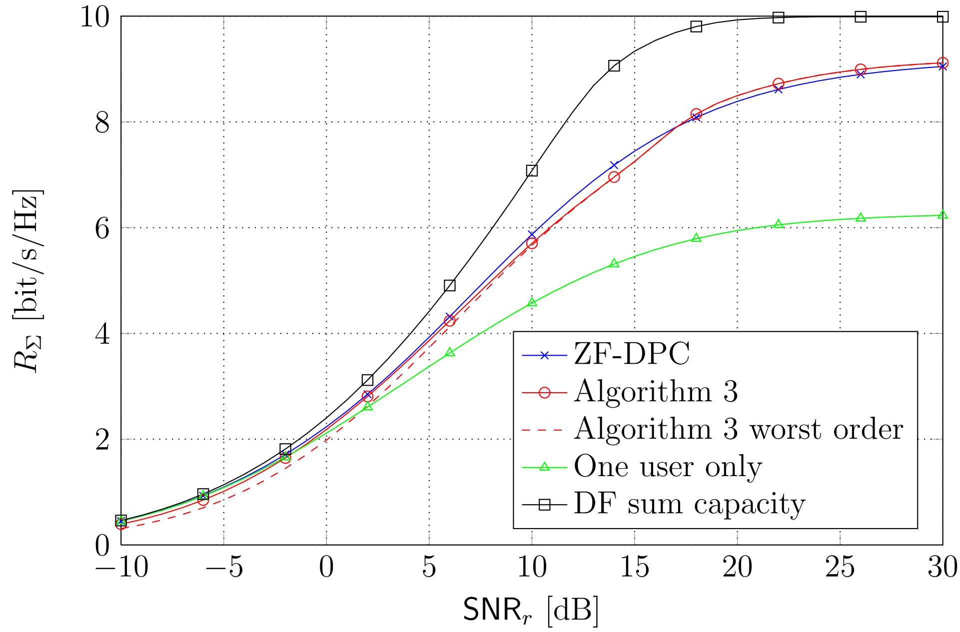

Figure 4.6: Achievable rates in MISO BRC for $\mathsf{SNR}_t = 10$ dB and $K = 2$

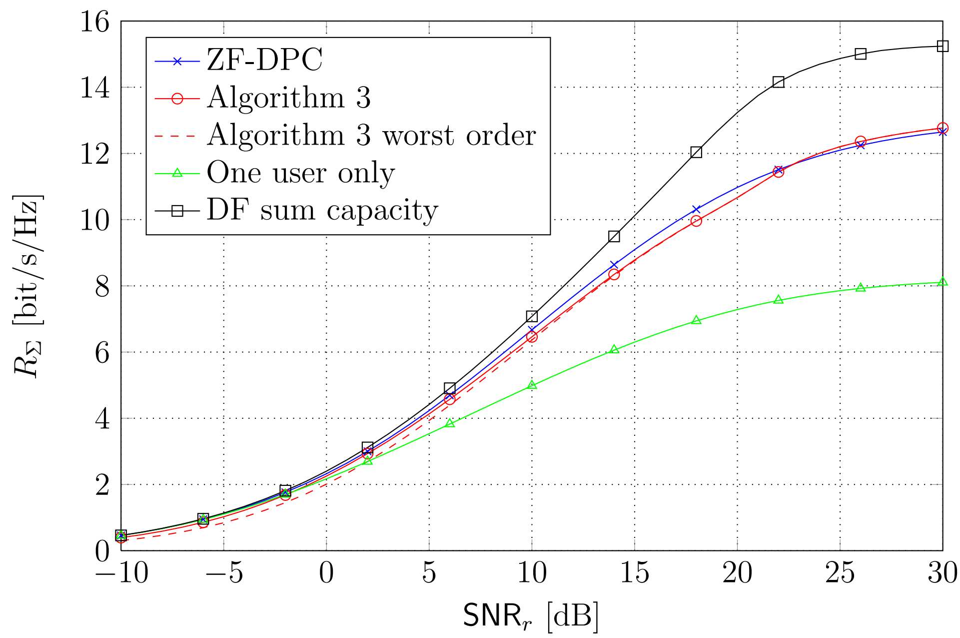

Figure 4.7: Achievable rates in MISO BRC for $\mathsf{SNR}_t = 16$ dB and $K = 2$

values of P_r. At higher values, the performance loss is significant, i.e., a TDMA-based relaying scheme does not make sense in the BRC. Finally, comparing the sum-rates to the sum capacity achieved by DF, it can be seen that the performance gap is not too big. Contrary to the MARC, the maximum performance loss is not obtained at high but at medium P_r. The reason for this might be that in the BRC the channel from the relay to the transmitter is not always the bottleneck, such that at high values of P_r, also the performance of DF is limited by the channel from the transmitter to the relay.

5

AF-Relaying for MISO Gaussian MAC with Direct Links

In the preceding chapter, we were able to find the optimal relaying and transmit covariance matrices for the MISO MARC without direct links both for our proposed TDMA-based relaying scheme and for joint relaying. In this section, we consider also the direct links between the transmitters and the receiver, such that the channel model explained in Section 2.1 holds without simplifications except for the single antenna at the receiver. Of course, this makes the problem of finding the optimal relay and transmit covariance matrices much harder to solve.

As the solutions for this problem could only be found for a few special cases, our approach is to find upper and lower bounds on the achievable sum-rate, where the upper bounds are only valid given the constraint that AF is used as relaying technique. Contrary to the MARC without direct links, neither does DF necessarily achieve the cut-set bound nor it is clear whether the cut-set bound is achievable at all or which relaying technique achieves the optimal sum-rate.

In order to obtain upper bounds, we relax the relay power constraint and find the optimal relaying matrix either in closed-form or with some kind of brute-force maximization for both joint relaying and TDMA. For the lower bounds, we first consider an approach which uses both brute-force and convex optimization methods to tackle the problem. However, as this approach suffers from an immense computational complexity, it does not seem feasible to deliver useful results in reasonable computation time. Therefore, we focus on an iterative algorithm, which has a much lower complexity and gets close to the upper bounds for a large set of parameters. This algorithm is designed for use with joint relaying but can easily be adapted for use with TDMA. Although the question for the exact achievable sum-rate with AF remains open, the question whether TDMA is also superior in case of direct links can be investigated.

This chapter is structured as follows: In Section 5.1, we derive three upper bounds for the MISO MARC with direct links. While the first upper bound is for single-user channels to be used with TDMA, the second and third upper bounds are for multi-user channels. Subsequently, in Section 5.2 we explain why brute-force methods are not suitable for lower bounds and describe our iterative algorithm, which is used to obtain achievable sum rates for joint relaying and TDMA. The special case of a SISO MARC is considered in Section 5.3, where the optimal solution for TDMA can be found with very low complexity and an analytic comparison of joint relaying and TDMA is possible for large relay power. Finally, Section 5.4 gives a set of simulation results. These allow some insights how tight upper and lower bounds are and whether the superiority of TDMA persists for the MARC with direct links.

5.1 Upper Bounds on the Achievable Sum-Rate

Recalling the definitions from Section 2.1 and considering the single antenna at the receiver, the receive signal in a MISO MARC is given by

$$\widetilde{\mathbf{y}} = \sum_{k=1}^{K} \mathbf{H}_{\text{eff}}^{(k)} \mathbf{x}^{(k)} + \widetilde{\mathbf{z}},$$

where $\widetilde{\mathbf{z}} \sim \mathcal{CN}(0, \mathbf{I})$, and

$$\mathbf{H}_{\text{eff}}^{(k)} = \begin{bmatrix} s^{-1/2}\mathbf{h}^H \mathbf{F} \mathbf{H}_r^{(k)} \\ \mathbf{h}_d^{(k)H} \end{bmatrix} \tag{5.1}$$

with $s = 1 + \mathbf{h}^H \mathbf{F}\mathbf{F}^H \mathbf{h}$. Using the sum-rate formula for the MAC, we can formulate the optimization problem for the MISO MARC with direct links as

$$\begin{aligned} \max_{\mathbf{F}, \mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}} \quad & R_\Sigma = \log_2 \left| \mathbf{I} + \sum_{k=1}^{K} \mathbf{H}_{\text{eff}}^{(k)} \mathbf{Q}^{(k)} \mathbf{H}_{\text{eff}}^{(k)H} \right| \\ \text{s.t.} \quad & \mathbf{Q}^{(k)} \succeq 0 \quad \forall k \\ & \operatorname{tr}\left(\mathbf{Q}^{(k)}\right) \leq P^{(k)} \ \ \forall k \\ & \operatorname{tr}\left(\mathbf{F}\left(\mathbf{I} + \mathbf{R}\right)\mathbf{F}^H\right) \leq P_r, \end{aligned} \tag{5.2}$$

where $\mathbf{R} = \sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{Q}^{(k)} \mathbf{H}_r^{(k)H}$. In the following subsections, this problem is considered for the cases of $K = 1$ and $K > 1$, where algorithms are found that compute the solution for a relaxation of the relay power constraint. These solutions can be used as an upper bound for the general problem.

5.1.1 Single-User Upper Bound

The first upper bound that we derive is for the case of $K = 1$, i.e., for a single-user relay channel. The technique of relaxing the relay power constraint is similar to the approach from [TH07] explained in Subsection 3.3.3. However, contrary to [TH07], the challenge here is that the covariance matrix is not fixed to a scaled identity matrix, but also subject to optimization.

For this upper bound, we assume

$$\operatorname{tr}\left(\mathbf{F}\mathbf{F}^H\right) \leq P_r \tag{5.3}$$

as relaxed relay power constraint and use the definitions

$$\begin{aligned}
\mathbf{h} &= \mathbf{U}_h \mathbf{\Sigma}_h \mathbf{V}_h^H = \mathbf{U}_h \mathbf{\Sigma}_h \\
\mathbf{H}_r^{(1)} &= \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \\
\mathbf{\Sigma} &= \operatorname{diag}\left(\sigma_1, \ldots, \sigma_{N_t^{(1)}}\right) \\
\mathbf{a} &\triangleq \frac{\mathbf{V}^H \mathbf{h}_d^{(1)}}{||\mathbf{h}_d^{(1)}||} \\
\mathbf{F} &= \mathbf{U}_h \widetilde{\mathbf{F}} \mathbf{U}^H,
\end{aligned} \tag{5.4}$$

where the first two equations denote the SVDs of $\mathbf{h}$ and $\mathbf{H}_r^{(1)}$. As $\mathbf{h}$ is a vector, we have $\mathbf{V}_h^H = 1$ and $\mathbf{\Sigma}_h = [||\mathbf{h}||, 0, \ldots, 0]^H$. Moreover, the last line defines a structure of $\mathbf{F}$, which does not reduce it's generality as no assumptions are made about $\widetilde{\mathbf{F}}$ and both $\mathbf{U}_h$ and $\mathbf{U}^H$ are unitary.

Considering (5.1) and that $\mathbf{h}^H \mathbf{F} \mathbf{H}_r^{(k)} = \mathbf{\Sigma}_h \widetilde{\mathbf{F}} \mathbf{\Sigma} \mathbf{V}^H$, it can be seen that only the first row of $\widetilde{\mathbf{F}}$ influences the sum-rate. As the power constraint can be rewritten as $\operatorname{tr}\left(\widetilde{\mathbf{F}}\widetilde{\mathbf{F}}^H\right) \leq P_r$, it is not beneficial to set any of the elements outside the first row of $\widetilde{\mathbf{F}}$ to a nonzero value. Therefore, we can restrict $\widetilde{\mathbf{F}}$ to be

$$\widetilde{\mathbf{F}} = \begin{bmatrix} f_1 & f_2 & \cdots & f_{N_f} \\ 0 & \cdots & & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & & 0 \end{bmatrix}, \tag{5.5}$$

where $f_1, \ldots, f_{N_f} \in \mathbb{C}$. Moreover, we make use of the result that for single-user channels the capacity of the Hermitian channel, which is obtained by taking the Hermitian of the channel matrix, is the same [VJG03, Tel99]. Hence, for the case of $K = 1$, problem (5.2) can be rewritten as

$$\begin{aligned}
\max_{\mathbf{F}, \widetilde{\mathbf{Q}}^{(1)}} \quad & R_\Sigma = \log_2 \left| \mathbf{I} + \mathbf{H}_{\text{eff}}^{(1)\,H} \widetilde{\mathbf{Q}}^{(1)} \mathbf{H}_{\text{eff}}^{(1)} \right| \\
\text{s.t.} \quad & \widetilde{\mathbf{Q}}^{(1)} \succeq 0 \\
& \operatorname{tr}\left(\widetilde{\mathbf{Q}}^{(1)}\right) \leq P^{(1)} \\
& \sum_{n=1}^{N_f} |f_n|^2 \leq P_r,
\end{aligned} \tag{5.6}$$

where $\widetilde{\mathbf{Q}}^{(1)}$ is the transmit covariance of the Hermitian channel which can be written as

$$\widetilde{\mathbf{Q}}^{(1)} = \begin{bmatrix} q_1 & \widetilde{q} \\ \widetilde{q}^H & q_2 \end{bmatrix}.$$

As $\widetilde{\mathbf{Q}}^{(1)}$ is a covariance matrix, the parameters can not be chosen freely. While the restrictions $q_1, q_2 \geq 0$ and $q_1 + q_2 \leq P^{(1)}$ follow directly from the constraints, the condition $|\widetilde{q}|^2 \leq q_1 q_2$ follows implicitly.

Using all the definitions from above, the sum-rate R_Σ can be rewritten as

$$\begin{aligned} R_\Sigma &= \log_2 \left| \mathbf{I} + \mathbf{H}_{\text{eff}}^{(1)} \mathbf{H}_{\text{eff}}^{(1)H} \widetilde{\mathbf{Q}}^{(1)} \right| \\ &= \log_2 \left| \mathbf{I} + \begin{bmatrix} s^{-1/2} \mathbf{\Sigma}_h \widetilde{\mathbf{F}} \mathbf{\Sigma} \\ ||\mathbf{h}_d^{(1)}|| \mathbf{a}^H \end{bmatrix} \mathbf{V}^H \mathbf{V} \begin{bmatrix} s^{-1/2} \mathbf{\Sigma}_h \widetilde{\mathbf{F}} \mathbf{\Sigma} \\ ||\mathbf{h}_d^{(1)}|| \mathbf{a}^H \end{bmatrix}^H \begin{bmatrix} q_1 & \widetilde{q} \\ \widetilde{q}^H & q_2. \end{bmatrix} \right| \\ &= \log_2 \left| \mathbf{I} + \begin{bmatrix} s^{-1/2} ||\mathbf{h}|| \left[f_1 \sigma_1, \ldots, f_n \sigma_n \right] \\ ||\mathbf{h}_d^{(1)}|| \left[a_1^H, \ldots, a_n^H \right] \end{bmatrix} \begin{bmatrix} s^{-1/2} ||\mathbf{h}|| \left[f_1 \sigma_1, \ldots, f_n \sigma_n \right] \\ ||\mathbf{h}_d^{(1)}|| \left[a_1^H, \ldots, a_n^H \right] \end{bmatrix}^H \begin{bmatrix} q_1 & \widetilde{q} \\ \widetilde{q}^H & q_2 \end{bmatrix} \right|, \end{aligned} \tag{5.7}$$

where for a simpler notation we used $n = N_t^{(1)}$ and set $f_{N_f+1} = \ldots = f_{N_t^{(1)}} = 0$ for the case of $N_t^{(1)} > N_f$. These definitions allow us to give a reformulation of the sum-rate in the following lemma.

Lemma 6

A reformulation of the sum-rate under the aforementioned definitions and the constraints specified in (5.6) is given by

$$\begin{aligned} R_\Sigma = \log_2 \Bigg(& 1 + s^{-1} ||\mathbf{h}||^2 q_1 \Phi + ||\mathbf{h}_d^{(1)}||^2 q_2 + 2 \frac{||\mathbf{h}||^2 ||\mathbf{h}_d^{(1)}||^2 |\Psi|^2 (q_1 - q_2)}{||\mathbf{h}||^2 \Phi - ||\mathbf{h}_d^{(1)}||^2 s} \\ & + s^{-1} ||\mathbf{h}||^2 ||\mathbf{h}_d^{(1)}||^2 \left(\Phi - |\Psi|^2 \right) \left(q_1 q_2 - \frac{s ||\mathbf{h}||^2 ||\mathbf{h}_d^{(1)}||^2 |\Psi|^2 (q_1 - q_2)^2}{\left(||\mathbf{h}||^2 \Phi - ||\mathbf{h}_d^{(1)}||^2 s \right)^2} \right) \Bigg), \end{aligned} \tag{5.8}$$

where $\Phi \triangleq \sum_{i=1}^n |f_i|^2 \sigma_i^2$ and $\Psi \triangleq \sum_{i=1}^n f_i \sigma_i a_i$.

Proof: *The proof is given in Appendix B.1.* ■

In the following, (5.8) shall be used to optimize the upper bound. This is done by an algorithm which optimizes $f_1, \ldots, f_n$ by a brute-force method first and then uses the classical single-user water-filling method to find the optimal covariance matrix $\widetilde{\mathbf{Q}}^{(1)}$.

However, contrary to the MISO MARC without direct links, the optimal structure of $\mathbf{F}$ is unknown, especially $\widetilde{\mathbf{F}}$ is not diagonal. Moreover, $f_1, \ldots, f_n$ can be complex, such that not only their absolute value but also their phases are subject to optimization. Therefore, we introduce the notation

$$f_i \triangleq |f_i|\, e^{j\theta_i} \quad \forall i = 1, \ldots, n$$

to denote absolute value and phase separately. Moreover, we denote the phase of a complex number a by $\angle(a)$, i.e., $\angle(f_i) = \theta_i$. As it can be seen from (5.6), the absolute values are restricted by the relay power constraints, while the phases can be chosen freely. However, optimizing both the phases and the absolute values in a brute-force fashion requires an enormous complexity. Therefore, considering that the phases only influence $|\Psi|$ but not Φ, we give the possible optimal solutions for $\theta_1, \ldots, \theta_n$ and the corresponding value of $|\Psi|$ in the following theorem.

Theorem 7

Consider the optimization problem

$$\max_{\theta_1,\ldots,\theta_n} \qquad R_\Sigma,$$

where for R_Σ we consider the expression from (5.8) and assume all variables except $\theta_1, \ldots, \theta_n$ are fixed. Then the optimal solution of this problem is to choose $\theta_1, \ldots, \theta_n$ such that $|\Psi|$ is either minimized or maximized, which is achieved by one of the following choices of $\theta_1, \ldots, \theta_n$:

- $\theta_i = -\angle(a_i) \;\; \forall i$, *such that* $|\Psi| = \sum_{i=1}^{n} |f_i a_i| \sigma_i$.
- $\theta_1, \ldots, \theta_n$ *are chosen such that* $|\Psi| = 0$.
- $\theta_k = -\angle(a_k)$ *for* $k = \arg\max_l |f_l a_l| \sigma_l$ *and* $\theta_i = -\angle(a_i) + \pi \;\forall i \neq k$*, such that* $|\Psi| = 2\max_k |f_k a_k| \sigma_k - \sum_{i=1}^{n} |f_i a_i| \sigma_i$.

From these choices, the first one maximizes $|\Psi|$ and the latter two minimize $|\Psi|$, where the choice that sets $|\Psi| = 0$ is only feasible if $\max_k |f_k a_k| \sigma_k \leq \frac{1}{2} \sum_{i=1}^{n} |f_i a_i| \sigma_i$ and $n > 2$. The third choice is only made if one of these condition does not hold.

Proof: *The proof is given in Appendix B.2.* ■

Although the above theorem does not give a general solution for the phases, it reduces the number of possible solutions to three, which is a significant simplification compared to a brute-force search over all possible values. Using this simplification, we can construct Algorithm 4, which is given here in terms of pseudo code. This algorithm divides the relay power by the parameters $\beta_1, \ldots, \beta_n$, where $\beta_i = |f_i|^2 P_r^{-1}$. As an example, we assume $N_f = 3$ and $N_t^{(1)} \geq 3$ in this algorithm. The algorithm can be extended to larger values of N_f and $N_t^{(1)}$. But of course, due to the brute-force search for the optimal values of β_i, the complexity grows exponentially.

Algorithm 4 Upper bound for the MISO single-user relay channel with direct links

for $\beta_1 = 0 : \Delta_\beta : 1$ **do**
  for $\beta_2 = 0 : \Delta_\beta : 1 - \beta_1$ **do**
    $\beta_3 = 1 - \beta_1 - \beta_2$
    for $i = 1 : 3$ **do**
      $f_i = \sqrt{\beta_i P_r} e^{-j\angle(a_i)}$
    end for
    $\mathbf{F} = \mathbf{U}_h \widetilde{\mathbf{F}} \mathbf{U}^H$
    $\mathbf{H}_{\text{eff}}^{(1)} = \begin{bmatrix} s^{-1/2}\mathbf{h}^H \mathbf{F} \mathbf{H}_r^{(1)} \\ \mathbf{h}_d^{(1)H} \end{bmatrix}$
    $\widetilde{\mathbf{Q}}^{(1)} = \mathtt{WF}\left(\mathbf{H}_{\text{eff}}^{(1)}, P^{(1)}\right)$
    $R_{\Sigma,1} = \log_2 \left(\mathbf{I} + \mathbf{H}_{\text{eff}}^{(1)} \mathbf{H}_{\text{eff}}^{(1)H} \widetilde{\mathbf{Q}}^{(1)}\right)$
    $k = \arg\max_l |f_l a_l| \sigma_l$
    if $|f_k a_k| \sigma_k \geq \frac{1}{2} \sum_{i=1}^{n} |f_i a_i| \sigma_i$ **then**
      for $i = 1 : 3$ **do**
        $f_i = -f_i$
      end for
      $f_k = -f_k$
    else
      Calculate $\theta_1, \ldots, \theta_k$ such that $|\Psi| = 0$ (see Appendix B.2)
      for $i = 1 : 3$ **do**
        $f_i = \sqrt{\beta_i P_r} e^{-j\theta_i}$
      end for
    end if
    $\mathbf{F} = \mathbf{U}_h \widetilde{\mathbf{F}} \mathbf{U}^H$
    $\mathbf{H}_{\text{eff}}^{(1)} = \begin{bmatrix} s^{-1/2}\mathbf{h}^H \mathbf{F} \mathbf{H}_r^{(1)} \\ \mathbf{h}_d^{(k)H} \end{bmatrix}$
    $\widetilde{\mathbf{Q}}^{(1)} = \mathtt{WF}\left(\mathbf{H}_{\text{eff}}^{(1)}, P^{(1)}\right)$
    $R_{\Sigma,2} = \log_2 \left(\mathbf{I} + \mathbf{H}_{\text{eff}}^{(1)} \mathbf{H}_{\text{eff}}^{(1)H} \widetilde{\mathbf{Q}}^{(1)}\right)$
    $\widetilde{R}_\Sigma(\beta_1, \beta_2) = \max(R_{\Sigma,1}, R_{\Sigma,2})$
  end for
end for
$R_\Sigma = \max_{\beta_1,\beta_2} \widetilde{R}_\Sigma(\beta_1, \beta_2)$

As it can be seen in the algorithm, the absolute values of f_i are fixed at the beginning by the brute-force search over $\beta_1, \ldots, \beta_n$. Subsequently, it is checked which of the three possible phase selections from Theorem 7 provides the best rate. The phase selection of $\theta_i = -\angle(a_i)$, which maximizes $|\Psi|$ is assigned in lines 4-6. Afterwards the resulting effective channel is calculated and the single-user water-filling algorithm from Appendix A.2, referred to as `WF`, is executed to compute the optimal covariance matrix $\widetilde{\mathbf{Q}}^{(1)}$. Finally, the achievable sum-rate $R_{\Sigma,1}$ with this selection of $\theta_1, \ldots, \theta_n$ is computed. In lines 11-26 this procedure is repeated to compute the rate $R_{\Sigma,2}$, which is achieved by choosing the phases such that $|\Psi|$ is minimized. For this purpose it is checked in line 12 whether it is possible to achieve $|\Psi| = 0$. If this is not the case, we set f_i to $-f_i$ for all $i \neq k$. Otherwise, the phases are chosen as explained in the proof of Theorem 7 in Appendix B.2. While the sum-rate for the current selection of β_1, β_2 is computed as the maximum of $R_{\Sigma,1}$ and $R_{\Sigma,2}$, the final upper bound is calculated as the maximum over all β_1, β_2 in the final line of the algorithm.

Although this derived upper bound is only valid for $K = 1$, it is not only useful for single-user channels. If the number of antennas is not too large, it can be combined with TDMA to be used also in the "real" MISO MARC. However, a closed-form optimal solution for the duration of the time slots does not seem feasible. Hence, this solution needs to be approximated in a brute-force fashion as well.

5.1.2 Upper Bounds for Joint Relaying

If we consider joint relaying instead of the TDMA-based approach, the upper bound derived in the preceding section can not be used. Even worse, it does not seem feasible to obtain a bound for joint relaying in a similar way. The reason for this is that in the sum-rate expression the rule $|\mathbf{I} + \mathbf{AB}| = |\mathbf{I} + \mathbf{BA}|$ can not be applied. Moreover, the determinant inside the logarithm contains a sum and multiple covariance matrices, which makes it harder to bring it in a tractable scalar form. Therefore, we present two upper bounds in this subsection. The first one is obtained by an algorithm which follows a similar approach as Algorithm 4, while the second one is based on an analytic reformulation of the sum-rate.

For both bounds, we fix $\mathbf{F} = \mathbf{U}_h \widetilde{\mathbf{F}}$ as structure of the relay matrix, where $\mathbf{U}_h \mathbf{\Sigma}_h \mathbf{V}_h^H$ is still the SVD of $\mathbf{h}$. As for the single-user bound, $\mathbf{\Sigma}_h$ contains only one eigenvalue. From (5.1) and (5.2), it is obvious that choosing $\widetilde{\mathbf{F}}$ as in (5.5) is sufficient.

For the first bound, we use the relaxed relay power constraint (5.3) as in the single-user bound. The main problem in adapting Algorithm 4 for $K > 1$ is that it seems not feasible to derive something similar to Theorem 7. Hence, the optimal phases can not be computed and have to be approximated in a brute-force fashion, such that we obtain the code which is listed in Algorithm 5 for the case of $N_f = 3$.

Again, we use $\beta_i = |f_i|^2 P_r^{-1}$ to distribute the relay power between the variables $f_1, \ldots, f_n$. Additionally, there are now three for-loops for choosing the optimal phases

Algorithm 5 Upper bound for the MISO MARC with direct links

for $\beta_1 = 0 : \Delta_\beta : 1$ **do**
for $\beta_2 = \beta_1 : \Delta_\beta : 1$ **do**
$\beta_3 = 1 - \beta_1 - \beta_2$
for $\theta_1 = 0 : \Delta_\theta : \pi$ **do**
for $\theta_2 = 0 : \Delta_\theta : \pi$ **do**
for $\theta_3 = 0 : \Delta_\theta : \pi$ **do**
for $i = 1 : 3$ **do**
$f_i = \sqrt{\beta_i P_r} e^{-j\theta_i}$
end for
$\mathbf{F} = \mathbf{U}_h \widetilde{\mathbf{F}}$
for $k = 1 : K$ **do**
$\mathbf{H}_{\text{eff}}^{(k)} = \begin{bmatrix} s^{-1/2} \mathbf{h}^H \mathbf{F} \mathbf{H}_r^{(k)} \\ {\mathbf{h}_d^{(k)}}^H \end{bmatrix}$
end for
$\left[\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}\right] = \texttt{IWF}\left(\left[\mathbf{H}_{\text{eff}}^{(1)}, \ldots, \mathbf{H}_{\text{eff}}^{(K)}\right], \left[P^{(1)}, \ldots, P^{(K)}\right]\right)$
$\widetilde{R}_\Sigma\left(\beta_1, \beta_2, \theta_1, \theta_2, \theta_3\right) = \log_2\left(\mathbf{I} + \sum_{k=1}^{K} \mathbf{H}_{\text{eff}}^{(k)} \mathbf{Q}^{(k)} {\mathbf{H}_{\text{eff}}^{(k)}}^H\right)$
end for
end for
end for
end for
end for
$R_\Sigma = \max_{\beta_1, \beta_2, \theta_1, \theta_2, \theta_3} \widetilde{R}_\Sigma\left(\beta_1, \beta_2, \theta_1, \theta_2, \theta_3\right)$

by brute force. By these loops the possible intervals for the absolute values and phases are scanned with accuracy Δ_β and Δ_θ, respectively, for the optimal solution. As soon as $\mathbf{F}$ is set to the current value, the effective channel matrices $\mathbf{H}_{\text{eff}}^{(k)}$ are calculated in lines 11-13. Subsequently, the optimal transmit covariance matrices for the current choice of $\mathbf{F}$ are determined by Algorithm 1, which is abbreviated by `IWF` here. This allows a calculation of the achievable rate for the current selection of the absolute values and phases of f_i. The final upper bound is again calculated as maximum over all those values in the last line of the algorithm.

The second upper bound is derived in the following theorem.

Theorem 8

An upper bound to the achievable sum-rate of the MISO MARC with direct links is given by

$$R_\Sigma \leq \log_2 \left(\left(1 + \sum_{k=1}^{K} ||\mathbf{h}_d^{(k)}||^2 P^{(k)} \right) \left(1 + \frac{||\mathbf{h}||^2 P_r \lambda_{max}\left(\widetilde{\mathbf{R}}\right)}{1 + ||\mathbf{h}||^2 P_r + \lambda_{max}\left(\widetilde{\mathbf{R}}\right)} \right) \right),$$

where $\widetilde{\mathbf{R}} = \sum_{k=1}^{K} \mathbf{H}_r^{(k)} \mathbf{H}_r^{(k)^H} P^{(k)}$.

Proof: *Given the abbreviations*

$$\mathbf{T}^{(k)} \triangleq \mathbf{H}_r^{(k)} \mathbf{Q}^{(k)^{1/2}} \tag{5.9}$$

$$\mathbf{w}^{(k)} \triangleq \mathbf{Q}^{(k)^{H/2}} \mathbf{h}_d^{(k)}. \tag{5.10}$$

and using $\widetilde{\mathbf{f}}^H = [f_1, \ldots, f_n]$ *to denote the first row of* $\widetilde{\mathbf{F}}$, *we can write*

$$\begin{aligned}
R_\Sigma &= \log_2 \left| \mathbf{I} + \sum_{k=1}^{K} \begin{bmatrix} s^{-1/2}\mathbf{h}^H \mathbf{F} \mathbf{H}_r^{(k)} \\ \mathbf{h}_d^{(k)^H} \end{bmatrix} \mathbf{Q}^{(k)} \begin{bmatrix} s^{-1/2}\mathbf{h}^H \mathbf{F} \mathbf{H}_r^{(k)} \\ \mathbf{h}_d^{(k)^H} \end{bmatrix}^H \right| \\
&= \log_2 \begin{vmatrix} 1 + s^{-1}||\mathbf{h}||^2 \widetilde{\mathbf{f}}^H \sum_{k=1}^{K} \mathbf{T}^{(k)} \mathbf{T}^{(k)^H} \widetilde{\mathbf{f}} & s^{-1/2}||\mathbf{h}|| \widetilde{\mathbf{f}} \sum_{k=1}^{K} \mathbf{T}^{(k)} \mathbf{w}^{(k)} \\ \sum_{k=1}^{K} \mathbf{w}^{(k)^H} \mathbf{T}^{(k)^H} \widetilde{\mathbf{f}} ||\mathbf{h}|| s^{-1/2} & 1 + \sum_{k=1}^{K} \mathbf{w}^{(k)^H} \mathbf{w}^{(k)} \end{vmatrix} \\
&= \log_2 \left(1 + \sum_{k=1}^{K} \mathbf{w}^{(k)^H} \mathbf{w}^{(k)} + s^{-1} ||\mathbf{h}||^2 \widetilde{\mathbf{f}}^H \mathbf{D} \widetilde{\mathbf{f}} \right),
\end{aligned} \tag{5.11}$$

where

$$\mathbf{D} = \left(1 + \sum_{k=1}^{K} \mathbf{w}^{(k)^H} \mathbf{w}^{(k)} \right) \sum_{k=1}^{K} \mathbf{T}^{(k)} \mathbf{T}^{(k)^H} - \left(\sum_{k=1}^{K} \mathbf{T}^{(k)} \mathbf{w}^{(k)} \right) \left(\sum_{k=1}^{K} \mathbf{w}^{(k)^H} \mathbf{T}^{(k)^H} \right). \tag{5.12}$$

The upper bound on R_Σ is now established by observing that

$$\mathbf{w}^{(k)H}\mathbf{w}^{(k)} \leq ||\mathbf{h}_d^{(k)}||^2 P^{(k)}$$

$$\mathbf{D} \preceq \left(1 + \sum_{k=1}^{K} \mathbf{w}^{(k)H}\mathbf{w}^{(k)}\right) \sum_{k=1}^{K} \mathbf{T}^{(k)}\mathbf{T}^{(k)H}.$$

While the first inequation is obtained by using Lemma 2 from Subsection 4.2.1, the second inequation is due to the fact that the subtracted matrix in the definition of $\mathbf{D}$ *is a product of two matrices that are mutually Hermitian. Therefore, the subtracted matrix term is positive semidefinite. Hence, we can formulate the upper bound as*

$$R_\Sigma \leq \log_2\left(\left(1 + \sum_{k=1}^{K} ||\mathbf{h}_d^{(k)}||^2 P^{(k)}\right)\left(1 + s^{-1}||\mathbf{h}||^2 \widetilde{\mathbf{f}}^H \left(\sum_{k=1}^{K} \mathbf{T}^{(k)}\mathbf{T}^{(k)H}\right)\widetilde{\mathbf{f}}\right)\right).$$

In this expression, only the last term is subject to optimization. As $\sum_{k=1}^{K} \mathbf{T}^{(k)}\mathbf{T}^{(k)H} = \sum_{k=1}^{K} \mathbf{H}_r^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_r^{(k)H} = \mathbf{R}$, *it can be seen that the optimization of this expression is equivalent to the optimization for the MISO MARC without direct links from (4.2). Adapting the corresponding optimal relay matrix from (4.3) it follows that*

$$\widetilde{\mathbf{f}}^H = \sqrt{\frac{P_r}{1 + \lambda_{max}(\mathbf{R})}}\mathbf{v}_{max}^H(\mathbf{R}).$$

When also the optimal covariance matrices from (4.4) are used, it follows that the upper bound is given by the expression in the theorem. ■

Although this bound is clearly not tight, it is a reasonable addition to the preceding bound computed with Algorithm 5. As the preceding bound relaxes the power constraint, it is good especially for high values of P_r. However, for lower values of P_r the overestimation of the relay power makes the bound very loose. On the contrary, the second bound overestimates the terms in the sum-rate formula. This makes the bound loose at high P_r, while at low P_r it is much closer to the actually achievable rates than the first bound.

5.2 Algorithms to Approach the Optimal Sum-Rate

In this section, we consider strategies to compute rates that are definitely achievable in the MISO MARC with direct links. The main problem for these strategies is that the optimization problem (5.2) becomes very challenging when the relay power constraint is not relaxed as in the preceding section. The reason for this is that the covariance matrices also appear in the relay power constraint. This makes it impossible to select $\mathbf{F}$ by brute-force methods and find $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ separately by water-filling as in the preceding section, because $\mathbf{F}$ and $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ are now strongly interdependent.

Therefore, we consider two different approaches to the problem. As a first approach, we try to modify the brute-force approach of Algorithm 5, such that it can be used when all constraints are considered. Although this algorithm is theoretically capable of finding the optimal solution, its complexity is too high to achieve a reasonable accuracy. The reasons for this are explained in Subsection 5.2.1. A different philosophy is followed by the algorithm presented in Subsection 5.2.2, which is possibly suboptimal but keeps the computational complexity low. It uses a heuristic approach and is designed for use with joint relaying. However, with some modifications, which are explained at the end of Subsection 5.2.2, it can also be used with TDMA.

5.2.1 On the Complexity of Brute-Force Algorithms

Contrary to the upper bounds calculated in the preceding section, a calculation of achievable rates by brute-force methods seems infeasible, because the complexity becomes too high. The main reason for this is that, if the relay power constraint has to be fulfilled in its original version, there is a interdependence between $\mathbf{F}$ and $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ which makes it hard to find the solution of the relay or the covariance matrices without considering the other matrices.

To our knowledge, no efficient possibility of calculating the optimal covariance matrices exists, even if $\mathbf{F}$ is fixed. This is due to the existence of both individual and sum power constraints on the covariance matrices. As the Algorithms 1 and 2 support either sum or individual power constraints but not both together, they can not be used. Even worse, it is not clear whether the optimal covariance matrices diagonalize the determinant of the sum-rate, such that a brute-force search over $\mathbf{Q}$ would be very complex.

Also the other part of the solution, finding the optimal relay matrix $\mathbf{F}$, is harder although setting $\mathbf{F} = \mathbf{U}_h\widetilde{\mathbf{F}}$ with $\widetilde{\mathbf{F}}$ as defined in (5.5) can be used without loss of generality. In general, even for single-user channels, Lemma 6 and Theorem 7 do not hold, such that there is no solution how to optimally chose the phases of $f_1, \ldots, f_n$. Furthermore, f_n can not be computed from $f_1, \ldots, f_{n-1}$ as in Algorithm 5, such that the number of parameters that has to be found by brute-force increases.

Considering all these properties, we present an approach that could find the optimal solution of our problem. However, the complexity is so high, that the brute-force search has to scan the search space with very big steps. Therefore, the approximation of the solution is too imprecise to be considered as a benchmark. Nevertheless, we briefly explain the idea of the algorithm here. By either improving the algorithm or by better hardware, it might be possible to compute solutions with this algorithm in the future.

From the basic structure, this brute-force algorithm is the same as Algorithm 5. Therefore, we use the same assumptions and definitions as in Subsection 5.1.2 (except the relaxed relay power constraint) and describe only the differences to Algorithm 5. These are:

- As mentioned above, β_3 can not be computed as $\beta_3 = 1 - \beta_1 - \beta_2$. Therefore, line 3 in Algorithm 5 is replaced by a further for-loop to calculate also β_3 in a brute-force fashion. However, the range of β_3 is $[0, 1 - \beta_1 - \beta_2]$ instead of $[0, 1]$.

- As also described above, Algorithm 1 (abbreviated by `IWF` in Algorithm 5) can not be used any more because the relay power constraint has to be considered as well when computing the covariance matrices. Fortunately, the resulting optimization problem is at least convex. Therefore, line 14 in Algorithm 5 is replaced by a convex optimization program which computes the optimal solution of $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$.

- Of course, R_Σ is now also a function of β_3, which has to be considered in the final maximization.

The main issue that makes the complexity enormous is of course the convex optimization, which is not as efficient as the iterative Algorithms 1 and 2 and executed for each possible combination of absolute values and phases of $f_1, \ldots, f_n$. While optimization of the convex program or solver (in this case CVX [CVX12]) is not expected to reduce the performance significantly, the introduction of an iterative algorithm that converges to the optimal solution, if possible, would be much more beneficial.

5.2.2 An Efficient Iterative Algorithm

As finding the optimal solution with brute-force methods seems infeasible, even for a low number of users and antennas, we present a heuristic iterative algorithm to find a possibly suboptimal solution in this section. The targets of this algorithm are a low computational complexity as well as a performance that is as close as possible to the optimal solution. As the optimal solution is not known, we will compare the achieved rates with the derived upper bounds in Section 5.4.

The algorithm is given in terms of pseudo code in Algorithm 6. The basic strategy of the algorithm is to alternately optimize $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ and $\mathbf{F}$ until they converge. As the optimal solution of $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ is hard to find, a suboptimal solution is found by ignoring the relay power constraint. Thus, Algorithm 1 (abbreviated by `IWF`) can be used to efficiently find the covariance matrices in line 6. However, as the relay power constraint might be violated, $\mathbf{F}$ has to be rescaled by a scalar. This is done in line 8, where the root at the end of the expression ensures that the relay power constraint is fulfilled with equality. Moreover, the general structure of $\mathbf{F}$ is heuristically chosen as the outer product of $\mathbf{h}$ and the eigenvector corresponding to the maximum eigenvalue of $\mathbf{D}$. Using the notation from the previous subsection, this is equivalent to setting $\widetilde{\mathbf{f}}^H = \mathbf{v}_{\max}^H(\mathbf{D})\,\gamma$, where

$$\gamma = \sqrt{\frac{P_r}{\operatorname{tr}\left(\mathbf{v}_{\max}^H(\mathbf{D})\left(\mathbf{I}+\mathbf{R}\right)\mathbf{v}_{\max}(\mathbf{D})\right)}}.$$

Algorithm 6 Iterative algorithm for the MISO MARC with direct links

Initialize: $\mathbf{Q}^{(k)} = \mathbf{I}\frac{P^{(k)}}{N_t^{(k)}}\ \forall k$; $\mathbf{F} = \mathbf{I}\sqrt{\frac{P_r}{\mathrm{tr}(\mathbf{I}+\mathbf{R})}}$
for k=1:K **do**
  $\mathbf{H}_{\text{eff}}^{(k)} = \begin{bmatrix} s^{-1/2}\mathbf{h}^H\mathbf{F}\mathbf{H}_r^{(k)} \\ \mathbf{h}_d^{(k)H} \end{bmatrix}$
end for
repeat
  $\left[\mathbf{Q}^{(1)}, \dots, \mathbf{Q}^{(K)}\right] = \mathtt{IWF}\left(\left[\mathbf{H}_{\text{eff}}^{(1)}, \dots, \mathbf{H}_{\text{eff}}^{(K)}\right], \left[P^{(1)}, \dots, P^{(K)}\right]\right)$
  Calculate $\mathbf{T}^{(k)}$, $\mathbf{w}^{(k)}$, and $\mathbf{D}$ as given in (5.9),(5.10), and (5.12)
  $\mathbf{F} = \frac{\mathbf{h}}{||\mathbf{h}||}\mathbf{v}_{\max}^H(\mathbf{D})\sqrt{\frac{P_r}{\mathrm{tr}(\mathbf{v}_{\max}^H(\mathbf{D})(\mathbf{I}+\mathbf{R})\mathbf{v}_{\max}(\mathbf{D}))}}$
  for k=1:K **do**
    $\mathbf{H}_{\text{eff}}^{(k)} = \begin{bmatrix} s^{-1/2}\mathbf{h}^H\mathbf{F}\mathbf{H}_r^{(k)} \\ \mathbf{h}_d^{(k)H} \end{bmatrix}$
  end for
  $R_\Sigma = \log_2\left|\mathbf{I} + \sum_{k=1}^K \mathbf{H}_{\text{eff}}^{(k)}\mathbf{Q}^{(k)}\mathbf{H}_{\text{eff}}^{(k)H}\right|$
until sum-rate convergence or decline

The reasons for this choice of $\mathbf{F}$ is seen when considering the last line of (5.11). If $\mathbf{Q}^{(1)}, \dots, \mathbf{Q}^{(K)}$ are fixed and P_r is large, we can write

$$\lim_{P_r\to\infty} s^{-1}||\mathbf{h}||^2\widetilde{\mathbf{f}}^H\mathbf{D}\widetilde{\mathbf{f}} = \lim_{P_r\to\infty}\frac{||\mathbf{h}||^2\widetilde{\mathbf{f}}^H\mathbf{D}\widetilde{\mathbf{f}}}{1+||\mathbf{h}||^2\widetilde{\mathbf{f}}^H\widetilde{\mathbf{f}}} = \frac{\widetilde{\mathbf{f}}^H\mathbf{D}\widetilde{\mathbf{f}}}{\widetilde{\mathbf{f}}^H\widetilde{\mathbf{f}}} \le \lambda_{\max}(\mathbf{D}),$$

where the last inequality is due to the Rayleigh quotient and we have equality for $\widetilde{\mathbf{f}}^H = \mathbf{v}_{\max}(\mathbf{D})$. Thus, for the given covariance matrices this choice of $\mathbf{F}$ is optimal when P_r tends to infinity.

As it can be seen, the complexity of Algorithm 6 does not scale exponentially with the number of antennas as the brute-force approaches. Also the number of users only influences the complexity of `IWF`, which is low anyway. A much more important point for the complexity is the speed of convergence of the solutions. As the choices of $\mathbf{F}$ and $\mathbf{Q}^{(1)}, \dots, \mathbf{Q}^{(K)}$ are heuristic, an analytic proof of convergence seems infeasible. However, the termination of Algorithm 6 is ensured by stopping it as soon as R_Σ decreases. Concerning the speed of convergence, we will show by means of simulation results in Section 5.4 that the algorithm converges very fast and that already after one iteration, the achieved rates are quite good.

Although Algorithm 6 is intended for joint relaying it can easily be redesigned in order to use it in combination with TDMA. As TDMA turned out to be superior in channels without direct links, this might be beneficial for some cases with direct links as well. However, the additional optimization of the duration of the time slots $\tau^{(1)}, \ldots, \tau^{(k)}$ is not as easy as in the MARC without direct links. In detail, it seems infeasible to obtain a closed-form solution for the optimal values of $\tau^{(1)}, \ldots, \tau^{(k)}$. Therefore, we approximate the optimal values by a brute-force search. If the number of users is not too large, the complexity of the resulting algorithm still remains reasonable.

In detail, Algorithm 6 can be redesigned for use with TDMA by the following changes:

- Outer loops are added, which select the possible values of $\tau_1, \ldots, \tau^{(K)}$ between 0 and 1 with intervals of Δ_τ in a way that $\sum_{k=1}^{K} \tau^{(k)} = 1$.
- As each user is considered separately and has a separate relaying matrix $\mathbf{F}^{(k)}$, a further outer loop is added that iterates over the users k. In return the two for-loops to calculate $\mathbf{H}_{\text{eff}}^{(k)}$ are no longer necessary.
- The use of Algorithm 1 (`IWF`) in line 6 can be replaced by a simpler single-user water-filling as explained in Appendix A.2, which is not iterative and therefore reduces the complexity.
- Due to the partial inactivity of the users, the transmit powers $P^{(1)}, \ldots, P^{(K)}$ can be replaced by $\frac{P^{(1)}}{\tau^{(1)}}, \ldots, \frac{P^{(K)}}{\tau^{(K)}}$. Also the rates have to be multiplied with $\tau^{(k)}$.
- Finally, the best sum-rate is found by a maximization over all tested values of $\tau_1, \ldots, \tau^{(K)}$.

Overall, from a computational complexity point of view, it can be stated that the use of TDMA is not as desirable as in channels without direct links. The reason for this is that especially the brute-force search of the optimal duration of the time slots makes the computation time of the TDMA version of Algorithm 6 much longer than in its original version. Whether the performance of TDMA is better at all in channels with direct links will be discussed in more detail in the following sections.

5.3 A Simplified Case: Single-Antenna Transmitters

An interesting special case is obtained if also the transmitters have only a single antenna, i.e., a SISO MARC is considered. Although this case is less general, the optimal rates are easier to obtain. The reason for this is that the covariance matrices $\mathbf{Q}^{(1)}, \ldots, \mathbf{Q}^{(K)}$ are no longer subject to optimization because they are simply the scalar power values $P^{(1)}, \ldots, P^{(K)}$.

For joint relaying this means that the iteration between $\mathbf{F}$ and the covariance matrices in Algorithm 6 is no longer necessary. Instead, the heuristic choice of $\mathbf{F}$ is made only once and the achievable rate is directly calculated without any iterations. Moreover, as we will see in Section 5.4, the upper bounds and the achievable rates become much tighter.

If TDMA is used, the simplifications obtained by the single transmit antennas have even higher effects, such that it is possible to compute the optimal transmit strategy with a very efficient iterative algorithm. Using the same steps as in (5.11), the rate of user k can be written as

$$R_{\text{TDMA}}^{(k)} = \tau^{(k)} \log_2 \left(1 + |h_d^{(k)}|^2 \frac{P^{(k)}}{\tau^{(k)}} + \frac{\mathbf{h}^H \mathbf{F}^{(k)} \mathbf{h}_r^{(k)} \frac{P^{(k)}}{\tau^{(k)}} {\mathbf{h}_r^{(k)}}^H {\mathbf{F}^{(k)}}^H \mathbf{h}}{1 + \mathbf{h}^H \mathbf{F}^{(k)} {\mathbf{F}^{(k)}}^H \mathbf{h}} \right).$$

As the first two terms inside the logarithm do not depend on $\mathbf{F}$ and the optimization of the third term is similar to the one discussed in Subsection 4.2.1, the optimal solution is similar to the one given in (4.3). Hence, in this case, the optimal $\mathbf{F}$ is given by

$$\mathbf{F}^{(k)} = \frac{\mathbf{h} {\mathbf{h}_r^{(k)}}^H}{||\mathbf{h}|| ||\mathbf{h}_r^{(k)}||} \sqrt{\frac{P_r}{1 + ||\mathbf{h}_r^{(k)}||^2 \frac{P^{(k)}}{\tau^{(k)}}}}$$

and the optimal rate of user k can be written as

$$R_{\text{TDMA}}^{(k)} = \tau^{(k)} \log_2 \left(1 + |h_d^{(k)}|^2 \frac{P^{(k)}}{\tau^{(k)}} + \frac{||\mathbf{h}||^2 ||\mathbf{h}_r^{(k)}||^2 \frac{P^{(k)}}{\tau^{(k)}} P_r}{1 + ||\mathbf{h}||^2 P_r + ||\mathbf{h}_r^{(k)}||^2 \frac{P^{(k)}}{\tau^{(k)}}} \right). \tag{5.13}$$

As in the MARC without direct links it remains to find the optimal duration of the time slots that maximize the sum-rate $R_{\Sigma,\text{TDMA}} = \sum_{k=1}^{K} R_{\text{TDMA}}^{(k)}$, which is given by the following optimization problem (cf. Theorem 4)

$$\max_{\tau^{(1)},\ldots,\tau^{(K)}} R_{\Sigma,\text{TDMA}}(\boldsymbol{\tau})$$
$$\text{s.t.} \;\; h(\boldsymbol{\tau}) = 1 - \sum_{k=1}^{K} \tau^{(k)} = 0,$$

where $\boldsymbol{\tau} = \left[\tau^{(1)}, \ldots \tau^{(K)}\right]$. The derivative of $R_{\Sigma,\text{TDMA}}(\boldsymbol{\tau})$ can be calculated as

$$\frac{\partial R_{\Sigma,\text{TDMA}}}{\partial \tau^{(k)}} = \frac{\partial R_{\text{TDMA}}^{(k)}}{\partial \tau^{(k)}} = \log_2 \left(1 + |h_d^{(k)}|^2 \frac{P^{(k)}}{\tau^{(k)}} + \frac{||\mathbf{h}||^2 ||\mathbf{h}_r^{(k)}||^2 \frac{P^{(k)}}{\tau^{(k)}} P_r}{1 + ||\mathbf{h}||^2 P_r + ||\mathbf{h}_r^{(k)}||^2 \frac{P^{(k)}}{\tau^{(k)}}} \right) +$$
$$\log_2(e) \frac{-\frac{|h_d^{(k)}|^2 P^{(k)}}{{\tau^{(k)}}^2} - \frac{||\mathbf{h}||^2 P^{(k)} ||\mathbf{h}_r^{(k)}||^2 P_r \left(1 + ||\mathbf{h}||^2 P_r\right)}{\left(||\mathbf{h}||^2 P_r \tau^{(k)} + P^{(k)} ||\mathbf{h}_r^{(k)}||^2 + \tau^{(k)}\right)^2}}{1 + \frac{|h_d^{(k)}|^2 P^{(k)}}{\tau^{(k)}} + \frac{||\mathbf{h}^2|| P^{(k)} ||\mathbf{h}_r^{(k)}||^2 P_r}{||\mathbf{h}||^2 P_r \tau^{(k)} + P^{(k)} ||\mathbf{h}_r^{(k)}||^2 + \tau^{(k)}}}.$$

As the calculation of the second derivative results in a very lengthy expression, it is omitted here. However, it is straightforward to reformulate the second derivative such that $\frac{\partial^2 R_{\Sigma,\mathrm{TDMA}}}{\partial {\tau^{(k)}}^2} = \frac{\partial^2 R^{(k)}_{\mathrm{TDMA}}}{\partial {\tau^{(k)}}^2} < 0$ can be observed if $0 \leq \tau^{(k)} \leq 1 \; \forall k$, i.e., the problem is convex.

Hence, the KKT conditions provide necessary and sufficient conditions for a solution $\boldsymbol{\tau}^*$ with dual variable ν^* to be optimal. For this problem, these conditions can be written as

$$h(\boldsymbol{\tau}^*) = 0 \tag{5.14}$$

$$\nabla R_{\Sigma,\mathrm{TDMA}}(\boldsymbol{\tau}^*) + \nu^* \nabla h(\boldsymbol{\tau}^*) = 0 \tag{5.15}$$

Unfortunately, a closed-form solution can not be directly obtained from the derivatives as in Theorem 4 for the MARC without direct links. However, considering that $\frac{\partial h}{\partial \tau^{(k)}} = -1 \; \forall k$, it follows from (5.15) that the optimal solution has to fulfill the property

$$\left.\frac{\partial R^{(1)}_{\mathrm{TDMA}}}{\partial \tau^{(1)}}\right|_{\tau^{(1)} = {\tau^{(1)}}^*} = \ldots = \left.\frac{\partial R^{(K)}_{\mathrm{TDMA}}}{\partial \tau^{(K)}}\right|_{\tau^{(K)} = {\tau^{(K)}}^*} .$$

Moreover, it can be seen that the first derivatives are strictly monotonic decreasing with $\left.\frac{\partial R^{(k)}_{\mathrm{TDMA}}}{\partial \tau^{(k)}}\right|_{\tau^{(k)}=0} \to \infty$ and $\left.\frac{\partial R^{(k)}_{\mathrm{TDMA}}}{\partial \tau^{(k)}}\right|_{\tau^{(k)}=1} = 0 \; \forall k$. Therefore, the solution can always be found by an iterative algorithm, which is given in Algorithm 7 in terms of pseudo code.

Algorithm 7 Iterative optimization of ${\tau^{(k)}}^*$ for the MISO MARC with direct links

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Set ${\tau^{(k)}}^* = \frac{1}{K} \; \forall k = 1, \ldots, K$
while true **do**
  $i = \arg\min_k \left.\frac{\partial R^{(k)}_{\mathrm{TDMA}}}{\partial \tau^{(k)}}\right|_{\tau^{(k)} = {\tau^{(k)}}^*}$
  $j = \arg\max_k \left.\frac{\partial R^{(k)}_{\mathrm{TDMA}}}{\partial \tau^{(k)}}\right|_{\tau^{(k)} = {\tau^{(k)}}^*}$
  if $\left.\frac{\partial R^{(j)}_{\mathrm{TDMA}}}{\partial \tau^{(j)}}\right|_{\tau^{(j)} = {\tau^{(j)}}^*} - \left.\frac{\partial R^{(i)}_{\mathrm{TDMA}}}{\partial \tau^{(i)}}\right|_{\tau^{(i)} = {\tau^{(i)}}^*} > \varepsilon$ **then**
    Find $0 < \delta < \min\{\tau^{(i)}, 1 - \tau^{(j)}\}$, such that

$$\left.\frac{\partial R^{(j)}_{\mathrm{TDMA}}}{\partial \tau^{(j)}}\right|_{\tau^{(j)} = {\tau^{(j)}}^* + \delta} = \left.\frac{\partial R^{(i)}_{\mathrm{TDMA}}}{\partial \tau^{(i)}}\right|_{\tau^{(i)} = {\tau^{(i)}}^* - \delta}$$

    ${\tau^{(j)}}^* = {\tau^{(j)}}^* + \delta$
    ${\tau^{(i)}}^* = {\tau^{(i)}}^* - \delta$
  else
    break
  end if
end while

The main idea of the algorithm is to iteratively equalize the derivatives of $R_{\mathrm{TDMA}}^{(k)}$ by changing the duration of the time slots ${\tau^{(k)}}^*$. Therefore, the users i and j with the smallest and largest derivative are selected. Their derivatives are equalized by numerically finding a value δ, which is added to $\tau^{(j)}$ and subtracted from $\tau^{(i)}$. Due to the properties of the functions discussed above this value can always be found in the interval $(0, \min\{\tau^{(i)}, 1-\tau^{(j)}\})$, and the other derivatives remain unchanged. This procedure is repeated iteratively until the difference of the largest and smallest derivative is at most ε, which can be selected very small to approximate the optimal solution as good as desired.

Although the simplifications obtained by single-antenna transmitters allow a much simpler calculation of the achievable rates, a closed-form solution is obtained neither for joint relaying nor for TDMA. Hence, a direct analytic comparison as for the MARC without direct links can not be made. However, it is possible to derive the sum-rates that are achieved for $P_r \to \infty$ as closed-form solution. Although the relay will never have infinite power, it follows from the cut-set bounds that the sum-rates have to saturate for high values of P_r. Thus, the rates at infinite P_r, which are given in the theorem below provide a good approximation for finite but large enough values of P_r. Moreover, the following corollary allows a comparison of TDMA and joint relaying for $P_r \to \infty$.

Theorem 9

As $P_r \to \infty$, the achievable sum-rates of joint relaying and TDMA in the SISO MARC with direct links are given by

$$R_{\Sigma,\infty} = \log_2 \left(1 + \sum_{k=1}^{K} |h_d^{(k)}|^2 P^{(k)} + \lambda_{max}(\mathbf{D}) \right)$$

and

$$R_{\Sigma,TDMA,\infty} = \log_2 \left(1 + \sum_{k=1}^{K} P^{(k)} \left(|h_d^{(k)}|^2 + ||\mathbf{h}_r^{(k)}||^2 \right) \right),$$

respectively.

Proof: *The proof is given in Appendix B.3.* ■

Corollary 10

The superiority of TDMA obtained for the MARC without direct links persists in the SISO MARC with direct links at $P_r \to \infty$ if and only if

$$\sum_{k=1}^{K} ||\mathbf{h}_r^{(k)}||^2 P^{(k)} \geq \lambda_{max}\left(\mathbf{D}\right),$$

Proof: *The corollary follows by comparing the rate formulas of Theorem 9.* ■

A relation to Theorem 5 can be obtained by rewriting the expression for $\mathbf{D}$. Using (5.12) and considering that $\mathbf{w}^{(k)}$ reduces to a scalar $w^{(k)} = |\mathbf{h}_d^{(k)}|{P^{(k)}}^{1/2}$ and $\mathbf{T}^{(k)}$ reduces to a vector $\mathbf{t}^{(k)} = \mathbf{h}_r^{(k)}{P^{(k)}}^{1/2}$, we can write

$$\begin{aligned}
\mathbf{D} &= \left(1 + \sum_{k=1}^{K} {h_d^{(k)}}^H h_d^{(k)} P^{(k)}\right)\left(\sum_{k=1}^{K} \mathbf{h}_r^{(k)} {\mathbf{h}_r^{(k)}}^H P^{(k)}\right) \\
&\qquad\qquad - \left(\sum_{k=1}^{K} \mathbf{h}_r^{(k)} h_d^{(k)} P^{(k)}\right)\left(\sum_{k=1}^{K} {h_d^{(k)}}^H {\mathbf{h}_r^{(k)}}^H P^{(k)}\right) \\
&= \widetilde{\mathbf{R}} + \frac{1}{2}\sum_{l=1}^{K}\sum_{k=1}^{K}\Big(h_d^{(k)} {h_d^{(k)}}^H \mathbf{h}_r^{(l)} {\mathbf{h}_r^{(l)}}^H P^{(k)} P^{(l)} + h_d^{(l)} {h_d^{(l)}}^H \mathbf{h}_r^{(k)} {\mathbf{h}_r^{(k)}}^H P^{(k)} P^{(l)} \\
&\qquad\qquad - h_d^{(k)} {h_d^{(l)}}^H \mathbf{h}_r^{(k)} {\mathbf{h}_r^{(l)}}^H P^{(k)} P^{(l)} - h_d^{(l)} {h_d^{(k)}}^H \mathbf{h}_r^{(l)} {\mathbf{h}_r^{(k)}}^H P^{(k)} P^{(l)}\Big) \\
&= \widetilde{\mathbf{R}} + \mathbf{W},
\end{aligned}$$

where

$$\widetilde{\mathbf{R}} = \sum_{k=1}^{K} \mathbf{h}_r^{(k)} P^{(k)} {\mathbf{h}_r^{(k)}}^H \tag{5.16}$$

$$\mathbf{W} = \frac{1}{2}\sum_{l=1}^{K}\sum_{k=1}^{K}\left(h_d^{(l)}\mathbf{h}_r^{(l)} - h_d^{(k)}\mathbf{h}_r^{(k)}\right)\left(h_d^{(l)}\mathbf{h}_r^{(l)} - h_d^{(k)}\mathbf{h}_r^{(k)}\right)^H P^{(l)} P^{(k)}. \tag{5.17}$$

Obviously, for the case of zero direct links ($h_d^{(k)} = 0\ \forall k$), we obtain $\mathbf{D} = \widetilde{\mathbf{R}}$ and the condition of Corollary 10 is the same as in the proof of Theorem 5. This condition is always fulfilled, which shows the superiority of TDMA. Hence, Theorem 5 is a special case of Theorem 9 and Corollary 10. However, if the strength of the direct links is increased, $\mathbf{W}$ and thus also $\lambda_{\max}\left(\widetilde{\mathbf{R}} + \mathbf{W}\right)$ increases rapidly, which compensates the disadvantage of joint relaying. More details about the comparison of TDMA and joint relaying are given in the following section.

5.4 Simulation Results

In this section, we evaluate the achievable rates and the upper bounds for both joint relaying and TDMA by means of simulation results. The goal of these simulations is to obtain a quantitative estimation of the gap between achievable schemes and upper bounds as well as the gain or loss of TDMA compared to joint relaying. Although an existing gap between upper bounds and achievable schemes leaves some uncertainty about the exact rate, we will be able to state whether TDMA or joint relaying is better for many cases.

While the achievable rates are computed with Algorithm 6 and its modified version for TDMA as explained in Subsection 5.2.2, the upper bounds that will be plotted in the following figures are always the minimum of two upper bounds. For joint relaying, we use Algorithm 5 (with $\Delta_\beta = 0.02$ and $\Delta_\theta = 0.1\pi$) and Theorem 8 to derive upper bounds, where the former provides a tighter upper bound for high values of P_r and the latter is tighter for moderate and low values of P_r. For TDMA, we simply use the single-user upper bounds, which are obtained from Algorithm 4 (with $\Delta_\beta = 0.01$) and Theorem 8 with $K = 1$. These bounds are computed for every user, where the duration of the time slots $\tau^{(1)}, \ldots, \tau^{(K)}$ are set by an outer loop in a brute-force fashion. In detail, $\tau^{(1)}, \ldots, \tau^{(K)}$ are selected between 0 and 1 with intervals of $\Delta_\tau = 0.05$ such that $\sum_{k=1}^{K} \tau^{(k)} = 1$. Although the brute-force scheme only allows an approximation of the bounds, the precision can be made arbitrarily high by decreasing Δ_τ.

The further assumptions for the simulations are mostly the same as in the preceding sections. Thus, we consider independent Rayleigh fading channels, i.e., all entries of the channel matrices are $\sim \mathcal{CN}(0, 1)$ and independent from each other. An exception are the entries of the vectors $\mathbf{h}_d^{(k)}$. In order to be able to evaluate the influence of the strength of the direct links, their entries are independent but have a variance of σ_d^2. We assume that the number of users is $K = 2$ and that both users have the same number of antennas $N_t^{(1)} = N_t^{(2)} = N_t$ and the same transmit power $P^{(1)} = P^{(2)} = P$. Unless otherwise specified, we set the number of transmitter and relay antennas to $N_t = N_f = 3$, while, as we consider MISO channels, we have a single receive antenna. Furthermore, all results are obtained by averaging over 100 channel realizations and, as we assume unit noise variance, the SNRs at transmitter and relay are given by $\mathsf{SNR}_t = P$ and $\mathsf{SNR}_r = P_r$, respectively. Finally, in order to allow for a compact notation in the plots, we abbreviate the terms "upper bound" and "joint relaying" by "UB" and "JR", respectively.

First, we present the sum-rates that are achieved for the simplified case with $N_t = 1$, i.e., for single-antenna transmitters. As explained in Section 5.3, the TDMA rates can be computed exactly for this case with Algorithm 7, while for joint relaying we have an upper and lower bound. These rates are plotted in Figure 5.1 for $\mathsf{SNR}_t = 4$ dB and in Figure 5.2 for $\mathsf{SNR}_t = 16$ dB. As it can be seen, the upper and lower bounds of joint relaying are tight enough to make a clear comparison between joint relaying and TDMA. Obviously, the superiority of TDMA, which was found for channels without direct links in Section 4.2, persists only for the case of $\mathsf{SNR}_t = 4$ dB and $\sigma_d^2 = 0.1$. For all other cases joint relaying clearly outperforms TDMA, where the gap grows dramatically with the strength of the direct links and the relay power. Moreover, the gap also grows with the transmit power, but this influence is not as significant as the one of σ_d^2 and SNR_r.

A mathematical explanation for this observation is given in Theorem 9, where it can be seen that the matrix $\mathbf{W}$ grows enormously with the direct links, which rapidly compensates the disadvantage of joint relaying if the direct links are zero. A more in-

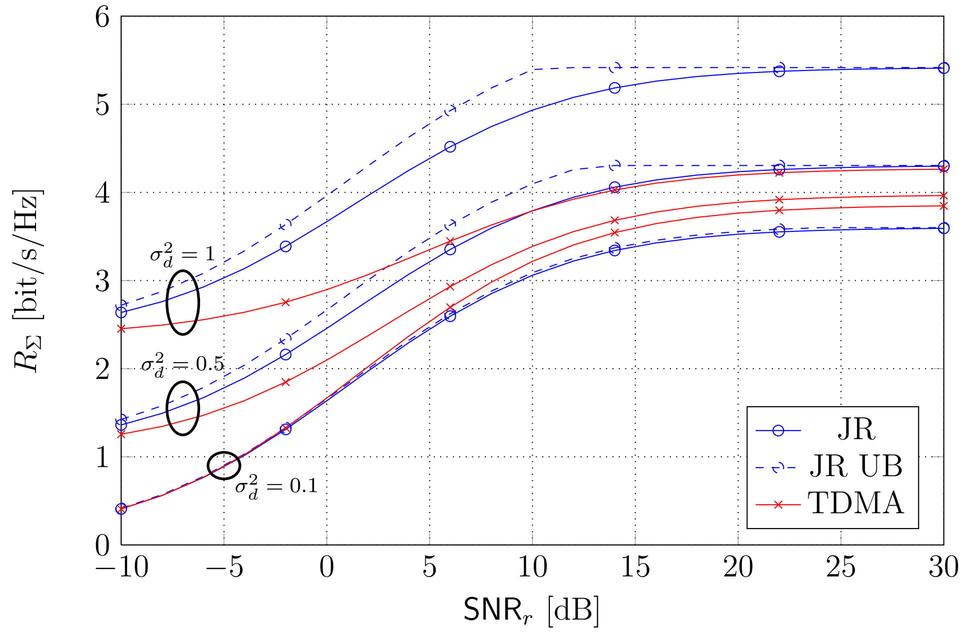


Figure 5.1: Sum-rates in the SISO MARC with direct links at $\mathsf{SNR}_t = 4$ dB

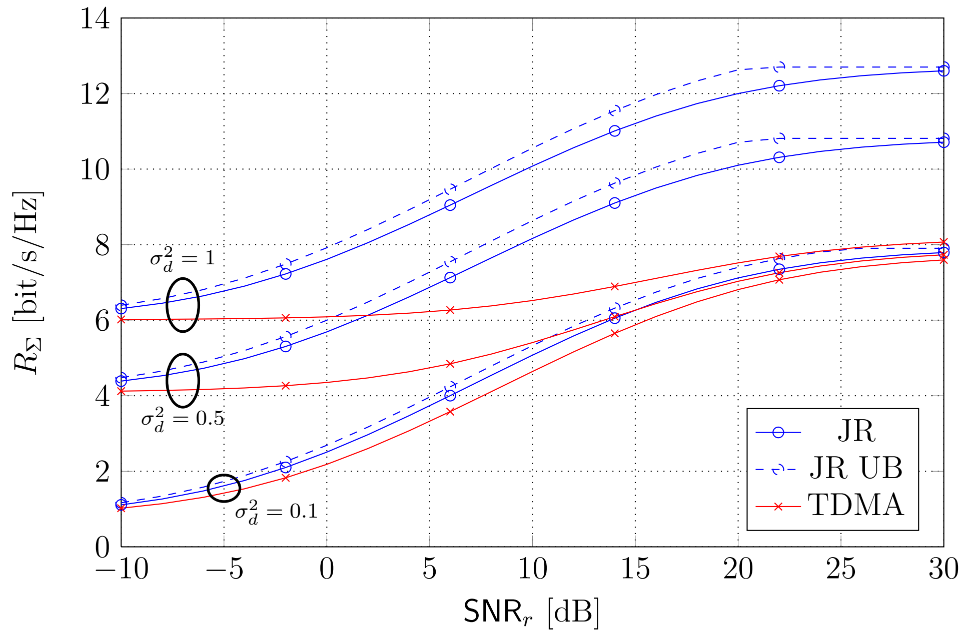


Figure 5.2: Sum-rates in the SISO MARC with direct links at $\mathsf{SNR}_t = 16$ dB

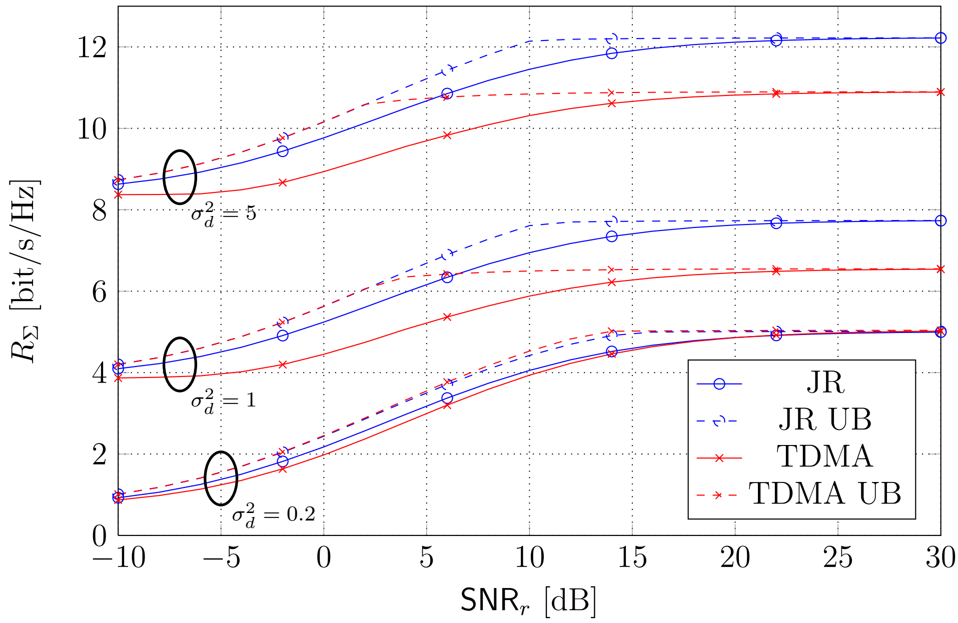


Figure 5.3: Sum-rates in the MISO MARC with direct links at $\mathsf{SNR}_t = 4$ dB

tuitive explanation follows when looking at the system from a MIMO multi-user point of view: If the direct links are added, the receiver virtually has a second antenna, i.e., the effective channel is a SIMO MAC. As already mentioned at the end of Subsection 3.1.2, this allows the parallel reception of two dimensions, i.e., we gain spatial diversity. As in TDMA, only one single-antenna user accesses the channel at a time, the spatial diversity can not be exploited. On the contrary, multiple users access the channel in joint relaying, such that the spatial diversity can be used, which is an enormous advantage.

If multi-antenna transmitters, i.e., the MISO MARC with direct links is considered, the situation changes. First, the exact rates of TDMA are not known for this case, such that they can only be estimated by upper and lower bounds, too. Of course, this makes it harder to give a clear statement whether TDMA or joint relaying is better. The achieved sum-rates as well as the upper bounds are plotted in Figure 5.3 for $\mathsf{SNR}_t = 4$ dB and in Figure 5.4 for $\mathsf{SNR}_t = 16$ dB.

It can be observed that, while the gap between upper and lower bounds grows only slightly for joint relaying, it is considerably larger for TDMA. Nevertheless, the suboptimality of TDMA obtained for single-antenna transmitters can also be shown for multi-antenna transmitters if the achievable rate of joint relaying lies above the upper bound of TDMA. As it can be seen, this is the case if P_r is large enough. However, compared to the single-antenna case, the gaps between joint relaying and TDMA have decreased considerably, even for much stronger direct links. An explanation for this

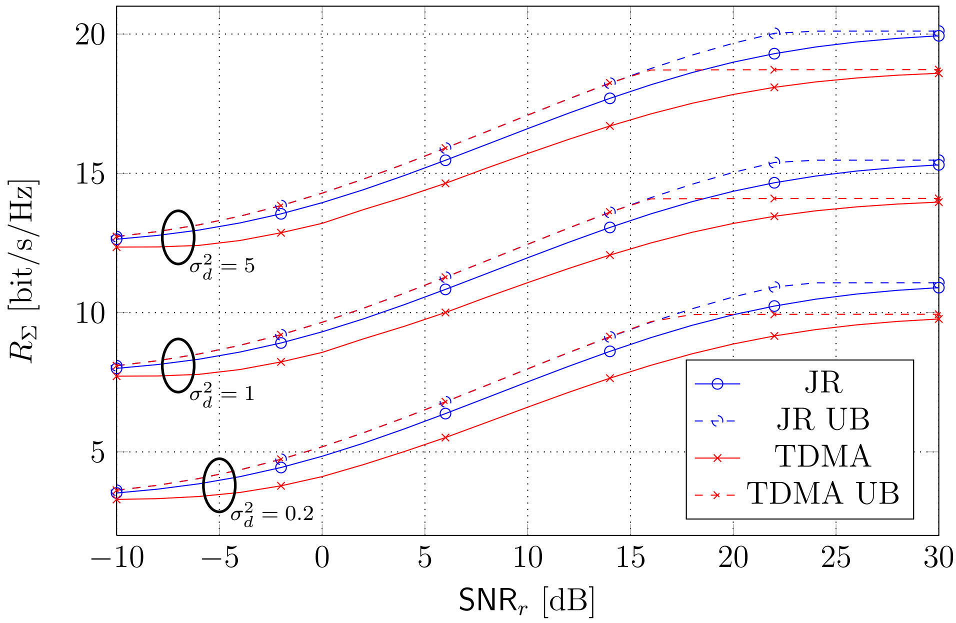


Figure 5.4: Sum-rates in the MISO MARC with direct links at $\mathsf{SNR}_t = 16$ dB

is that with multiple transmit antennas, the effective channel is now a MIMO MAC. Thus, also TDMA, where one user uses the channel exclusively can benefit from the spatial diversity by transmitting as in a MIMO single-user channel.

More insights about the difference of TDMA and joint relaying are found if the achievable sum-rates for $P_r \to \infty$ are considered. In Figures 5.5, 5.6, and 5.7 the loss of TDMA compared to joint relaying is plotted over several parameters.

In Figure 5.5 a medium SNR of $\mathsf{SNR}_t = 10$ dB is considered, while σ_d^2 is varied between 0 and 1 and the number of transmit antennas N_t is varied between 1 and 10. Obviously, the rate loss of TDMA reduces significantly with the number of transmit antennas, especially if the direct links are strong. While the rate loss for $\sigma_d^2 = 1$ is about 30% for $N_t = 1$, it reduces to about 5% for $N_t = 10$. Moreover, TDMA can also achieve higher rates than joint relaying. However, this is only the case for low values of σ_d^2.

The influence of the transmit power is considered in Figures 5.6 and 5.7, where for Figure 5.6 we consider the SISO channel with $N_t = 1$ and for Figure 5.7 we consider the MISO channel with $N_t = 3$. The observation from Figures 5.1 and 5.4 that the rate loss of TDMA grows with the transmit power is confirmed by Figure 5.6. However, the dependence on the strength of the direct links is much stronger, such that for $\sigma_d^2 = 1$ and $\mathsf{SNR}_t = 20$ dB the loss of TDMA is almost 40%. On the other hand, if the transmit power is low such that $\mathsf{SNR}_t = 2$ dB, TDMA can achieve gains as long as $\sigma_d^2 \leq 0.35$.

As it can be expected from the previous plots, the situation is different if the MISO channel is considered. Figure 5.7 shows that multiple antennas do not only reduce the

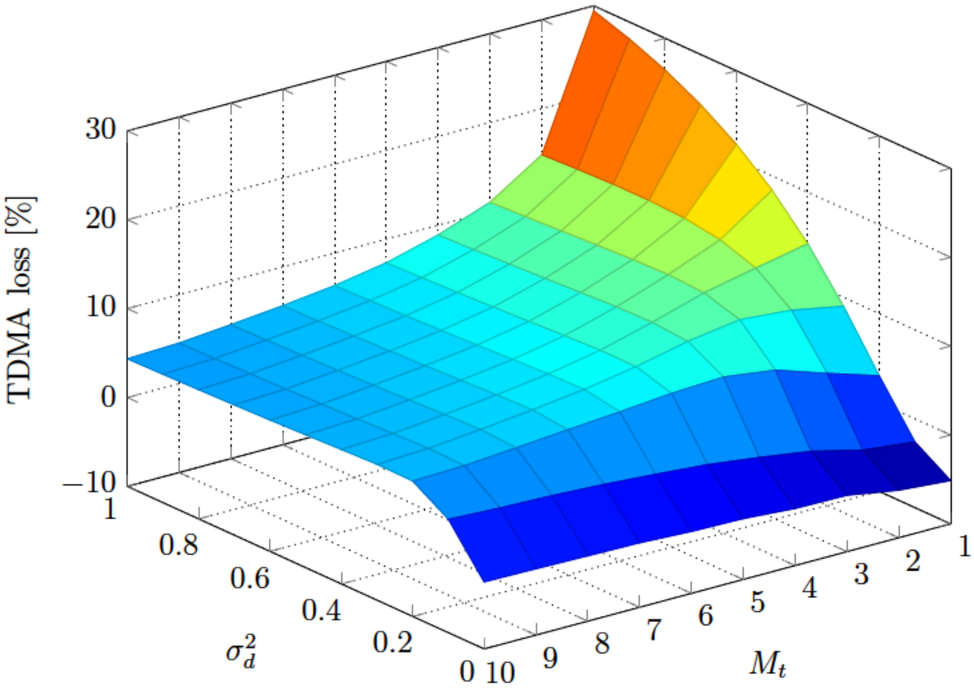


Figure 5.5: TDMA loss for $P_r \to \infty$ at $\mathsf{SNR}_t = 10$ dB

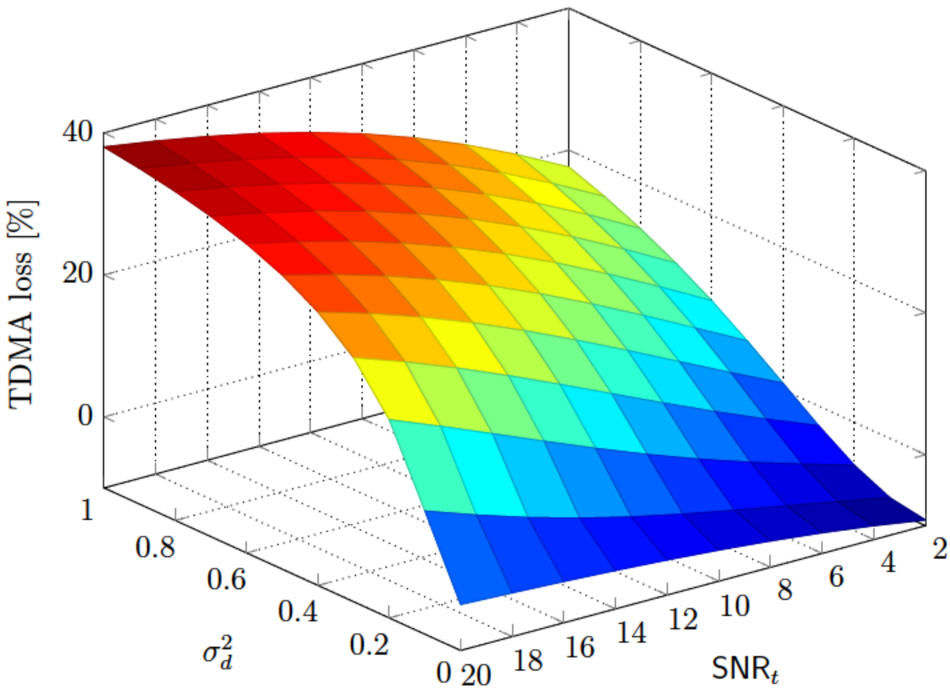


Figure 5.6: TDMA loss for $P_r \to \infty$ with $N_t = 1$ (SISO)

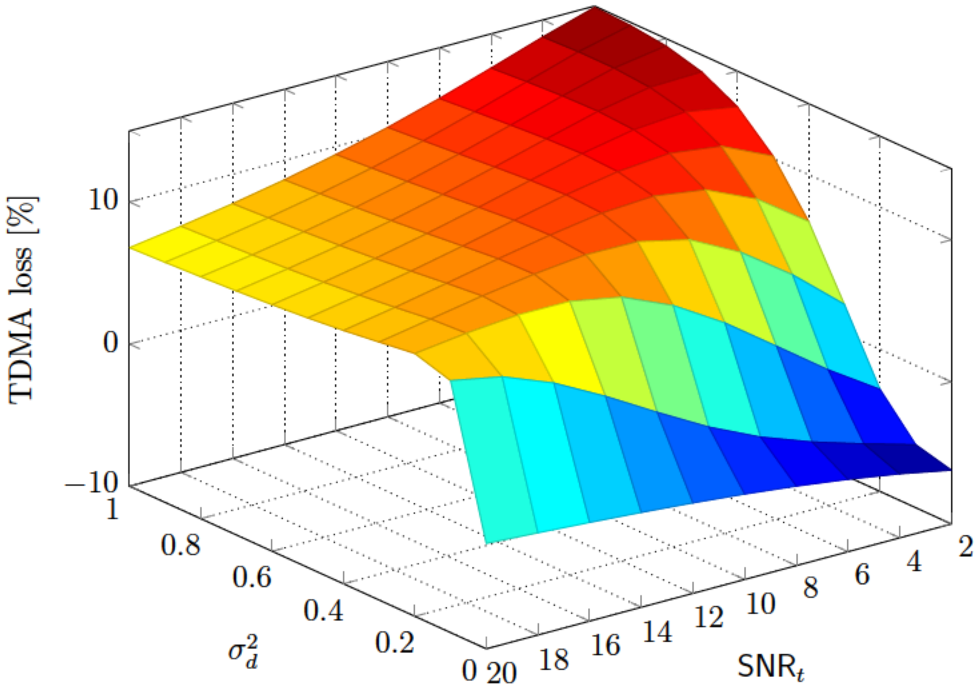


Figure 5.7: TDMA loss for $P_r \to \infty$ with $N_t = 3$

maximum loss of TDMA to approximately 15%, but also reverse the dependency on the transmit power partly. If the direct links are strong, the maximum loss of TDMA is obtained at $\mathsf{SNR}_t = 2$ dB, while for growing transmit power the loss reduces to approximately 7% at $\mathsf{SNR}_t = 20$ dB. Even more interesting, for $\mathsf{SNR}_t = 20$ dB, the maximal rate loss of TDMA is obtained for $\sigma_d^2 = 0.2$ (about 8.5%), while the loss at $\sigma_d^2 = 1$ is lower. A reason for this might be that strong direct links and high transmit power allow the TDMA scheme to benefit more from the gained spatial diversity, while joint relaying can already exploit this advantage for lower SNR_t and σ_d^2.

Finally, we want to give a short evaluation of the convergence behavior of Algorithm 6. As explained in Subsection 5.2.2, the number of iterations that are necessary to compute the optimal solution influences the speed of the algorithm essentially. In Figure 5.8, the average ratio of the achieved sum-rate $\widetilde{R}_\Sigma$ after 1, 2, 3, 5, and 10 iterations to the finally achieved sum-rate R_Σ is plotted for $\mathsf{SNR}_t = 10$ dB, $\sigma_d^2 = 1$ and several values of SNR_r. Additionally, if we define convergence as a rate increase of less than 10^{-3} bit/s/Hz, the average number of iterations N_{iter} is given by the following table:

SNR_r	-10	-6	-2	2	6	10	14	18	22	26	30
N_{iter}	2.53	3.31	3.75	4.02	4.31	4.54	4.73	5.25	5.42	5.63	5.85

From these results it can be seen that the convergence speed decreases with SNR_r.

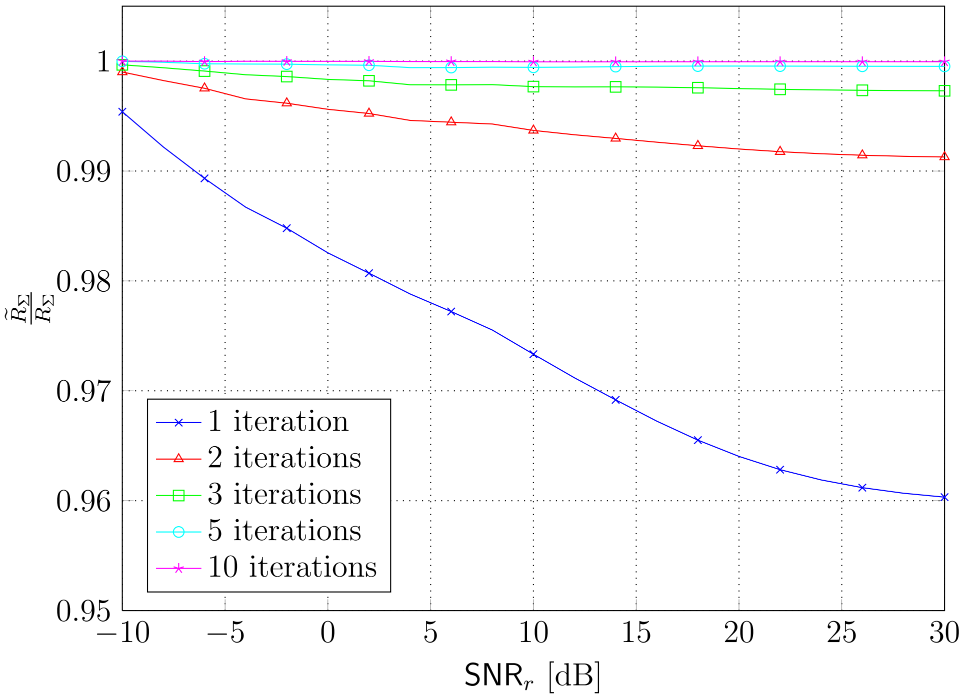


Figure 5.8: Convergence of Algorithm 6 for $\mathsf{SNR}_t = 10$ dB and $\sigma_d^2 = 1$

However, also for high values of SNR_r, the rate increase after 5 iterations is negligible. Furthermore, already after one iteration, the achieved rate lies only 4% below the finally achieved rate for high SNR_t, for low SNR_t the performance decrease is only 0.5%. Thus, using no iterations at all would also deliver results which are not much worse. For the case considered here, using 3 iterations would be sufficient to have a good trade-off between convergence time and performance.

As the convergence behavior for other values of SNR_t and σ_d^2 does not differ much from the one shown here, we omit further plots for these cases. The TDMA version of Algorithm 6 has a similar convergence behavior but manages to converge with fewer iterations. The reason for this is that the iteration only optimizes one instead of K covariance matrices. However, it has to be considered that the TDMA version optimizes the relaying matrix and the covariance matrix of each user separately, such that the inner iteration is executed K times more often than with joint relaying. Moreover, the outer loop which optimizes the duration of the time slots in a brute-force fashion has to be considered which increases the complexity even further. Thus, as already stated in Subsection 5.2.2, the complexity of the TDMA version of Algorithm 6 is much higher.

6

Characteristics of Finite Alphabets in Low-SNR Broadcast Channels

This chapter considers the use of finite instead of Gaussian alphabets for transmission. Although Gaussian alphabets are optimal for the channels considered here, they are continuous and unbounded, which makes them unusable for practical systems. Instead those systems have to use finite alphabets like M-PSK or M-QAM, where one of M constellation points is transmitted. Contrary to Gaussian alphabets, the finite constellation generally limits the achievable rate by $\log_2 M$. Nevertheless, if M is chosen large enough, it is possible to achieve rates that are close to the optimal rates of Gaussian alphabets, although a shaping loss [FGL$^+$84] has to be tolerated.

Here, we consider a SISO broadcast channel, where the users are operating at low SNR level. For simplicity, the channel does not contain a relay. As described in chapter 4.3, fixing the relaying matrix of the AF-relay in a BRC effectively delivers a broadcast channel, such that for the problems considered here, it is not of importance whether the channel contains a relay or not.

The low SNR level entails that the energy per bit of each user is close to the minimum that allows for reliable transmission. In [Ver02], the wideband slope was introduced, which describes the behavior of the capacity as SNR tends to zero. It has already been shown [Ver02] that in single-user channels, quadrature phase shift keying (QPSK) achieves the optimum slope reached by Gaussian alphabets, while binary phase shift keying (BPSK) only achieves half the slope. In this chapter, we show that the same holds for the slope region of the broadcast channel. Furthermore, we consider the performance of TDMA and show that the suboptimality of TDMA, which was already shown for multi-user channels with Gaussian inputs in [CTV04], persists.

This chapter is structured as follows: In Section 6.1, we introduce the minimum energy per bit and the wideband slope as quality criteria for channels at low SNR. Subsequently, we consider previous results for broadcast channels at low SNR with Gaussian alphabets in Section 6.2. Similar results are established for the broadcast channel with finite alphabets in Section 6.3, where we consider BPSK and QPSK as modulation schemes. Finally, Section 6.4 evaluates the obtained slope regions and presents an example, for which the achievable slope regions are plotted.

6.1 Minimum Energy Per Bit and Wideband Slope

Considering channels at low SNR is of special importance if, e.g., the bandwidth is increased considerably while the transmit power remains constant, such as in ultra wideband channels. Contrary to the preceding chapters in this work, taking the channel capacity as quality criterion is not suitable, because as the SNR tends to zero the channel capacity also tends to zero. In order to have a criterion also for low-SNR channels, Verdú introduced the capacity per unit cost [Ver90] first, and considered the spectral efficiency in channels at low SNR later in [Ver02].

This spectral efficiency C equals the capacity C, but is a function of the energy per information bit to noise ratio instead of SNR, i.e., we have

$$\mathsf{C}\left(\frac{E_b}{N_0}\right) = C\left(\mathsf{SNR}\right), \tag{6.1}$$

where SNR is defined as the ratio of the transmit power per symbol P and the noise power N_0. Hence, assuming transmission at the capacity, the energy per information bit is given by the ratio of the transmit power P and the capacity, i.e., $E_b = \frac{P}{C(\mathsf{SNR})}$. Using (6.1), the relationship between SNR and $\frac{E_b}{N_0}$ can also be described as (cf. [Ver02])

$$\frac{E_b}{N_0} C\left(\mathsf{SNR}\right) = \mathsf{SNR}. \tag{6.2}$$

Of course, as SNR tends to zero, also the spectral efficiency tends to zero. However, it can be used to define two quality criteria for channels at low SNR [Ver02].

First, the minimum required energy per information bit is of interest. As $C\left(\mathsf{SNR}\right)$ is a monotonically increasing concave function, it follows from (6.2) that

$$\frac{E_b}{N_0}_{\min} = \lim_{\mathsf{SNR}\to 0} \frac{\mathsf{SNR}}{C\left(\mathsf{SNR}\right)} = \frac{\ln 2}{\dot{C}\left(0\right)},$$

where $\dot{C}\left(0\right)$ denotes the first derivative of the capacity with respect to SNR.

Second, for comparing setups with equal minimum energy per bit, Verdú introduced the wideband slope $\mathcal{S}_0$. It is defined as the slope of the spectral efficiency $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ in bit/s/Hz/(3 dB) at $\frac{E_b}{N_0}_{min}$, i.e.,

$$\mathcal{S}_0 = \lim_{\frac{E_b}{N_0} \downarrow \frac{E_b}{N_0}_{\min}} \frac{\mathsf{C}\left(\frac{E_b}{N_0}\right)}{10\log_{10}\frac{E_b}{N_0} - 10\log_{10}\frac{E_b}{N_0}_{\min}} 10\log_{10} 2,$$

where $\downarrow$ indicates convergence from above. Moreover, it is shown in [Ver02, Theorem 9] that the wideband slope $\mathcal{S}_0$ can also be calculated from the capacity by

$$\mathcal{S}_0 = -2\frac{\dot{C}\left(0\right)^2}{\ddot{C}\left(0\right)}.$$

In the following sections, we will use these quality criteria to evaluate the performance of Gaussian and finite alphabets in the broadcast channel at low SNR with and without TDMA.

6.2 Slope Region of Broadcast Channels with Gaussian Alphabets

In [CTV04], the minimum energy per bit and the wideband slope were derived for two-user broadcast channels as described in Subsection 3.2.1, for the case that Gaussian alphabets are used for transmission. In the following, we will describe the results without proofs, which can be found in [CTV04].

For the sake of consistency, we use the notation introduced in Subsection 3.2.1 for the SISO broadcast channel in the remainder of this chapter. Moreover, we assume without loss of generality that $|h^{(1)}| \geq |h^{(2)}|$, i.e., receiver 1 has a higher channel gain and receiver 2 is degraded. Therefore, the capacity region of this channel, achieved by superpositioning, is given by (cf. [CTV04])

$$\bigcup_{0\leq\alpha\leq 1} \Bigg\{ R^{(1)} \leq \log_2\left(1+\alpha|h^{(1)}|^2\mathsf{SNR}\right) \qquad R^{(2)} \leq \log_2\left(1+\frac{(1-\alpha)|h^{(2)}|^2\mathsf{SNR}}{1+\alpha|h^{(2)}|^2\mathsf{SNR}}\right)\Bigg\},$$

while the rate region achievable with TDMA is

$$\bigcup_{0\leq\tau\leq 1} \Bigg\{ R^{(1)} \leq \tau\log_2\left(1+|h^{(1)}|^2\mathsf{SNR}\right) \qquad R^{(2)} \leq (1-\tau)\log_2\left(1+|h^{(2)}|^2\mathsf{SNR}\right)\Bigg\}.$$

As in [CTV04] the energy per information bit $E_b^{(k)}$ of user k as well as the received energy per information bit ${E_b^{(k)}}^r$ of user k can be calculated as

$$\frac{{E_b^{(k)}}^r}{N_0} \triangleq |h^{(k)}|^2\frac{E_b^{(k)}}{N_0} \triangleq \frac{\ln 2}{\dot{R}^{(k)}\left(0\right)} \qquad k=1,2 \tag{6.3}$$

and the slopes of the users are given by

$$\mathcal{S}_0^{(k)} = -2\frac{\dot{R}^{(k)}\left(0\right)^2}{\ddot{R}^{(k)}\left(0\right)}. \tag{6.4}$$

In analogy to the capacity region at nonzero SNR, we obtain a slope region at zero SNR. In order to parameterize this region properly, the ratio of the rates is fixed to $\vartheta = \frac{R^{(1)}}{R^{(2)}}$ and one point of the slope region is found for each $\vartheta \in [0, \infty]$.

Using these definitions, the minimum received energies per bit found in [CTV04, Theorem 5] for both superpositioning and TDMA with Gaussian alphabets are given by

$$\begin{aligned} \frac{E_b^{(1)\,r}}{N_0} &= \left(1 + \frac{|h^{(1)}|^2}{|h^{(2)}|^2 \vartheta}\right) \ln 2 \\ \frac{E_b^{(2)\,r}}{N_0} &= \left(1 + \frac{|h^{(2)}|^2 \vartheta}{|h^{(1)}|^2}\right) \ln 2. \end{aligned} \tag{6.5}$$

Moreover, the slope region of TDMA was shown to be (cf. [CTV04, Theorem 6])

$$\bigcup_{\vartheta \in [0,\infty)} \left\{ \left(\mathcal{S}_0^{(1)}, \mathcal{S}_0^{(2)}\right) : 0 \le \mathcal{S}_0^{(1)} \le \frac{2\vartheta}{1+\vartheta},\ 0 \le \mathcal{S}_0^{(2)} \le \frac{2}{1+\vartheta} \right\}, \tag{6.6}$$

while the slope region achieved by superpositioning can be formulated as (cf. [CTV04, Theorem 7])

$$\bigcup_{\vartheta \in [0,\infty)} \left\{ \left(\mathcal{S}_0^{(1)}, \mathcal{S}_0^{(2)}\right) : 0 \le \mathcal{S}_0^{(1)} \le \frac{2\vartheta\left(\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}\right)}{\vartheta^2 + 2\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}},\ 0 \le \mathcal{S}_0^{(2)} \le \frac{2\left(\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}\right)}{\vartheta^2 + 2\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}} \right\}. \tag{6.7}$$

6.3 Slope Region of Broadcast Channels with Finite Alphabets

In this section, we consider M-PSK constellations for transmission, where $M \in \{2, 4\}$, i.e., we use BPSK and QPSK. For superpositioning the BPSK alphabets of the users are given by $\mathcal{A}_2^{(1)} = \{\pm\sqrt{\alpha}A\}$ and $\mathcal{A}_2^{(2)} = \left\{\pm\sqrt{1-\alpha}A\right\}$, respectively, while for QPSK we use $\mathcal{A}_4^{(1)} = \{\sqrt{\alpha}A(\pm 1 \pm j)\}$ and $\mathcal{A}_4^{(2)} = \left\{\sqrt{1-\alpha}A(\pm 1 \pm j)\right\}$, respectively. For the case of TDMA, both users use the alphabet $\mathcal{A}_2 = \{\pm A\}$ for BPSK and $\mathcal{A}_4 = \{A(\pm 1 \pm j)\}$ for QPSK. Note that for BPSK the SNR is given by $\mathsf{SNR} = \frac{A^2}{N_0}$, while for QPSK, we have $\mathsf{SNR} = \frac{2A^2}{N_0}$.

Contrary to transmissions with Gaussian alphabets, the achievable rates can not be calculated by simple formulas if finite alphabets are used. Instead, the rates have to be calculated through the mutual information of the channel input and output at the respective receiver. Using the random variables $X^{(1)}$,$X^{(2)}$ for the channel inputs $x^{(1)}$,$x^{(2)}$ and $Y^{(1)}$, $Y^{(2)}$ for the channel outputs $y^{(1)}$, $y^{(2)}$, the rates can be written as $R^{(1)} = I\left(X^{(1)}; Y^{(1)}|Y^{(2)}\right)$ and $R^{(2)} = I\left(X^{(2)}; Y^{(2)}\right)$. These expressions have been calculated for broadcast channels in [DR09]. However, they are very lengthy, contain expected values, and their derivatives can not directly be calculated. Therefore, we follow a different approach to calculate the rates achievable by BPSK in the following subsection. Subsequently, the rates achievable by QPSK are obtained by exploiting a simple relationship between the BPSK and QPSK rates.

Throughout this section, we write the rates as $R_M^{(k)}$, where M denotes the cardinality of the alphabet ($M \in \{2,4\}$). Moreover, we use the rate ratio $\vartheta = \frac{R_M^{(1)}}{R_M^{(2)}}$ as defined in the preceding section. As a consequence of this, the power dividing variable α in superpositioning depends not only on SNR but also on ϑ, which will be indicated by writing α_ϑ instead. The same holds for the time dividing variable τ in TDMA, which will be denoted as τ_ϑ.

6.3.1 Slope Region with BPSK

In superpositioning, the first user does not experience any interference from user two and effectively uses a single-user channel. Hence, for the expression of the mutual information we can take the result from [Ver02], where single-user channels were considered. In this work, a Taylor expansion of the mutual information was given, which can be written as

$$R_2^{(1)}(\mathsf{SNR}) = |h^{(1)}|^2 \alpha_\vartheta(\mathsf{SNR})\mathsf{SNR} - |h^{(1)}|^4 \alpha_\vartheta(\mathsf{SNR})^2 \mathsf{SNR}^2 + o(\mathsf{SNR}^2)$$

for the channel considered here. As only the first two derivatives of the rates are of importance, this expression is sufficient for the calculation of the minimum energy per bit and the wideband slope.

However, for the second user, which is subject to interference from the first user, this expression can not be used. Instead, we can use a reformulation of the mutual information [Ver02] given by

$$I\left(X^{(2)};Y^{(2)}\right) = E_{x^{(2)}}\{D\left(p_{Y^{(2)}|X^{(2)}=x^{(2)}} \| p_{Y^{(2)}|X^{(2)}=0}\right)\} - D\left(p_{Y^{(2)}} \| p_{Y^{(2)}|X^{(2)}=0}\right), \quad (6.8)$$

where $D(p\|q)$ denotes the Kullback-Leibler divergence between the distributions p and q, $p_{Y^{(2)}|X^{(2)}=x^{(2)}}$ denotes the conditional output distribution given input $x^{(2)}$, and $E_{x^{(2)}}\{T\}$ denotes the expectation value of the term T with respect to $x^{(2)}$. Using (6.8), we can write

$$\begin{aligned} R_2^{(2)} = \frac{1}{2} \sum_{x^{(2)} \in \mathcal{A}_2^{(2)}} \int_{\mathbb{C}} p_{Y^{(2)}|X^{(2)}=x^{(2)}}(y) \log_2 \frac{p_{Y^{(2)}|X^{(2)}=x^{(2)}}(y)}{p_{Y^{(2)}|X^{(2)}=0}(y)}\, dy \\ - \int_{\mathbb{C}} p_{Y^{(2)}}(y) \log_2 \frac{p_{Y^{(2)}}(y)}{p_{Y^{(2)}|X^{(2)}=0}(y)}\, dy, \end{aligned} \quad (6.9)$$

where

$$\begin{aligned} p_{Y^{(2)}|X^{(2)}=x^{(2)}}(y) &= \frac{1}{2} \sum_{x^{(1)} \in \mathcal{A}_2^{(1)}} \frac{1}{\pi} e^{-|y-h^{(2)}x^{(2)}-h^{(2)}x^{(1)}|^2} \\ p_{Y^{(2)}}(y) &= \frac{1}{2} \sum_{x^{(2)} \in \mathcal{A}_2^{(2)}} p_{Y^{(2)}|X^{(2)}=x^{(2)}}(y) \end{aligned} \tag{6.10}$$

are the probability densities appearing inside the integrals.

Through some reformulations[1], it is possible to express $R_2^{(2)}$ as

$$R_2^{(2)}(\mathsf{SNR}) = |h^{(2)}|^2(1-\alpha_\vartheta(\mathsf{SNR}))\mathsf{SNR} - |h^{(2)}|^4(1-\alpha_\vartheta(\mathsf{SNR})^2)\mathsf{SNR}^2 + o(\mathsf{SNR}^2).$$

Therefore, the derivatives of the rates at $\mathsf{SNR} = 0$ can be calculated as

$$\begin{aligned} \dot{R}_2^{(1)}(0) &= |h^{(1)}|^2\alpha_\vartheta(0) \\ \dot{R}_2^{(2)}(0) &= |h^{(2)}|^2(1-\alpha_\vartheta(0)) \\ \ddot{R}_2^{(1)}(0) &= -2|h^{(1)}|^4\alpha_\vartheta(0)^2 + 2|h^{(1)}|^2\dot{\alpha}_\vartheta(0) \\ \ddot{R}_2^{(2)}(0) &= -2|h^{(2)}|^4 + 2|h^{(2)}|^4\alpha_\vartheta(0)^2 - 2|h^{(2)}|^2\dot{\alpha}_\vartheta(0). \end{aligned} \tag{6.11}$$

As $\vartheta = \frac{R_M^{(1)}}{R_M^{(2)}}$ holds for all SNR, it follows that $\frac{\dot{R}_2^{(1)}(0)}{\dot{R}_2^{(2)}(0)} = \frac{\ddot{R}_2^{(1)}(0)}{\ddot{R}_2^{(2)}(0)} = \vartheta$ and we can further derive

$$\begin{aligned} \alpha_\vartheta(0) &= \frac{\vartheta|h^{(2)}|^2}{|h^{(1)}|^2 + \vartheta|h^{(2)}|^2} \\ \dot{\alpha}_\vartheta(0) &= -\vartheta|h^{(2)}|^4|h^{(1)}|^2\frac{(1-\vartheta)|h^{(1)}|^2 + 2\vartheta|h^{(2)}|^2}{(|h^{(1)}|^2 + \vartheta|h^{(2)}|^2)^3}. \end{aligned} \tag{6.12}$$

Combining (6.11),(6.12), and (6.3), the minimum required received energies per bit are given by the same expressions as in (6.5), i.e., they are the same as for Gaussian alphabets. The slope region follows from (6.4) and is given by

$$\bigcup_{\vartheta \in [0,\infty)} \left\{ \left(\mathcal{S}_0^{(1)}, \mathcal{S}_0^{(2)}\right) : 0 \le \mathcal{S}_0^{(1)} \le \frac{\vartheta\left(\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}\right)}{\vartheta^2 + 2\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}},\ 0 \le \mathcal{S}_0^{(2)} \le \frac{\left(\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}\right)}{\vartheta^2 + 2\vartheta + \frac{|h^{(1)}|^2}{|h^{(2)}|^2}} \right\},$$

i.e., the slope of each user is halved compared to the slope region of Gaussian alphabets from (6.7).

If TDMA is used instead of superpositioning, the Taylor expansions of both users' rates can be derived from the single-user Taylor expansion [Ver02] and are given by

$$\begin{aligned} R_2^{(1)}(\mathsf{SNR}) &= |h^{(1)}|^2\tau_\vartheta(\mathsf{SNR})\mathsf{SNR} - |h^{(1)}|^4\tau_\vartheta(\mathsf{SNR})\mathsf{SNR}^2 + o(\mathsf{SNR}^2) \\ R_2^{(2)}(\mathsf{SNR}) &= |h^{(2)}|^2(1-\tau_\vartheta(\mathsf{SNR}))\mathsf{SNR} - |h^{(2)}|^4(1-\tau_\vartheta(\mathsf{SNR}))\mathsf{SNR}^2 + o(\mathsf{SNR}^2). \end{aligned}$$

[1] due to their lengthiness, these reformulations are not mentioned here, for details see [KWHK09]

Hence, the derivatives at $\mathsf{SNR} = 0$ can be calculated as

$$\begin{aligned}
\dot{R}_2^{(1)}(0) &= |h^{(1)}|^2 \tau_\vartheta(0) \\
\dot{R}_2^{(2)}(0) &= |h^{(2)}|^2 (1 - \tau_\vartheta(0)) \\
\ddot{R}_2^{(1)}(0) &= -2|h^{(1)}|^4 \tau_\vartheta(0) + 2|h^{(1)}|^2 \dot{\tau}_\vartheta(0) \\
\ddot{R}_2^{(2)}(0) &= -2|h^{(2)}|^4 + 2|h^{(2)}|^4 \tau_\vartheta(0) - 2|h^{(2)}|^2 \dot{\tau}_\vartheta(0).
\end{aligned} \tag{6.13}$$

Again, the ratio $\vartheta = \frac{R_M^{(1)}}{R_M^{(2)}}$ holds for all derivatives, such that $\tau_\vartheta(0)$ and $\dot{\tau}_\vartheta(0)$ are given by

$$\begin{aligned}
\tau_\vartheta(0) &= \frac{\vartheta |h^{(2)}|^2}{|h^{(1)}|^2 + \vartheta |h^{(2)}|^2} \\
\dot{\tau}_\vartheta(0) &= \vartheta |h^{(1)}|^2 |h^{(2)}|^2 \frac{|h^{(1)}|^2 - |h^{(2)}|^2}{(|h^{(1)}|^2 + \vartheta |h^{(2)}|^2)^2}.
\end{aligned} \tag{6.14}$$

Thus, also for TDMA it follows from the combination of (6.11),(6.12), (6.3), and (6.4) that the minimum required received energies per bit are the same as in (6.5), while the slope region is given by

$$\left\{ \left(\mathcal{S}_0^{(1)}, \mathcal{S}_0^{(2)} \right) : 0 \leq \mathcal{S}_0^{(1)} \leq \frac{\vartheta}{1+\vartheta},\ 0 \leq \mathcal{S}_0^{(2)} \leq \frac{1}{1+\vartheta} \right\},$$

i.e., the slope of each user is again halved compared to the slope region achieved with TDMA and Gaussian alphabets from (6.6).

6.3.2 Slope Region with QPSK

As QPSK can be interpreted as two orthogonal BPSK transmissions, where one takes place in the real and the other in the complex dimension, it seems natural that there is a fix relationship between the rates. As the number of dimensions doubles, while the power per dimensions is halved one can conjecture that

$$R_4^{(k)}(\mathsf{SNR}) = 2 R_2^{(k)} \left(\frac{\mathsf{SNR}}{2} \right) \tag{6.15}$$

holds for the rates of both users in the broadcast channel. As this relationship was shown for single-user channels in [Ver02], this also holds for the first user in superpositioning and both users in TDMA. For the second user in superpositioning, (6.15) was shown in our work [KWHK09]. The main idea of the proof is to use the rate expression from (6.9) and reformulate the probability distributions as products of two distributions, one for the real and one for the imaginary part, like the ones in (6.10). This allows to write the logarithms as the sum of two logarithms, which finally results in a doubling of the rate with half the SNR. However, as the proof is lengthy, we refer to our work [KWHK09] for details.

Using (6.15), it follows that

$$\begin{aligned}\dot{R}_4^{(k)}(0) &= \dot{R}_2^{(k)}(0)\\ \ddot{R}_4^{(k)}(0) &= \frac{1}{2}\ddot{R}_2^{(k)}(0)\end{aligned}$$

for $k = 1, 2$ and for both superpositioning and TDMA. Hence, as the first derivatives at $\mathsf{SNR} = 0$ are the same, QPSK achieves the same minimum received energies per bit as BPSK and Gaussian alphabets described in (6.5). Moreover, as the second derivatives are halved compared to BPSK, it follows for the slopes of QPSK that

$$\mathcal{S}_{0,\mathrm{QPSK}}^{(k)} = -2\frac{\dot{R}_4^{(k)}(0)^2}{\ddot{R}_4^{(k)}(0)} = -4\frac{\dot{R}_2^{(k)}(0)^2}{\ddot{R}_2^{(k)}(0)} = 2\mathcal{S}_{0,\mathrm{BPSK}}^{(k)},$$

i.e., the slope of each user is doubled compared to BPSK. Thus, the slopes of QPSK are again the same as that of Gaussian alphabets described in (6.7) for superpositioning and (6.6) for TDMA. The superiority of QPSK compared to BPSK is not surprising, because QPSK uses the whole complex space, while BPSK only uses the real dimension.

6.4 Evaluation of Slope Regions

As shown in the preceding section, BPSK and QPSK achieve the same minimum received energy per bit as Gaussian alphabets. Therefore, those constellations can be compared by their wideband slopes, for which we found the relation

$$2\mathcal{S}_{0,\mathrm{BPSK}}^{(k)} = \mathcal{S}_{0,\mathrm{QPSK}}^{(k)} = \mathcal{S}_{0,\mathrm{Gauss}}^{(k)},$$

for $k = 1, 2$ and for both superpositioning and TDMA. Thus, BPSK achieves half the slope of QPSK and Gaussian alphabets. Up to now, this relation was only known for the single-user extreme cases obtained by setting $\vartheta = 0$ and $\vartheta = \infty$ from [Ver02]. The results from above show that this relation also holds for all intermediate values of ϑ.

As an example, the slope regions of a broadcast channel with $h^{(1)} = 10$ and $h^{(2)} = 7$ are plotted in Figure 6.1. It can be seen that the slope of each user halves when using BPSK instead of QPSK as modulation scheme. Furthermore, it is observed that the slope region of superpositioning is larger than the one of TDMA. Comparing their slope regions in detail, it can be seen that the advantage of superpositioning grows with $\frac{|h^{(1)}|^2}{|h^{(2)}|^2}$. For the extreme case of $|h^{(1)}| = |h^{(2)}|$, the slope regions of TDMA and superpositioning are equivalent, which was already shown in [CTV04] for Gaussian alphabets.

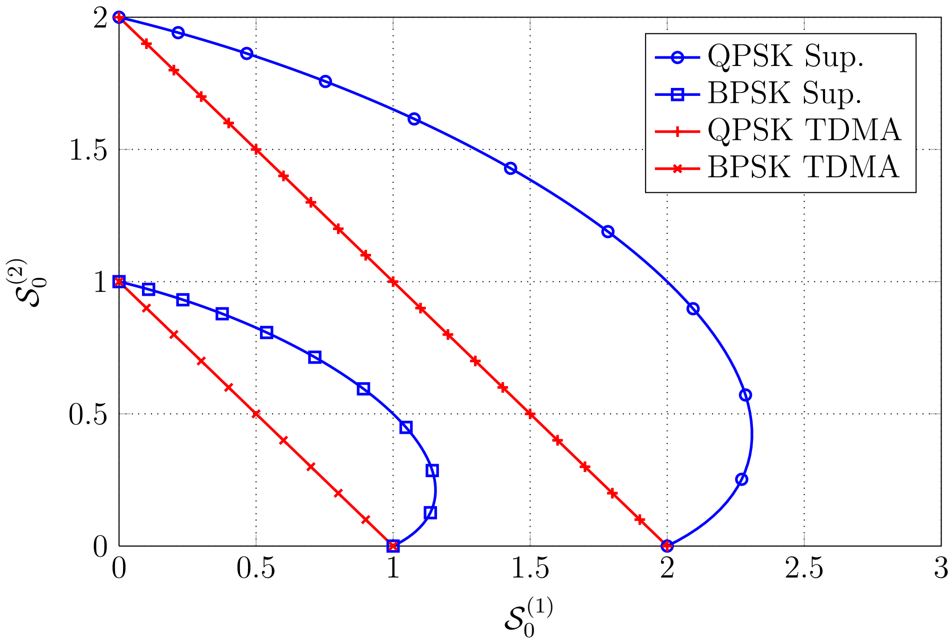


Figure 6.1: QPSK and BPSK slope regions for $h^{(1)} = 10$, $h^{(2)} = 5$

7

Conclusion

In this thesis, we considered different types of multi-user MIMO channels with an AF-relay. Our goal was to find transmit strategies, which allow to achieve sum-rates that are close to the optimum.

A fundamental question in this problem is how the users should access the channel. In existing works, it is mostly assumed that all users access the channel jointly and the relay forwards the sum of the signals to the receiver. On the contrary, we suggested a scheme based on TDMA, where each user is assigned one time slot, in which it can use the channel and the relay exclusively. While for both schemes it is necessary to optimize the transmit covariance matrices as well as the relay matrix, an additional optimization of the duration of the time slots is required for TDMA.

These optimization problems are another very important aspect, which was considered in this thesis. Concerning the performance of the overall system, it is of course desirable to find the global optimal solution of this problems. However, if this requires algorithms with too large complexity, suboptimal solutions with lower complexity and small performance loss might be more desirable.

For the MISO MARC without direct links, closed-form solutions could be found for joint relaying and TDMA. Thus, the effort to compute the optimal transmit covariance and relay matrices is low for both schemes. Comparing the achievable rates of TDMA and joint relaying, we could show that the TDMA-based scheme achieves higher rates. By means of simulation results, we could show that, if the number of users and antennas is large, the sum-rate gain of TDMA can be up to 20%.

A different situation emerges in the MISO BRC without direct links. As the optimization problems are much harder to solve and a duality with the MARC exists only for special cases, a closed-form solution for the problem seems infeasible. Therefore, our main focus in the BRC was to reduce the complexity of existing algorithms. We created an algorithm that uses joint relaying and is based on the duality of MAC and broadcast channel without relay. As it is shown by simulation results, this algorithm achieves the same rates as existing strategies, which require a higher complexity. Moreover, we could show that the use of TDMA is not beneficial in the BRC.

The most involved channel that we considered was the MISO MARC with direct links. As a closed-form solution for the achievable rates seems infeasible for both joint relaying and TDMA, we derived two upper bounds for both schemes. While these upper bounds are partly based on brute-force methods, using such methods for computing an achievable solution turned out to be too complex. Instead, we constructed an efficient, iterative, and heuristic algorithm to compute an achievable solution. This algorithm can be used with both joint relaying and TDMA and achieves rates which are close to the upper bounds. A special case is obtained for single-antenna transmitters, for which we found an algorithm that computes the optimal rates of TDMA. For large relay power, we were even able to formulate a condition for the superiority of TDMA.

Through a set of simulation results, we could show that the superiority of TDMA only persists if the direct links are weak. Especially for single-antenna transmitters, the loss of TDMA can be up to 40%. For multi-antenna transmitters, the loss of TDMA is rather around 10%, depending on the strength of the direct links and the transmit powers. However, as multiple antennas increase the complexity of finding the duration of the time slots for TDMA considerably, we can state that the use of TDMA in the MARC with direct links is mostly not advantageous. Finally, we have shown that the number of iterations that our constructed algorithm requires to converge is low and already after one iteration the rates are close to the finally achieved ones.

A step towards practical systems has been made by considering the use of finite alphabets in broadcast channels. Using the minimum energy per bit and the slope region as quality criteria, it could be shown that QPSK achieves the same performance as Gaussian alphabets, while BPSK achieves only half the slope. Furthermore, the suboptimality of TDMA for broadcast channels was shown to hold also with finite alphabets.

Outlook

Due to the variety of schemes and possible assumptions in MIMO multi-user systems, especially in combination with relays, it is possible to extend the results of this thesis in many directions. For example, an extension to complete MIMO channels as considered in [YH10, TH07], where also the receiver has more than one antenna, would be desirable. As finding closed-form solutions for this case might be difficult, the schemes presented here can already be used as lower bounds. Other interesting extensions might be the assumption of no or only partial channel knowledge, the consideration of the whole capacity region instead of only the sum-rate, or the investigation of the interference channel (IC) [Sha61, Ahl74, Car78] with an AF-relay. However, the latter one could be really challenging, because even the capacity region of the SISO IC is only known partially [Car75, Sat81, HK81, AV09] or to within one bit [ETW08]. With multiple antennas, this problem is even harder to solve.

For the open problems in the MARC with direct links, namely the gap between upper and lower bounds, it might be possible to find solutions in the near future. As, at least for some parameters, the upper bounds derived in this thesis are clearly not tight, one approach might be to find tighter upper bounds. But of course, also better lower bounds can possibly be found. The latter could be done by investing high computational power to find solutions in a brute-force fashion, or maybe by new approaches using methods from optimization theory, which could result in the derivation of new algorithms with low complexity.

Finally, the role of TDMA in multi-user channels with AF-relay could be further considered. Due to the superiority of TDMA for the MISO MARC, it can be stated that joint relaying is not generally sufficient to achieve the best possible rates in the MARC with AF-relay. However, considering the results for channels with direct links where the receiver virtually has two antennas, joint relaying could be better than TDMA in the MIMO MARC, which would be an interesting observation. In order to benefit from the advantages of both joint relaying and TDMA, also hybrid schemes could be constructed that create $L < K$ time slots for K users and allow that time slots are shared by two or more users.

Appendix

A

Water-Filling

In this appendix, we discuss the use of the famous water-filling algorithm for calculating the capacity in MIMO channels. In the first section, the scalar water-filling problem and its solution are described. Subsequently, we consider the problem of finding the capacity in a single-user MIMO channel. It is shown, that this problem can be reformulated as a scalar water-filling problem. As the algorithms for finding the capacity both for the MIMO MAC and MIMO broadcast channel are based on iteratively solving the MIMO single-user capacity problem, the importance of this problem goes far beyond single-user systems.

A.1 The Scalar Water-Filling Algorithm

Let $h_k, p_k \in \mathbb{R}_0^+$ $(k = 1, \ldots, K)$, then consider a problem of the form

$$\begin{aligned} \max_{p_1,\cdots,p_K} \quad & \sum_{k=1}^{K} \log_2 \left(1 + h_k p_k\right) \\ \text{s.t.} \quad & \sum_{k=1}^{K} p_k \leq P \\ & p_k \geq 0 \quad \forall k. \end{aligned} \tag{A.1}$$

First, it can be observed that a necessary condition for a solution is that $\sum_{k=1}^{K} p_k = P$, because the target function is strictly increasing with p_k $\forall k$. The inequality in the problem is only used for reasons of conformance with the matrix problems. Second, the problem is convex. Hence, the KKT conditions are necessary and sufficient for optimality of a solution $\mathbf{p} \triangleq [p_1, \ldots, p_K]$ [BV04]. Using μ and $\boldsymbol{\lambda} \triangleq [\lambda_1, \ldots, \lambda_K]$ as Lagrange multipliers, the Lagrangian of the problem is

$$L(\mathbf{p}, \boldsymbol{\lambda}, \mu) = \sum_{k=1}^{K} \log_2 \left(1 + h_k p_k\right) - \mu \left(\sum_{k=1}^{K} p_k - P \right) + \sum_{k=1}^{K} \lambda_i x_i.$$

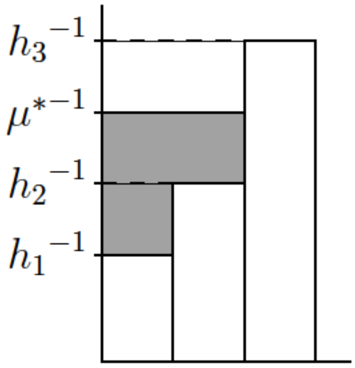


Figure A.1: Graphical Interpretation of Water-Filling for $K = 3$

Therefore, the KKT conditions for a solution $(\mathbf{p}^*, \boldsymbol{\lambda}^*, \mu^*)$ to be optimal can be written as

$$\begin{aligned} \sum_{k=1}^{K} p_k^* &= P \\ p_k^* &\geq 0 \qquad \forall k \\ \lambda_k^* &\geq 0 \qquad \forall k \\ \lambda_k^* p_k^* &= 0 \qquad \forall k \\ \nabla L(\mathbf{p}, \boldsymbol{\lambda}, \mu)_k = \frac{h_k}{1 + h_k p_k} - \mu + \lambda_k &= 0 \qquad \forall k, \end{aligned}$$

where $\nabla L(\mathbf{p}, \boldsymbol{\lambda}, \mu)_k$ denotes the k-th component of $\nabla L(\mathbf{p}, \boldsymbol{\lambda}, \mu)$.

The solution of these equations is given by the well-known water-filling method (for a detailed derivation see [BV04, Example 5.2])

$$p_k^* = \max\left\{0, \frac{1}{\mu^*} - \frac{1}{h_k}\right\} \qquad \forall k, \tag{A.2}$$

where μ^* can be uniquely determined from the power constraint

$$\sum_{k=1}^{K} p_k^* = \sum_{k=1}^{K} \max\left\{0, \frac{1}{\mu^*} - \frac{1}{h_k}\right\} = P.$$

The term water-filling is due to the graphical interpretation of the solution as shown in Figure A.1. We can think of a bin with different ground levels $h_1{}^{-1}, \ldots, h_K{}^{-1}$ and fill water of amount P in this bin. Associating the water level with μ^{*-1}, the optimal values p_k are found as the amount of water on the area with ground level $h_k{}^{-1}$. If the ground level is higher than the water level, i.e., $h_k{}^{-1} \geq \mu^{*-1}$, we have $p_k = 0$. This reflects exactly the solution described in (A.2).

A.2 Water-Filling for MIMO channels

The capacity of a standard single-user MIMO channel with channel matrix $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is obtained by solving the optimization problem[1]

$$\begin{aligned} \max_{\mathbf{Q}} \quad & \log_2 \left| \mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H \right| \\ \text{s.t.} \quad & \operatorname{tr}(\mathbf{Q}) \leq P \\ & \mathbf{Q} \succeq 0. \end{aligned}$$

The solution for this problem is found in [Tel99] and shall be quickly repeated here. Let $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$ be the SVD of $\mathbf{H}$ and optimize over the similar matrix $\boldsymbol{\Sigma}_q \triangleq \mathbf{V}^H\mathbf{Q}\mathbf{V}$ instead. Then, the optimization problem can be rewritten as

$$\begin{aligned} \max_{\boldsymbol{\Sigma}_q} \quad & \log_2 \left| \mathbf{I} + \boldsymbol{\Sigma}\boldsymbol{\Sigma}_q\boldsymbol{\Sigma}^H \right| \\ \text{s.t.} \quad & \operatorname{tr}(\boldsymbol{\Sigma}_q) \leq P \\ & \boldsymbol{\Sigma}_q \succeq 0, \end{aligned} \tag{A.3}$$

where the determinant identity $|\mathbf{I} + \mathbf{AB}| = |\mathbf{I} + \mathbf{BA}|$ was used. From the Hadamard inequality [HJ90], it follows that

$$\log_2 \left| \mathbf{I} + \boldsymbol{\Sigma}\boldsymbol{\Sigma}_q\boldsymbol{\Sigma}^H \right| \leq \log_2 \left(\prod_{i=1}^{n} 1 + \Sigma_{ii}\Sigma_{q_{ii}}\Sigma_{ii}^H \right) = \sum_{i=1}^{n} \log_2 \left(1 + \Sigma_{ii}\Sigma_{q_{ii}}\Sigma_{ii}^H \right),$$

where $n = \min\{N_r, N_t\}$ and equality only holds if $\boldsymbol{\Sigma}_q$ is diagonal. Hence, the optimization problem (A.3) is solved by a diagonal matrix $\boldsymbol{\Sigma}_q$. Using $q_1, \ldots, q_{M_t}$ and $\sigma_1, \ldots, \sigma_n$ as diagonal entries of $\boldsymbol{\Sigma}_q$ and $\boldsymbol{\Sigma}$, respectively, the problem can be rephrased as

$$\begin{aligned} \max_{q_1,\ldots,q_{M_t}} \quad & \sum_{i=1}^{n} \log_2 \left(1 + |\sigma_i|^2 q_i \right) \\ \text{s.t.} \quad & \sum_{i=1}^{M_t} q_i \leq P \\ & q_i \geq 0 \ \ \forall i, \end{aligned}$$

which is equivalent to the scalar water-filling problem (A.1) for $n = k$ and $h_k = |\sigma_k|^2$, $p_k = q_k \ \forall k$.

[1] If the noise covariance matrix $\mathbf{N} \in \mathbb{C}^{M_r \times M_r}$ is not a unit matrix, the same problem is obtained by considering the channel $\widetilde{\mathbf{H}} = \mathbf{N}^{-1/2}\mathbf{H}$ instead

B

Proofs

B.1 Proof of Lemma 6

Using the formulation (5.7) of R_Σ, we can write

$$\left|\mathbf{I} + \mathbf{H}_{\text{eff}}^{(1)} {\mathbf{H}_{\text{eff}}^{(1)}}^H \widetilde{\mathbf{Q}}^{(1)}\right| \tag{B.1}$$

$$= \begin{vmatrix} 1 + s^{-1}||\mathbf{h}||^2 q_1 \Phi + s^{-1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||\widetilde{q}^H \Psi & s^{-1}||\mathbf{h}||^2 \widetilde{q}\Phi + s^{-1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||q_2\Psi \\ s^{-1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||q_1\Psi^H + ||\mathbf{h}_d^{(1)}||^2 \widetilde{q}^H & 1 + s^{-1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||\widetilde{q}\Psi^H + ||\mathbf{h}_d^{(1)}||^2 q_2 \end{vmatrix} \tag{B.2}$$

$$\begin{aligned} &= 1 + 2s^{-1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||\mathrm{Re}\left(\widetilde{q}\Psi^H\right) + s^{-1}||\mathbf{h}||^2 q_1 \Phi + ||\mathbf{h}_d^{(1)}||^2 q_2 \\ &\qquad + s^{-1}||\mathbf{h}||^2||\mathbf{h}_d^{(1)}||^2 \left(q_1 q_2 - |\widetilde{q}|^2\right)\left(\Phi - |\Psi|^2\right) \end{aligned} \tag{B.3}$$

Considering the definitions of Φ and Ψ in the theorem, it follows that q_1, q_2, $\widetilde{q}$ and $f_1, \ldots, f_n$ remain as optimization variables. As in the preceding problems, these parameters depend on each other and are difficult to find. However, as we have modified the relay power constraint such that it does not depend on $\mathbf{Q}^{(1)}$ or $\widetilde{\mathbf{Q}}^{(1)}$, the problem can be simplified. The reason for this is that irrespective of the choice of the other variables, it is clear that $\widetilde{\mathbf{Q}}^{(1)}$ has to be chosen such that it diagonalizes the expression (B.1) (cf. Appendix A.2). Thus, if the EVD of $\mathbf{H}_{\text{eff}}^{(1)} {\mathbf{H}_{\text{eff}}^{(1)}}^H$ is $\mathbf{U}_{\text{eff}}\mathbf{\Sigma}_{\text{eff}}\mathbf{U}_{\text{eff}}^H$, the structure of $\widetilde{\mathbf{Q}}^{(1)}$ has to be $\widetilde{\mathbf{Q}}^{(1)} = \mathbf{U}_{\text{eff}}\mathbf{\Sigma}_q\mathbf{U}_{\text{eff}}^H$, where $\mathbf{\Sigma}_q$ is diagonal.

As a consequence of this, we have

$$\mathbf{H}_{\text{eff}}^{(1)} {\mathbf{H}_{\text{eff}}^{(1)}}^H \widetilde{\mathbf{Q}}^{(1)} = \mathbf{U}_{\text{eff}}\mathbf{\Sigma}_{\text{eff}}\mathbf{\Sigma}_q\mathbf{U}_{\text{eff}}^H = \mathbf{U}_{\text{eff}}\mathbf{\Sigma}_q\mathbf{\Sigma}_{\text{eff}}\mathbf{U}_{\text{eff}}^H = \widetilde{\mathbf{Q}}^{(1)}\mathbf{H}_{\text{eff}}^{(1)} {\mathbf{H}_{\text{eff}}^{(1)}}^H,$$

i.e, $\mathbf{H}_{\text{eff}}^{(1)}\mathbf{H}_{\text{eff}}^{(1)}$ and $\widetilde{\mathbf{Q}}^{(1)}$ commute [HJ90]. As both matrices are also Hermitian, it follows that their product is Hermitian as well. Also the identity matrix and the sum of Hermitian matrices are Hermitian. Hence, the expression inside the determinant in (B.2) has to be Hermitian as well. From this we can derive the condition that

$$s^{-1}||\mathbf{h}||^2\widetilde{q}\Phi + s^{-1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||q_2\Psi = s^{-1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||q_1\Psi + ||\mathbf{h}_d^{(1)}||^2\widetilde{q}$$

has to hold. Equivalently this can be written as the condition

$$\widetilde{q} = \frac{s^{1/2}||\mathbf{h}||||\mathbf{h}_d^{(1)}||\Psi(q_1 - q_2)}{||\mathbf{h}||^2\Phi - ||\mathbf{h}_d^{(1)}||^2 s}, \tag{B.4}$$

i.e, $\widetilde{q}$ is already determined by the condition on the structure of $\widetilde{\mathbf{Q}}^{(1)}$. Combining (B.3) and (B.4), the expression in Lemma 6 is obtained.

B.2 Proof of Theorem 7

It can be observed that R_Σ is a function of $|\Psi|^2$. Therefore, we use $\xi \triangleq |\Psi|^2$ in order to simplify the notation and derivations in this proof. As any phase of a_i can be compensated by choosing $\theta_i = \widetilde{\theta}_i - \angle(a_i)$, we assume $a_i \in \mathbb{R}$ and $a_i \geq 0\ \forall i$ without loss of generality. Moreover, throughout this proof we define

$$k = \arg\max_l |f_l| a_l \sigma_l,$$

and, as rotating the phase of the whole expression Ψ does not influence its absolute value, we can assume $\theta_k = 0$ without loss of generality.

First, we show that

$$\max\left\{0, \left(2|f_k|a_k\sigma_k - \sum_{i=1}^n |f_i|a_i\sigma_i\right)^2\right\} \leq \xi \leq \left(\sum_{i=1}^n |f_i|a_i\sigma_i\right)^2. \tag{B.5}$$

As $\xi = \left|\sum_{i=1}^n |f_i|e^{j\theta_i}\sigma_i a_i\right|^2$, the right side of the inequation follows from the triangle inequality. Concerning the left side, if $|f_k|a_k\sigma_k \leq \frac{1}{2}\sum_{i=1}^n |f_i|a_i\sigma_i$, $\xi \geq 0$ is clear from its definition. For the opposite case, we can write

$$|f_k|a_k\sigma_k \geq \sum_{i\neq k} |f_i|a_i\sigma_i \geq \left|\sum_{i\neq k} |f_i|e^{j\theta_i}a_i\sigma_i\right|,$$

where the first inequality is a reformulation of the condition $|f_k|a_k\sigma_k \geq \frac{1}{2}\sum_{i=1}^n |f_i|a_i\sigma_i$ and the second inequality is due to the triangle inequality again. Using this, it can be seen that

$$\begin{aligned}
|\Psi| &= \left||f_k|a_k\sigma_k + \sum_{i\neq k} |f_i|e^{j\theta_i}a_i\sigma_i\right| \geq \left||f_k|a_k\sigma_k - \left|\sum_{i\neq k} |f_i|e^{j\theta_i}a_i\sigma_i\right|\right| \\
&= |f_k|a_k\sigma_k - \left|\sum_{i\neq k} |f_i|e^{j\theta_i}a_i\sigma_i\right| \geq |f_k|a_k\sigma_k - \sum_{i\neq k} |f_i|a_i\sigma_i \\
&= 2|f_k|a_k\sigma_k - \sum_{i=1}^n |f_i|a_i\sigma_i
\end{aligned}$$

which shows the remaining case of (B.5).

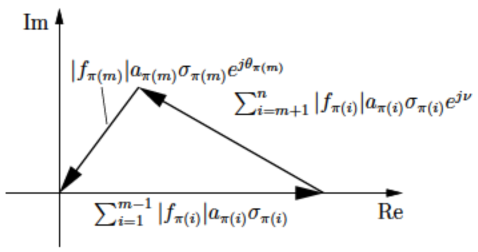
The corresponding values of $\theta_1, \ldots, \theta_n$ that achieve the values $|\Psi| = \sum_{i=1}^n |f_i a_i| \sigma_i$ and $|\Psi| = 2|f_k a_k| \sigma_k - \sum_{i=1}^n |f_i a_i| \sigma_i$ are given directly in the theorem. In order to show the achievability of $|\Psi| = 0$ for the case $|f_k| a_k \sigma_k \leq \frac{1}{2} \sum_{i=1}^n |f_i| a_i \sigma_i$, we introduce the order $\pi : \{1, n\} \to \{1, n\}$, which orders the values $|f_i| a_i \sigma_i$ such that

$$|f_{\pi(1)}| a_{\pi(1)} \sigma_{\pi(1)} \geq |f_{\pi(2)}| a_{\pi(2)} \sigma_{\pi(2)} \geq \ldots \geq |f_{\pi(n)}| a_{\pi(n)} \sigma_{\pi(n)}. \tag{B.6}$$

Moreover, we define m as the smallest integer that fulfills

$$\sum_{i=1}^{m} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)} \geq \sum_{i=m+1}^{n} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)}. \tag{B.7}$$

From the condition $|f_k| a_k \sigma_k \leq \frac{1}{2} \sum_{i=1}^n |f_i| a_i \sigma_i$ and (B.6) it follows that $2 \leq m \leq n-1$. Thus, we can set $\theta_{\pi(1)} = \theta_{\pi(2)} = \ldots = \theta_{\pi(m-1)} = 0$ and $\theta_{\pi(m+1)} = \theta_{\pi(m+2)} = \ldots = \theta_{\pi(n)} = \nu$. The remaining phases ν and $\theta_{\pi(m)}$ shall now be chosen such that $|\Psi| = 0$. This problem is illustrated by the following figure, which shows a triangle in $\mathbb{C}$ with edge points 0, $\sum_{i=1}^{m-1} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)}$, and $-|f_{\pi(m)}| a_{\pi(m)} \sigma_{\pi(m)} e^{j\theta_{\pi(m)}}$:



A necessary and sufficient condition for the existence of this triangle is that the triangle inequality holds for all combination of the side lengths. These inequalities are

$$\begin{aligned}
\sum_{i=1}^{m-1} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)} &\leq \sum_{i=m+1}^{n} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)} + |f_{\pi(m)}| a_{\pi(m)} \sigma_{\pi(m)} \\
\sum_{i=m+1}^{n} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)} &\leq \sum_{i=1}^{m-1} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)} + |f_{\pi(m)}| a_{\pi(m)} \sigma_{\pi(m)} \\
|f_{\pi(m)}| a_{\pi(m)} \sigma_{\pi(m)} &\leq \sum_{i=1}^{m-1} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)} + \sum_{i=m+1}^{n} |f_{\pi(i)}| a_{\pi(i)} \sigma_{\pi(i)}.
\end{aligned}$$

While the first two inequations hold because of the definition of m in (B.7), the final inequation holds because of (B.6) and $m > 1$. The phases can now be calculated by the law of cosines, which is omitted here.

Next, we show that R_Σ is convex with respect to ξ, which is done by calculating

$$\frac{\partial R_\Sigma}{\partial \xi} = 2\,\frac{||\mathbf{h}||^2||\mathbf{h}_d^{(1)}||^2(q_1-q_2)}{||\mathbf{h}||^2\Phi - ||\mathbf{h}_d^{(1)}||^2 s} - s^{-1}||\mathbf{h}||^2||\mathbf{h}_d^{(1)}||^2\left(q_1q_2 - \frac{s||\mathbf{h}||^2||\mathbf{h}_d^{(1)}||^2\xi(q_1-q_2)^2}{\left(||\mathbf{h}||^2\Phi - ||\mathbf{h}_d^{(1)}||^2 s\right)^2}\right)$$
$$- (\Phi-\xi)\,\frac{||\mathbf{h}||^4||\mathbf{h}_d^{(1)}||^4(q_1-q_2)^2}{\left(||\mathbf{h}||^2\Phi - ||\mathbf{h}_d^{(1)}||^2 s\right)^2}$$
$$\frac{\partial^2 R_\Sigma}{\partial \xi^2} = 2\frac{||\mathbf{h}||^4||\mathbf{h}_d^{(1)}||^4(q_1-q_2)^2}{\left(||\mathbf{h}||^2\Phi - ||\mathbf{h}_d^{(1)}||^2 s\right)^2}.$$

Hence, it is obvious that $\frac{\partial^2 R_\Sigma}{\partial \xi^2} > 0$ and the convexity is shown.

As R_Σ is convex and ξ lies in the interval specified in (B.5), it follows that the optimal value of ξ lies on the edge of the interval. Thus, it is optimal to make ξ either as small or as large as possible, which is achieved by the choices of $\theta_1, \ldots, \theta_n$ mentioned above.

B.3 Proof of Theorem 9

For joint relaying, we can use the sum-rate formulation of the last expression from (5.11). Contrary to the MISO case, $\mathbf{w}^{(k)}$ is now a scalar, such that

$$R_{\Sigma,\infty} = \lim_{P_r\to\infty} \log_2\left(1 + \sum_{k=1}^{K} |h_d^{(k)}|^2 P^{(k)} + \frac{||\mathbf{h}||^2\widetilde{\mathbf{f}}^H\mathbf{D}\widetilde{\mathbf{f}}}{1+||\mathbf{h}||^2\widetilde{\mathbf{f}}^H\widetilde{\mathbf{f}}}\right)$$
$$= \log_2\left(1 + \sum_{k=1}^{K} |h_d^{(k)}|^2 P^{(k)} + \frac{\widetilde{\mathbf{f}}^H\mathbf{D}\widetilde{\mathbf{f}}}{\widetilde{\mathbf{f}}^H\widetilde{\mathbf{f}}}\right),$$

where the second equality follows from the fact that as $P_r \to \infty$, $\widetilde{\mathbf{f}}$ can be made arbitrarily large. From the Rayleigh quotient, it follows that the last term in the logarithm is upper bounded by $\lambda_{\max}(\mathbf{D})$, which can be achieved by setting $\widetilde{\mathbf{f}} = \mathbf{v}_{\max}(\mathbf{D})$. Therefore, the optimal sum-rate with joint relaying is given by the expression in the theorem as $P_r \to \infty$. From (5.13), it follows that

$$R^{(k)}_{\mathrm{TDMA},\infty} = \lim_{Pr\to\infty} R^{(k)}_{\mathrm{TDMA}} = \tau^{(k)}\log_2\left(1 + \frac{P^{(k)}}{\tau^{(k)}}\left(|h_d^{(k)}|^2 + ||\mathbf{h}_r^{(k)}||^2\right)\right).$$

Considering the optimization of $R_{\Sigma,\mathrm{TDMA},\infty} = \sum_{k=1}^{K} R^{(k)}_{\mathrm{TDMA},\infty}$ over the duration of the time slots, we have a similar situation as for the SISO MAC without relay described in (3.3). Therefore, the solution is given by (cf. [CT06])

$$\tau^{(k)} = \frac{|h_d^{(k)}|^2 + ||\mathbf{h}_r^{(k)}||^2}{\sum_{l=1}^{K}\left(|h_d^{(l)}|^2 + ||\mathbf{h}_r^{(l)}||^2\right)} \tag{B.8}$$

and we obtain the optimal TDMA sum-rate as given in the theorem.

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Lebenslauf

Der Lebenslauf wurde aus Gründen des Datenschutzes entfernt.

CV

The CV has been removed for reasons of data protection.