



Spreading and Precoding for Wireless MIMO-OFDM Systems

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von

DORIS YASMOU YACOUB

AUS HELIOPOLIS/KAIRO

1. Gutachter: Prof. Dr.-Ing. Jürgen Lindner
 2. Gutachter: Prof. Dr. rer. nat. Dr. h. c. Hermann Rohling
- Amtierender Dekan: Prof. Dr. rer. nat. Helmuth Partsch

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Contents

1	Introduction	1
2	Theoretical Background	3
2.1	MIMO Channel Model	4
2.2	Orthogonal Frequency Division Multiplexing	5
2.2.1	SISO-OFDM	5
2.2.2	MIMO-OFDM	11
2.2.3	Spreading	14
2.3	Correlated MIMO-OFDM Channel Model	16
2.4	Effect of Antenna Correlations	19
2.4.1	Channel Condition Number	19
2.4.2	Diversity and Correlation Measures	19
2.5	Block Equalizers	22
2.5.1	Hard Decision Equalizers	23
2.5.2	Soft Decision Equalizers	25
2.6	Iterative Equalization and Decoding	29
2.7	Test Channels	34
2.8	Summary	40
3	Spreading	41
3.1	Diversity	42
3.2	Space Time and Space Frequency Codes	44
3.3	Spreading Criteria for MIMO-OFDM	46
3.4	MC-Code Division Multiplexing	48
3.5	MC-Cyclic Antenna Frequency Spreading	49
3.6	Spreading, Matched Filter Bound and Interference	59
3.6.1	Effect of Antenna Correlations on Interference and MFB	62
3.6.2	Asymptotic Behavior of α and β	66
3.6.3	Effect of Interference Distribution on the BER	70

Contents

3.7	Rotated MC-CAFS	72
3.7.1	Rotations Type I	73
3.7.2	Rotations Type II	73
3.8	Coded Transmission	81
3.9	Summary	84
3.A	Appendix to Chapter 3	85
3.A.1	Expected Values of Functions of Correlated Complex Random Variable for Kronecker Correlation Model	85
3.A.2	Expected values of α , β and the $\text{var}(\bar{r})$	87
4	Precoding	91
4.1	Linear Precoding	92
4.2	Full Channel Knowledge at the Transmitter	93
4.3	Partial Channel Knowledge at the Transmitter	94
4.3.1	Flat Fading Channels	95
4.3.2	Frequency Selective Channels	96
4.4	Uncoded Transmission	99
4.4.1	Imperfect Transmit Correlation Knowledge	102
4.4.2	Different Antenna Correlation at each Channel Tap	105
4.5	Coded Transmission	107
4.6	Summary	118
4.A	Appendix to Chapter 4	120
4.A.1	Antenna Correlations in Frequency Domain	120
4.A.2	BER curves and EXIT-Charts	121
5	MIMO Channel Capacity: Theory versus Measurement	131
5.1	MIMO Channel Capacity	131
5.1.1	No channel knowledge at the transmitter	132
5.1.2	Full channel knowledge at the transmitter	134
5.1.3	Partial channel knowledge at the transmitter	135
5.1.4	Impact of Antenna Correlations on MIMO Capacity	136
5.1.5	Impact of Line of Sight on MIMO Capacity	136
5.2	Measurement Setup	138
5.3	Measured Channel Capacity	141
5.4	Two Path Channel Model	145
5.5	Antenna Correlations	148
5.6	Summary	151
6	Summary and Conclusion	153

A	Matrix Basics	157
A.1	Matrix Basics	157
A.1.1	Inverse, Transpose and Hermitian	157
A.1.2	Trace, Determinant and Frobenious Norm	158
A.2	Mathematical Definitions	159
A.2.1	Kronecker Product	159
A.2.2	Hadamard Product	159
B	Mathematical Notations, Symbols and List of Abbreviations	161

Contents

Chapter *1*

Introduction

In recent years, multiple input multiple output (MIMO) systems have gained considerable attention due to their potential of achieving very high data rates and for providing a new diversity, *spatial diversity*, to the communication system. Multicarrier (MC) transmission schemes on the other hand are considered to be promising candidates for the fourth generation (4G) of mobile communications due to their efficient utilization of the available bandwidth, thus also allowing for high data rates. Orthogonal frequency division multiplexing (OFDM) is one of several MC variants and is a well-known technique used in broadcast media like, e.g. European terrestrial digital television (DVB-T) and digital audio broadcasting (DAB), and in wireless local area networks (WLAN). Thus, MIMO-OFDM transmission schemes, which offer both spatial and frequency diversity, have become an important area of research.

The goal of this work is to introduce and present new methods that exploit both the frequency and spatial diversities, i.e. utilize all diversity branches provided by MIMO-OFDM, in order to improve the system performance. Before we proceed to give an outline of this dissertation, we would like to give a short analogy between the system considered here and the game of chance, Roulette. Roulette is the french word for *small wheel* and is a gambling game

1 Introduction

where a wheel is spun in one direction, and a ball in the opposite. The ball finally falls on the wheel and into one of the 38 colored and numbered holes on it. Players can place their bets, for example, on the number of the hole the ball might land in or on a range of holes. Without any knowledge about the ball's speed or the Roulette wheel's rotational speed, any hole on the wheel is equally probable from the player's point of view and he/she might just as well bet on *any* of 38 holes or any range of holes. However, if – as the physics student Farmer did in 1978 – the player had knowledge of the initial ball's speed and the wheel's rotational speed, the range where the ball might fall can be limited to a small range, a sector of the wheel. The player then has a much better chance of winning. In the best case, when all the parameters are known, the hole where the ball falls can be fully predicted and the player then only needs to bet on this one hole. Our communication system can be compared to the Roulette wheel and ball and our transmitted symbols to the bets placed by the players. If nothing is known about the communication channel at the transmitter, the best one can do is to transmit all signals equally (bets) over all diversity branches (all Roulette holes). If partial channel knowledge is available (a sector of the wheel), then transmitting in that *approximate* direction can improve the system performance over the no knowledge case. Finally, if full channel knowledge is available, then the perfect direction of transmission is known and the performance can be improved even further. Of course, this is just a simplified analogy that serves as an example to aid the reader in understanding the idea and the structure behind this work, which is outlined as follows:

In Chapter 2, we present the theoretical background required for understanding this work. The MIMO-OFDM transmission model is presented in details which include, but not limited to, the modulation, demodulation, channel model and equalization. Chapter 3, deals with the case for which no channel knowledge is available at the transmitter. In this Chapter, we present the transmission scheme known as **spreading** and provide criteria for choosing spreading matrices that achieve the full diversity provided by MIMO-OFDM channel and introduce a family of spreading matrices satisfying those criteria. In Chapter 4, transmission with full or partial channel knowledge is presented. This transmission scheme is known as **precoding**. In this chapter, we will concentrate on the latter case, partial channel knowledge, and show the optimal direction for transmission. Finally, in Chapter 5, we present a theoretical overview of MIMO channel capacities for all of the afore described cases of channel knowledge at the transmitter. In addition, the capacities of measured MIMO channels for an outdoor scenario are examined and compared to the theoretical ones. Last but not least, throughout this work, we always assume the channel to be fully known at the receiver.

Parts of this work were published in [62, 65, 69, 90, 100, 106].

Chapter 2

Theoretical Background

The aim of this Chapter is to present a theoretical background for multiple-input-multiple-output orthogonal frequency division multiplexing or for short MIMO-OFDM and suboptimum equalization techniques suitable for detecting symbols transmitted over such systems. We start by introducing the concept of MIMO systems as well as an overview and a theoretical background for OFDM. We then combine those two systems into one (MIMO-OFDM) and derive the equivalent matrix vector transmission model in Sec. 2.2.2. We shall also show how spreading can be incorporated into the MIMO-OFDM matrix vector model. A distinctive feature of MIMO systems –antenna correlations– and a mathematical model of it shall be presented in Sec. 2.3. Depending on the magnitude of the antenna correlations, significant reduction in system performance and capacity are to be expected. Finally, in Sec. 2.5, we describe different block equalizers that have been used in this work to detect symbols transmitted over MIMO-OFDM systems.

From now on, vectors and matrices will be denoted by underlined and doubly-underlined letters, respectively. All vectors are column vectors. Lowercase letters will be used for the time domain whereas the uppercase LETTERS for the frequency domain. Scalars will be simply designated by letters.

2 Theoretical Background

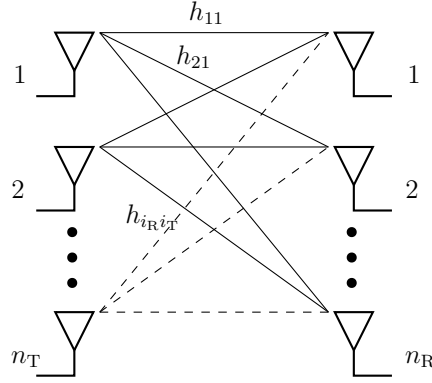


Figure 2.1: Illustration of an $n_R \times n_T$ MIMO Channel.

2.1 MIMO Channel Model

Multiple input multiple output (MIMO) systems have emerged in recent years due to their capability of tremendously increasing the system capacity [1, 2, 3, 4]. A MIMO system is comprised of n_T transmit antennas and n_R receive antennas providing spatial diversity that can be exploited at the transmitter and/or receiver to improve the system performance and/or to achieve higher data rates. In addition, as will be shown in Chapters 3 and 4, through appropriate coding or spreading the MIMO systems can lead to a more reliable transmission.

An $n_R \times n_T$ MIMO system is shown in Fig. 2.1. Usually Rayleigh fading between all transmit and receive antenna pairs is assumed. The MIMO channel in the time domain can thus be represented by an $n_R \times n_T$ matrix, $\underline{\underline{h}}$, as follows

$$\underline{\underline{h}} = \begin{bmatrix} h_{11} & \dots & \dots & \dots & h_{1n_T} \\ \vdots & \ddots & h_{i_R i_T} & \ddots & \vdots \\ h_{n_R 1} & \dots & \dots & \dots & h_{n_R n_T} \end{bmatrix}, \quad (2.1)$$

where the elements $h_{i_R i_T}$ represent the fading coefficients between transmit antenna i_T and receive antenna i_R and are assumed to be complex Gaussian with mean zero and variance σ^2 ($\text{CN}(0, \sigma^2)$). The variance σ^2 is usually normalized to 1.0. The received signal $\tilde{\underline{x}}$, as will be shown in details in Sec. 2.2.1, can be shown to be

$$\tilde{\underline{x}} = \underline{\underline{h}} \underline{x} + \underline{n}, \quad (2.2)$$

where $\underline{x}$ is the $n_T \times 1$ transmit vector, $\tilde{\underline{x}}$ is the $n_R \times 1$ receive vector and $\underline{n}$ is the additive white Gaussian noise (AWGN).

2.2 Orthogonal Frequency Division Multiplexing

Orthogonal frequency division multiplexing (OFDM) is a special case of frequency division multiplexing (FDM). The basic idea of OFDM is to divide the available spectrum into several subchannels narrow enough that signals transmitted over those subchannels experience flat fading. In addition, all of the subcarriers are transmitted simultaneously. As will be shown in the next section, the subchannels are chosen such that they are orthogonal to each other and thus no guard bands are required. In fact, the subchannels are overlapping and the resulting channel is a sum of the narrow orthogonal subchannels. OFDM is thus capable of achieving a high spectral efficiency while at the same time is able to combat multipath fading. The use of discrete Fourier transform (DFT) for modulation and demodulation, proposed by Weinstein and Ebert in 1971, eliminated the need for subcarrier oscillators [5]. However, since Weinstein and Ebert only used a guard space between the OFDM symbols, they could not obtain perfect orthogonality between the subcarriers over multipath channels. In 1980, Peled and Ruiz were able to solve the orthogonality problem by introducing the cyclic prefix (CP) to replace the guard space. In the following section, we shall first describe SISO-OFDM using a CP and show its equivalent matrix vector transmission model. We then extend this model to the MIMO case. In all that follows, we shall assume that the channel is constant during one OFDM symbol duration.

2.2.1 SISO-OFDM

OFDM is a combination of multiplexing and linear modulation. Multiplexing is a technique of using a single channel for parallel transmission. A general model of multiplex transmission is shown in Fig. 2.2 [6]. Every T_s seconds,

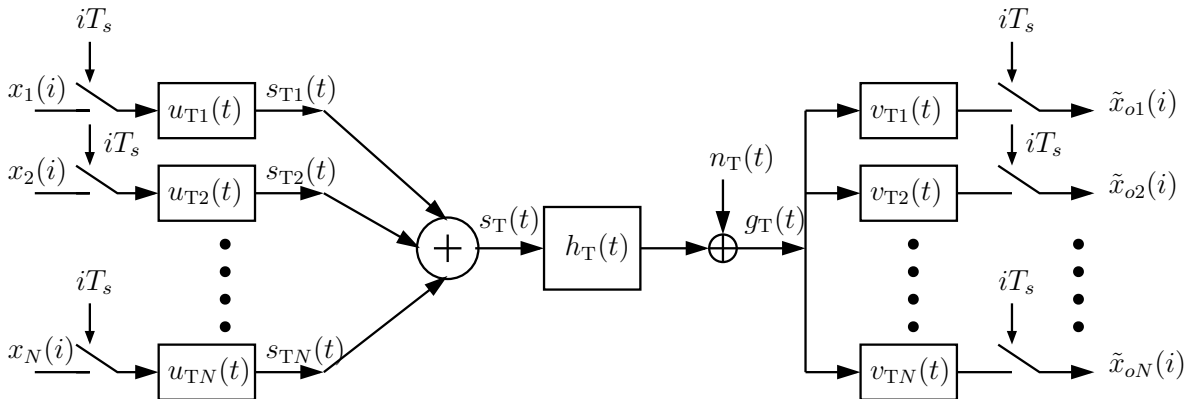


Figure 2.2: Multiplex transmission over a continuous time SISO channel.

2 Theoretical Background

the k -th symbol $x_k(i)$ is transmitted over the k -th multiplex channel, where i is the time instance. The transmission occurs at the same symbol rate in all multiplexing channels. In case of digital transmission with linear modulation schemes, each of the transmit signals in the low pass domain, $s_{Tk}(t)$, $k = 1, \dots, N$, is given by [6]

$$s_{Tk}(t) = \sum_i x_k(i) u_{Tk}(t - iT_s); \quad k = 1, 2, \dots, N, \quad (2.3)$$

where $u_{Tk}(t)$ are the basic waveforms at the transmitter. The transmit signal $s_T(t)$ is then the sum of all $s_{Tk}(t)$ signals, $s_T(t) = \sum_k s_{Tk}(t)$. The received signal $g_T(t)$ is thus the convolution (*) of the transmit signal $s_T(t)$ with the channel impulse response, $h_T(t)$, plus the additive noise $n_T(t)$,

$$g_T(t) = s_T(t) * h_T(t) + n_T(t). \quad (2.4)$$

At the receiver, $g_T(t)$ is convolved with the receive filter impulse response, $v_{Tk}(t)$, and sampled at time instances iT_s . The receive filter considered here is the channel matched filter (CMF). The output, $\tilde{x}_{ok}(i)$, after the Dirac delta sampler at time instance i is thus,

$$\tilde{x}_{ok}(i) = g_T(t) * v_{Tk}(t)|_{t=iT_s}. \quad (2.5)$$

The choice of the basic waveforms $u_{Tk}(t)$ for OFDM is based on a special property of linear time invariant (LTI) systems, which is: complex exponential functions are eigenfunctions of LTI systems. That is, for an input signal $s_T(t) = e^{j2\pi f_k t}$ of infinite duration, the output signal, $g_T(t)$, after the channel, $h_T(t)$, is

$$g_T(t) = H(f_k) e^{j2\pi f_k t}, \quad (2.6)$$

where $H(f_k)$ is the channel transfer function at frequency f_k . In other words, the resulting eigenvalue of the LTI system excited at frequency f_k is equal to the channel transfer function at that same frequency [6]. In case of OFDM, the basic waveforms $u_{Tk}(t)$ are also complex exponential functions, but they are time limited. The basic waveforms are given by

$$u_{Tk}(t) = \text{rect}\left(\frac{t}{T}\right) e^{j2\pi f_k t}, \quad k = 1, \dots, N. \quad (2.7)$$

Since the OFDM basic waveforms are time limited, intersymbol interference (ISI) can not be avoided if the channel is linearly distorting. Nonetheless, as we shall show next, through the use of a cyclic prefix and with the proper choice of its duration T_G , ISI can be avoided at the receiver.

2.2 Orthogonal Frequency Division Multiplexing

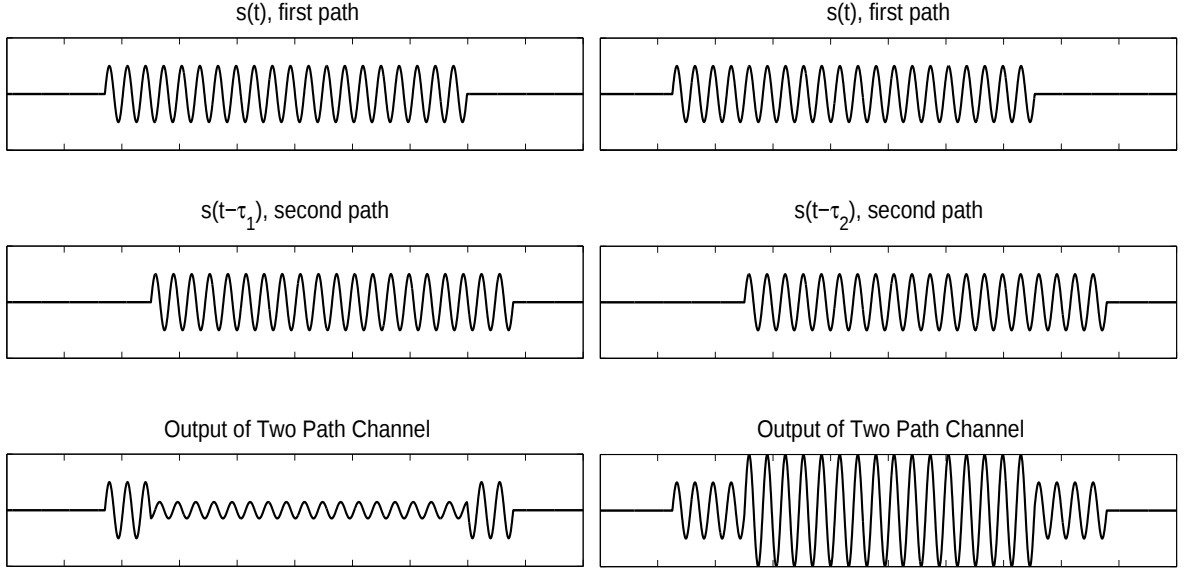


Figure 2.3: Output of a two path channel for different delay times.

Figure 2.3 shows the output signal, $g_T(t)$, for a time limited signal $s_T(t)$ over a two path channel for different delay times, τ_i , $i = 1, 2$, and $\tau_2 > \tau_1$. Transient conditions can be observed at the beginning and at the end of the received signal. The middle part, $g_{Tm}(t)$, is stationary and matches that of the unlimited signal in Eqn. 2.6. That is,

$$\begin{aligned} g_{Tm}(t) &= \text{rect}\left(\frac{t - \tau_i/2}{T_m}\right) g_T(t) \\ &= \text{rect}\left(\frac{t - \tau_i/2}{T_m}\right) H(f_k) e^{j2\pi f_k t}. \end{aligned} \quad (2.8)$$

where T_m is the duration of this stationary interval and is dependent on the delay, τ_i , of the two path channel, $T_m = T - \tau_i$. The larger the delay, the shorter T_m . Thus, in order to achieve that $T_m = T$, the cyclic prefix comes into play. The cyclic prefix is the copy of the last part of the OFDM symbol that is "prefixed" to the transmitted signal as shown in Fig. 2.4. The duration of the cyclic prefix, T_G , should be at least that of the maximum delay on the channel, i. e. $T_G \geq \tau_{max}$, to insure no ISI occurs and $T_m = T$. Accordingly, the OFDM transmit signal $s_T(t)$, is given by [5, 6, 7]

$$s_T(t) = \sum_{k=1}^N \sum_i x_k(i) \text{rect}\left(\frac{t - iT_S}{T_S}\right) e^{j2\pi f_k t} = \sum_{k=1}^N \sum_i x_k(i) u_{Tk}(t - iT_S), \quad (2.9)$$

where $T_S = T + T_G$. Note that in contrast to Eqn. 2.7, $u_{Tk}(t)$ is now of duration T_S . At the receiver, the CP is removed and the receive filters $v_{Tk}(t)$ are thus

2 Theoretical Background

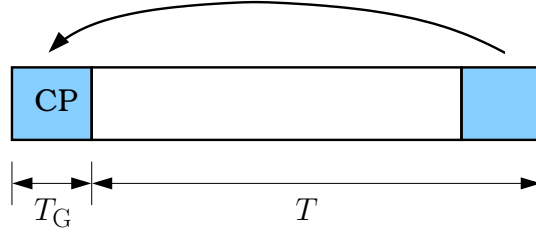


Figure 2.4: Addition of cyclic prefix to the OFDM symbol in time domain.

only matched to the last part of the transmit filters [5, 6, 7]. The received signal, $g_T(t)$, after CP removal is thus

$$g_T(t) = \sum_{k=1}^N H(f_k) x_k(i) \text{rect} \left(\frac{t}{T} \right) e^{j2\pi f_k t} + n_T(t). \quad (2.10)$$

That is, only the stationary part of the received signal is considered and the CMFs $v_{Tk}(t)$ are given by

$$v_{Tk}(t) = \text{rect} \left(\frac{t}{T} \right) e^{-j2\pi f_k t}, \quad k = 1, \dots, N. \quad (2.11)$$

Accordingly, $\tilde{x}_{ok}$ is

$$\tilde{x}_{ok}(i) = g_T(t) * v_{Tk}(t)|_{t=iT_s} = H(f_k) x_k(i) + n_{Tv}(i), \quad (2.12)$$

Note that the cyclic prefix also insures that no interblock interference (IBI) occurs between two OFDM symbols, since as mentioned above the CP is just removed at the receiver. To obtain an orthogonal system, the frequencies f_k must satisfy the following condition [6, 8]

$$f_k = f_c + \frac{k}{T}, \quad (2.13)$$

where f_c can be any chosen frequency.

OFDM Implementation

In practice, OFDM transmission systems usually employ digital signal processing. In discrete-time OFDM systems, modulation and demodulation are replaced by inverse discrete Fourier transform (IDFT) and discrete Fourier transform (DFT), respectively [9, 6]. Figure 2.5 shows the discrete time implementation of an OFDM transmission system. Each transmit vector, $\underline{x}(i)$, is modulated onto the N subcarriers using the IDFT,

$$\underline{s}(i) = \underline{F}_o^{-1} \underline{x}(i), \quad (2.14)$$

2.2 Orthogonal Frequency Division Multiplexing

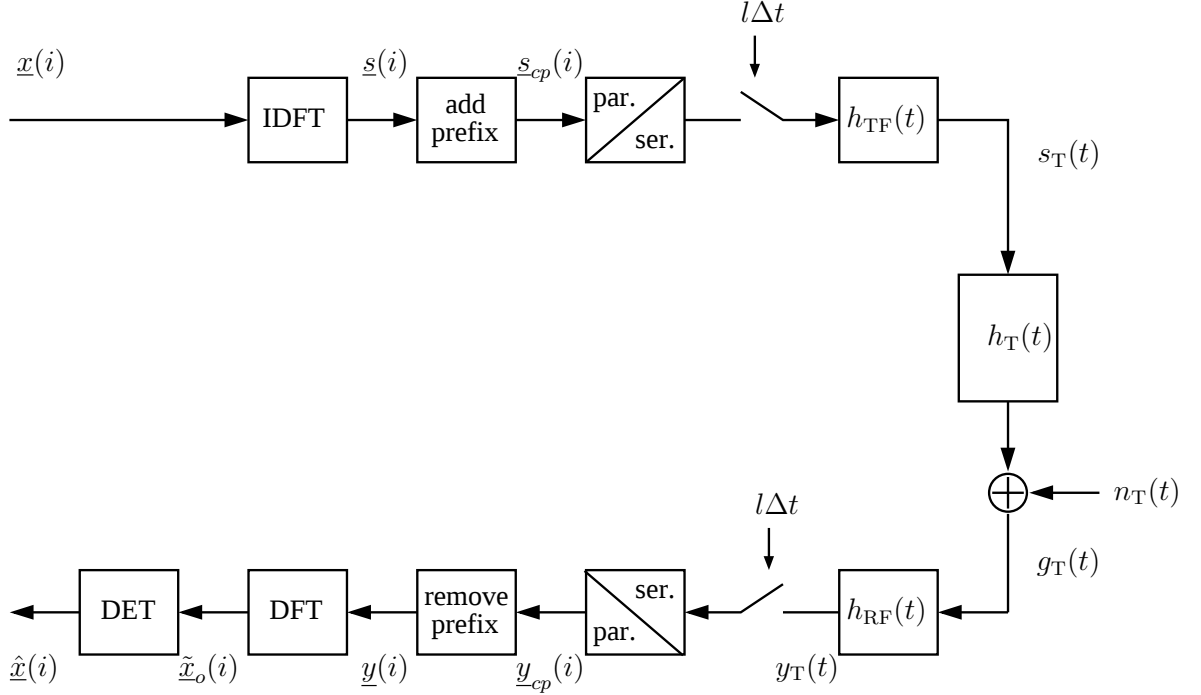


Figure 2.5: OFDM system model with conventional OFDM receiver.

where $\underline{F}_o$ is the Fourier matrix [10] and $\underline{F}_o^{-1} = \underline{F}_o^H$ and $\underline{F}_o^H \underline{F}_o = \underline{I}$. $\underline{s}(i)$ is then appended by the cyclic prefix of length L_{cp} resulting in

$$\underline{s}_{cp}(i) = [s_{N-L_{cp}+1}(i) \cdots s_N(i) \quad \underline{s}(i)^T]^T \quad (2.15)$$

which, after parallel to serial conversion and Dirac delta sampling, is filtered by the transmit low pass filter, $h_{TF}(t)$, which is bandlimited to the cutoff frequency f_g . The transmit signal $s_T(t)$ is then passed over the channel with impulse response $h_T(t)$ and WGN is added. The received signal $g_T(t)$ is then passed through the receive low pass filter $h_{RF}(t)$ also bandlimited to the cutoff frequency f_g . To find the discrete time channel, $h(l)$, the sampling rate must satisfy the sampling theorem [11]. Thus, assuming $\Delta t = 1/2f_g$, the discrete time channel is [9]

$$h(l) = h_{TF}(t) * h_T(t) * h_{RF}(t)|_{t=l\Delta t}. \quad (2.16)$$

The cyclic prefix functions in the same way in this discrete system. It transforms the convolution to a circular convolution. The discrete time received signal after the CP removal, $\underline{y}(i)$, is thus be given by [12, 13]

$$\underline{y}(i) = \underline{h}_c \underline{s}(i) + \underline{n}(i), \quad (2.17)$$

where $\underline{h}_c$ is an $N \times N$ matrix having cyclically shifted versions of the discrete time channel impulse response in its rows and the first row of $\underline{h}_c$ is

2 Theoretical Background

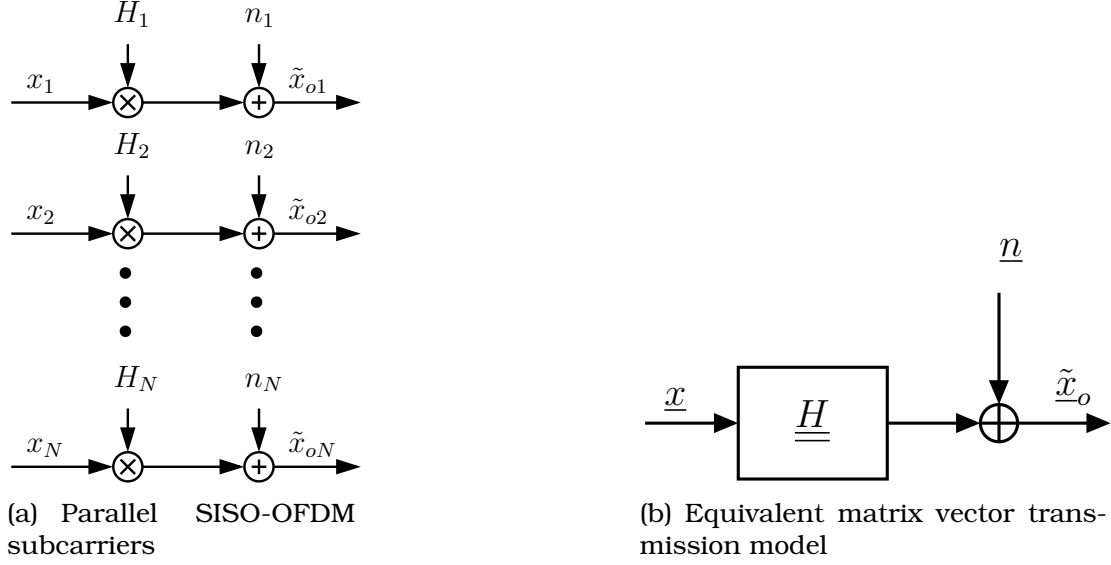


Figure 2.6: Matrix vector SISO-OFDM transmission model.

$(h_1 \ 0 \ 0 \cdots h_L \ h_{L-1} \cdots h_3 \ h_2)$ and L is the number of channel taps. Thus

$$\tilde{x}_o(i) = \underline{F}_o \underline{h}_c \underline{F}_o^H \underline{x}(i) + \underline{n}(i) = \underline{H} \underline{x}(i) + \underline{n}(i). \quad (2.18)$$

Since the eigenvectors of $N \times N$ cyclic matrices are given by the columns of the Fourier matrices of the same size [14], the $\underline{F}_o$ matrix diagonalize the cyclic matrix $\underline{h}_c$, resulting in the channel transfer function on the main diagonal of $\underline{H}$. Note that the CP represents a loss in the SNR at the receiver (parallel shift of the BER curves to the right). This loss is given as $\gamma = 10 \log_{10}(\frac{N+L_{cp}}{N})$. Unless otherwise specified, we assume that $L_{cp} = L - 1$.

Since we assume that the channel remains constant during one OFDM symbol, from now on, we shall drop the time index i . The SISO-OFDM channel matrix is then given by,

$$\underline{H} = \text{Diag}(H_k) = \begin{bmatrix} H_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots & \vdots \\ \vdots & 0 & H_k & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & H_N \end{bmatrix}. \quad (2.19)$$

We can thus represent the OFDM transmission, by an equivalent matrix vector transmission model as shown in Figure 2.6. Figure 2.6(a) shows the parallel OFDM subchannels, while Fig. 2.6(b) shows the equivalent matrix vector transmission model, where $\underline{H}$ is as described by Eqn. 2.19. Note that channel matched filtering can be applied at the receiver by multiplying $v_{Tk}(t)$

2.2 Orthogonal Frequency Division Multiplexing

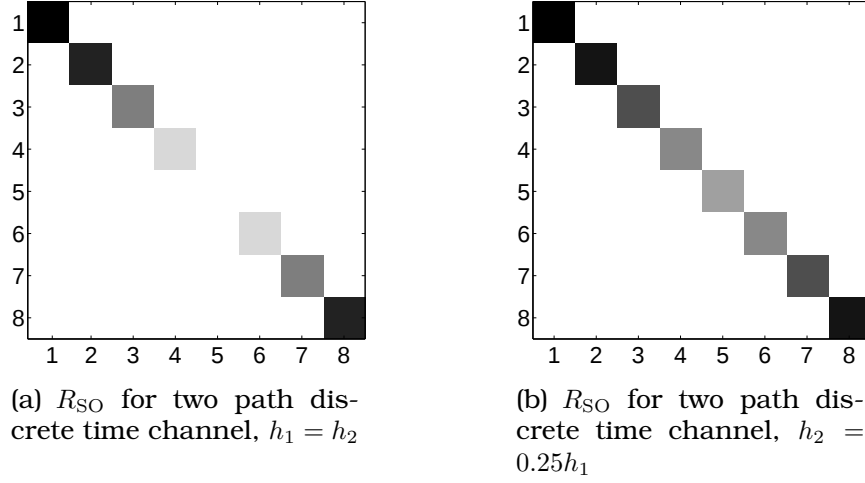


Figure 2.7: SISO-OFDM correlation matrix for different frequency selective channels.

in Eqn. 2.11 by the complex conjugate of the channel transfer functions, $H^*(f_k)$ [6]. Channel matched filtering can equivalently be applied in the frequency domain to obtain the channel correlation matrix for SISO-OFDM, $\underline{\underline{R}}_{SO}$, as follows [13, 15, 16],

$$\underline{\underline{R}}_{SO} = \underline{\underline{H}}^H \underline{\underline{H}}. \quad (2.20)$$

In SISO-OFDM, it can happen that one or more subcarriers suffer from a deep fade as can be seen in Fig. 2.7 for $h_1 = h_2$. Spreading and coding are two possible solutions to overcome this problem.

Assuming ideal lowpass filters (interpolation and anti-aliasing filters) at both the transmitter, $h_{TF}(t)$, and the receiver, $h_{RF}(t)$, with cutoff frequency, f_g , then,

$$N = \frac{1}{\Delta t \Delta f}, \quad \Delta f = \frac{1}{T}, \quad \Delta t = \frac{1}{2f_g} \quad (2.21)$$

where N is the DFT length, Δf the minimum distance between the OFDM subcarriers and Δt the sampling interval. Note that the optimum choice of the number of subcarriers, N , and the length of the guard interval, L_{cp} , is dependent on the Doppler frequency, f_D , the root mean square delay spread (RMS), τ_{RMS} , as well as the transmission rate [17].

2.2.2 MIMO-OFDM

In case of multipath channels, we represent our MIMO channels by L matrices of size $n_R \times n_T$. The channel matrices, $\underline{\underline{h}}(l)$, after sampling according to

2 Theoretical Background

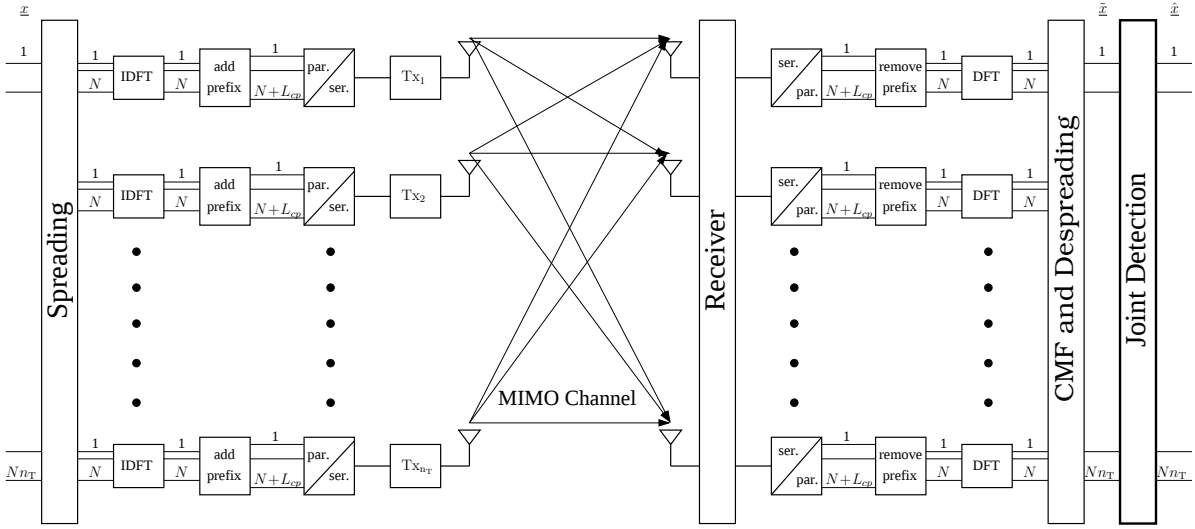


Figure 2.8: MIMO-OFDM system model with conventional OFDM receiver.

the sampling theorem are given as follows:

$$\underline{\underline{h}}(l) = \begin{bmatrix} h_{11}(l) & \dots & \dots & \dots & h_{1n_T}(l) \\ \vdots & \ddots & h_{i_R i_T}(l) & \ddots & \vdots \\ h_{n_R 1}(l) & \dots & \dots & \dots & h_{n_R n_T}(l) \end{bmatrix}, \quad (2.22)$$

where $h_{i_R i_T}(l)$ is the l -th tap of channel impulse response (CIR) between transmit antenna i_T ($i_T = 1, \dots, n_T$) and receive antenna i_R ($i_R = 1, \dots, n_R$), and ($l = 1, \dots, L$).

In the frequency domain, each transmit antenna can transmit over N OFDM subcarriers, with these being the same for all transmit antennas. One OFDM symbol, $\underline{x}$, is thus a vector of length $N \times n_T$. Figure 2.8 shows the system model for MIMO-OFDM. As in SISO-OFDM, modulation and demodulation can be efficiently implemented through the use of the inverse discrete Fourier transform (IDFT) and the discrete Fourier transform (DFT) respectively. We also assume that the length of the cyclic prefix (L_{cp}) is large enough to maintain orthogonality among the subcarriers. Accordingly, the channel correlation matrix, $\underline{\underline{R}}_{MO}$, for this MIMO-OFDM system can be described by the following equation [18, 19]:

$$\underline{\underline{R}}_{MO} = \frac{1}{n_R} \sum_{i_R=1}^{n_R} \underline{\underline{R}}_{MO, i_R} = \frac{1}{n_R} \sum_{i_R=1}^{n_R} \underline{\underline{H}}_{i_R}^H \underline{\underline{H}}_{i_R}, \quad (2.23)$$

where $\underline{\underline{H}}_{i_R}$ is an $N \times (N n_T)$ matrix containing the transfer functions of the channel impulse responses between the i_R -th receive antenna and all trans-

2.2 Orthogonal Frequency Division Multiplexing

mit antennas, $\underline{H}_{i_R} = [\underline{H}_{i_R 1}, \dots, \underline{H}_{i_R n_T}]$, where $(\cdot)^H$ denotes the complex conjugate transpose operation also known as Hermitian. The submatrices $\underline{H}_{i_R i_T}$ are $N \times N$ diagonal matrices and are given by:

$$\underline{H}_{i_R i_T} = \text{Diag}(\text{DFT}(h_{i_R i_T})) = \begin{bmatrix} H_{i_R i_T}(1) & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots & \vdots \\ \vdots & 0 & H_{i_R i_T}(k) & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & H_{i_R i_T}(N) \end{bmatrix}, \quad (2.24)$$

where $h_{i_R i_T} = [h_{i_R i_T}(1), \dots, h_{i_R i_T}(l), \dots, h_{i_R i_T}(L)]$ and represents the channel impulse response (CIR) between transmit antenna i_T and receive antenna i_R , and L is the maximum number of taps. DFT stands for the discrete Fourier transform and accordingly $H_{i_R i_T}(k)$ is the transfer function at frequency k , $k = 1, \dots, N$ for the channel between transmit antenna i_T and receive antenna i_R .

The channel correlation matrix, $\underline{R}_{\text{MO}}$, can alternatively be described by the following equation [19, 18]:

$$\underline{R}_{\text{MO}} = \frac{1}{n_R} \underline{H}^H \underline{H}, \quad (2.25)$$

where $\underline{H}$ is an $(N n_R) \times (N n_T)$ matrix containing the transfer functions of the channel impulse responses between all receive and transmit antennas. $\underline{H}$ is given by

$$\underline{H} = \begin{bmatrix} \underline{H}_{11} & \dots & \dots & \dots & \underline{H}_{1n_T} \\ \vdots & \ddots & \underline{H}_{i_R i_T} & \ddots & \vdots \\ \underline{H}_{n_R 1} & \dots & \dots & \dots & \underline{H}_{n_R n_T} \end{bmatrix}, \quad (2.26)$$

where $i_T = 1 \dots n_T$ and $i_R = 1 \dots n_R$. The submatrices $\underline{H}_{i_R i_T}$ are $N \times N$ diagonal matrices and are also given by Eqn. 2.24. Note that the diagonal elements, r_{ii} , of $\underline{R}_{\text{MO}}$ are always positive and contain the receive diversity. The diagonal elements are given by $r_{ii} = 1/n_R \sum_{i_R=1}^{n_R} H_{i_R i_T}^*(k) H_{i_R i_T}(k)$, where i represents the diagonal element belonging to transmit antenna i_T and frequency k . This weighted sum over all receive antennas obtained through the use of the CMF (Eqns. 2.25 and 2.23) is also known as maximum ratio combining (MRC) [6]. The MRC operation maximizes the SNR if the channel is perfectly known at the receiver [20].

Figure 2.9 shows the matrix vector transmission model for MIMO-OFDM. The channel matrices, $\underline{H}$, unlike for SISO-OFDM are not diagonal matrices. Intersymbol interference occurs between symbols that are transmitted from

2 Theoretical Background

different antennas at the same frequency. Figure 2.10 shows a typical correlation matrix for $\underline{\underline{R}}_{\text{MO}}$ with $n_{\text{R}} = n_{\text{T}} = 4$ and $N = 8$. The darker the blocks the higher the absolute value at that position (white corresponds to zero values). The off-diagonal elements represent the interference occurring in the MIMO-OFDM system. There are four off diagonal elements representing the interference from the four transmit antennas.

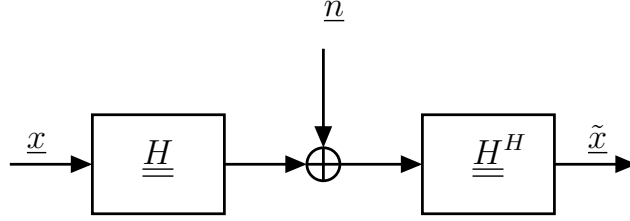


Figure 2.9: Matrix vector MIMO-OFDM transmission model with matched filter.

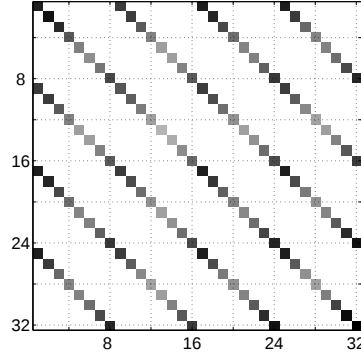


Figure 2.10: MIMO-OFDM correlation matrix, $n_{\text{T}} = n_{\text{R}} = 4$, $N = 8$.

2.2.3 Spreading

Spreading at the transmitter can be represented through premultiplication by a spreading matrix, $\underline{\underline{U}}$, and despreading at the receiver through postmultiplication by the despreading matrix, $\underline{\underline{U}}^H$ (Fig. 2.11), as follows [13, 16, 6]

$$\underline{\underline{R}}_{\text{S}} = \underline{\underline{U}}^H \underline{\underline{R}}_{\text{MO}} \underline{\underline{U}}, \quad (2.27)$$

where $\underline{\underline{R}}_{\text{S}}$ is the resulting channel correlation matrix. Based on Eqns. 2.25 and 2.27, we can define an effective channel matrix, $\underline{\underline{H}}_{\text{eff}}$, as follows

$$\underline{\underline{H}}_{\text{eff}} = \underline{\underline{H}} \underline{\underline{U}}, \quad (2.28)$$

2.2 Orthogonal Frequency Division Multiplexing

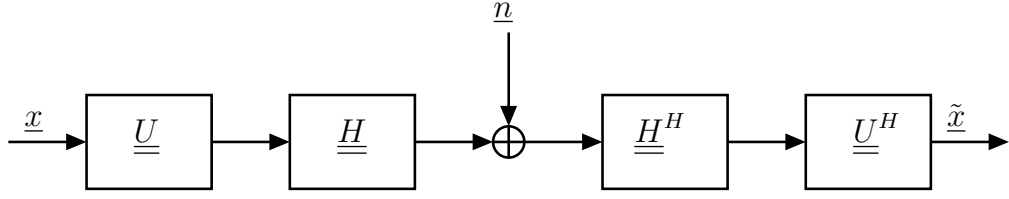


Figure 2.11: Matrix vector MIMO-OFDM transmission model with matched filter, spreading and despreading.

which includes the spreading matrix. The received OFDM symbol $\tilde{x}$ can thus be given by

$$\tilde{x} = \underline{R}x + \tilde{n}_c, \quad (2.29)$$

where $\underline{R}$ is either the unsprung, $\underline{R}_{\text{MO}}$, or spread channel matrix, $\underline{R}_{\text{S}}$, and $\tilde{n}_c$ the colored noise of variance $2N_o\underline{R}$. The equivalent matrix vector transmission model is shown in Fig. 2.12.

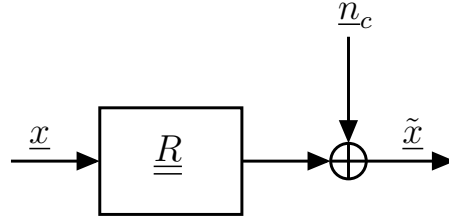


Figure 2.12: MIMO-OFDM transmission using the channel correlation matrix.

Channel Normalization

In all simulations results shown in this work, the MIMO channel has been normalized such that

$$\mathcal{E}\left\{\sum_{l=1}^L |h_{i_{\text{R}}i_{\text{T}}}(l)|^2\right\} = 1.0, \quad (2.30)$$

where $\mathcal{E}\{\cdot\}$ stands for the expected value. Unless otherwise specified, we assume block fading channels, where the channel remains constant during the transmission of one OFDM symbol and changes randomly from one symbol to the next. In the frequency domain, this normalization translates to

$$\mathcal{E}\{\text{Tr}(\underline{R}_{\text{MO}})\} = \mathcal{E}\left\{\sum_{i=1}^{n_{\text{T}}N} r_{ii}\right\} = \mathcal{E}\left\{\sum_{i=1}^{n_{\text{T}}N} \lambda_i(\underline{R}_{\text{MO}})\right\} = n_{\text{T}}N, \quad (2.31)$$

2 Theoretical Background

where λ_i and r_{ii} are the i -th eigenvalue and diagonal element of $\underline{\underline{R}}_{\text{MO}}$ respectively and Tr denotes its trace. Note that for any channel realization

$$n_{\text{R}} \text{Tr}(\underline{\underline{R}}_{\text{MO}}) = \text{Tr}(\underline{\underline{H}}^H \underline{\underline{H}}) = N \sum_{l=1}^L \|\underline{h}(l)\|_F^2, \quad (2.32)$$

where $\|\underline{h}(l)\|_F^2$ is the Frobenious norm of $\underline{h}(l)$, $l = 1, \dots, L$. Since only orthonormal spreading matrices are considered, the eigenvalues of $\underline{\underline{R}}_{\text{MO}}$ do not change after spreading. That is, Eqn.2.31 still applies to $\underline{\underline{R}}_{\text{S}}$. The eigenvectors do however change.

Another normalization method, would be to normalize each channel such that

$$\text{Tr}(\underline{\underline{R}}) = n_{\text{T}} N. \quad (2.33)$$

This type of normalization is suitable for time invariant channels or if power control is assumed at the transmitter.

2.3 Correlated MIMO-OFDM Channel Model

As mentioned earlier in this chapter, antenna correlations lead to reduced system performance and lower capacity. In this section, we present the Kronecker correlation model widely used to model antenna correlations in MIMO systems. The effect of correlations on the system BER performance and channel capacity shall be discussed in more details in Chapters 3, 4 and 5.

Again, we consider the MIMO channel in the time domain (Eqn.2.22) with n_{T} transmit antennas, n_{R} receive antennas and L taps. We define the channel vector, $\underline{h}_v$, as follows,

$$\underline{h}_v = [\text{vec}\{\underline{h}(1)\}^T \cdots \text{vec}\{\underline{h}(l)\}^T \cdots \text{vec}\{\underline{h}(L)\}^T]^T, \quad (2.34)$$

where the operator $\text{vec}\{\cdot\}$ stacks the columns of the $n_{\text{R}} \times n_{\text{T}}$ matrix $\underline{h}(l)$ to form a vector of length $n_{\text{R}} n_{\text{T}}$, and T denotes the matrix transpose. The correlation matrix that includes all spatial and path correlations is thus,

$$\underline{\underline{R}}_v = \mathcal{E}\{\underline{h}_v \underline{h}_v^H\}. \quad (2.35)$$

Although Eqn. 2.35 does capture any correlation effect between the elements of $\underline{h}(l) \forall l$ a simpler model is often used. That model assumes the transmit correlations are only affected by the immediate surroundings of the transmit antennas, i. e. the transmit antenna correlations have no influence on the receive antenna correlations and vice versa, and that the path correlations can

2.3 Correlated MIMO-OFDM Channel Model

be separated from spatial correlations, the correlation matrix in Eqn. 2.35 can be expressed by the Kronecker product of the transmit, receive and path correlations as follows [21],

$$\underline{\underline{R}}_v = \underline{\underline{k}}_p \otimes \underline{\underline{k}}_T^T(1) \otimes \underline{\underline{k}}_R(1), \quad (2.36)$$

where $\underline{\underline{k}}_p$ is the path correlation, $\underline{\underline{k}}_T(1)$, $\underline{\underline{k}}_R(1)$ transmit and receive correlations at the non-delay tap $l = 1$ respectively. This correlation model is well known as the Kronecker correlation model [21, 22]. If, in addition, we assume an equal power delay profile (PDP) and no path correlations (i. e. the different channel taps fade independently), $\underline{\underline{k}}_p = \underline{\underline{I}}$, the correlation matrix $\underline{\underline{R}}_v$ in Eqn. 2.36 becomes

$$\underline{\underline{R}}_v = \underline{\underline{I}} \otimes \underline{\underline{k}}_T^T(1) \otimes \underline{\underline{k}}_R(1) \quad (2.37)$$

If correlation matrices of the other channel taps are to be taken into consideration, again assuming uncorrelated paths and equal PDP,

$$\underline{\underline{R}}_v = \text{BlkDiag}([\underline{\underline{R}}_{kv}(1) \cdots \underline{\underline{R}}_{kv}(l) \cdots \underline{\underline{R}}_{kv}(L)]), \quad (2.38)$$

where $\underline{\underline{R}}_{kv}(l) = \underline{\underline{k}}_T^T(l) \otimes \underline{\underline{k}}_R(l)$, and BlkDiag forms a block diagonal matrix with the matrices $\underline{\underline{R}}_{kv}(l)$ on the main diagonal. Based on the Kronecker correlation model described above, the correlated channel matrices, $\underline{\underline{h}}^c(l)$, can thus be modeled as follows [21, 22],

$$\underline{\underline{h}}^c(l) = \underline{\underline{k}}_R^{1/2}(l) \underline{\underline{h}}(l) \underline{\underline{k}}_T^{1/2}(l), \quad (2.39)$$

where $\underline{\underline{k}}_R(l) = \underline{\underline{k}}_R^{1/2}(l) \underline{\underline{k}}_R^{1/2}(l)$ and $\underline{\underline{k}}_T(l) = \underline{\underline{k}}_T^{1/2}(l) \underline{\underline{k}}_T^{1/2}(l)$ are the $n_R \times n_R$ receive and $n_T \times n_T$ transmit correlation matrices for the l -th tap respectively. The transmit and receive correlation matrices can be calculated as follows,

$$\underline{\underline{k}}_T(l) = \frac{1}{n_R} \mathcal{E}\{\underline{\underline{h}}(l) \underline{\underline{h}}^H(l)\}, \quad (2.40)$$

$$\underline{\underline{k}}_R(l) = \frac{1}{n_T} \mathcal{E}\{\underline{\underline{h}}(l) \underline{\underline{h}}^H(l)\}. \quad (2.41)$$

If we assume that the transmit and receive correlation matrices are the same for all taps, i. e.,

$$\underline{\underline{k}}(l) = \underline{\underline{k}}^{1/2}(l) \underline{\underline{k}}^{1/2}(l) = \underline{\underline{k}}^{1/2} \underline{\underline{k}}^{1/2} = \underline{\underline{k}}, \quad (2.42)$$

where $\underline{\underline{k}}$ is either the transmit or receive correlation matrix, then $\underline{\underline{R}}_v$ in Eqn. 2.38 simplifies to $\underline{\underline{R}}_v = \underline{\underline{I}} \otimes \underline{\underline{k}}_T \otimes \underline{\underline{k}}_R$.

Since the correlation matrices are positive definite, the eigenvalue decomposition was used to calculate the square root of the correlation matrices as follows,

$$\begin{aligned} \underline{\underline{k}} &= \underline{\underline{v}} \underline{\underline{e}} \underline{\underline{v}}^H, \\ \underline{\underline{k}}^{1/2} &= \underline{\underline{v}} \underline{\underline{e}}^{1/2} \underline{\underline{v}}^H. \end{aligned} \quad (2.43)$$

2 Theoretical Background

$\underline{\underline{v}}$ is a matrix containing the eigenvectors of $\underline{k}$ and $\underline{\underline{e}}$ is a diagonal matrix containing its eigenvalues. The square root of a diagonal matrix is simply the square root of its diagonal elements. Thus, $\underline{\underline{H}}^c$ is then simply calculated by replacing $\underline{h}_{i_R i_T}$ by $\underline{h}_{i_R i_T}^c$ in Eqn. 2.24, i. e. $\underline{\underline{H}}_{i_R i_T}^c = \text{diag}[\text{DFT}(\underline{h}_{i_R i_T}^c)]$. In other words, the correlated channel matrices in Eqn. 2.39 are used instead of Eqn. 2.22 to obtain $\underline{\underline{H}}$ in Eqn. 2.26.

Antenna Correlation Models

We assume linear antenna arrays with uniformly spaced antennas at both the transmitter and receiver. Antennas at the same distance m are also assumed to experience the same correlations. Thus, $\underline{k}_R$ and $\underline{k}_T$ take the following form,

$$\begin{bmatrix} 1 & c_{12} & c_{13} & \cdots & c_{1n} \\ c_{21} & 1 & c_{23} & \cdots & c_{2n} \\ c_{31} & c_{32} & 1 & \cdots & c_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \cdots & \vdots & \vdots & \vdots \\ c_{n1} & \cdots & \cdots & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{n-1} \\ \rho_1^* & 1 & \rho_1 & \cdots & \rho_{n-2} \\ \rho_2^* & \rho_1^* & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \cdots & \vdots & \vdots & \vdots \\ \rho_{n-1}^* & \cdots & \cdots & \cdots & 1 \end{bmatrix}, \quad (2.44)$$

where n is either n_R or n_T , $c_{ii} = 1$ and $c_{ij} = \rho_{j-i}$ for $j > i$ and $c_{ji} = c_{ij}^*$ for $j < i$ ($i = 1, \dots, n, j = 1, \dots, n$). Accordingly, ρ_m is the correlation coefficient between antennas with spacing $m = |j-i|$. Throughout this work, we shall concentrate on two correlation models: constant and exponential correlation model. For the constant correlation model, the correlation coefficients are real and the same for all antennas, i. e. $\rho_m = \rho \ \forall m$ and $|\rho| < 1$. For the exponential correlation model, the correlation coefficients are defined as follows [23],

$$\begin{aligned} c_{ij} &= \rho_m = \rho^{j-i} \quad \forall i \leq j \\ c_{ji} &= c_{ij}^*, \end{aligned} \quad (2.45)$$

for real coefficients ρ , $|\rho| < 1$ and

$$\begin{aligned} c_{ij} &= \rho_m = (\rho + \sqrt{-1}\rho)^{j-i} \quad \forall i \leq j \\ c_{ji} &= c_{ij}^*, \end{aligned} \quad (2.46)$$

for complex coefficients $\rho + \sqrt{-1}\rho$. Note that $|\rho + \sqrt{-1}\rho| < 1$ for the correlation matrix to remain positive definite. Another example of a widely used correlation matrix is the tridiagonal correlation matrix [24]. The correlation matrix is given by Eqn. 2.44 where $\rho_m = \rho \ \forall m \leq M$ and $\rho_m = 0$ otherwise and $M < n$.

2.4 Effect of Antenna Correlations

In this section, we introduce three different measures that show how antenna correlations can affect the performance of the MIMO system.

2.4.1 Channel Condition Number

The condition number, χ , is defined as the ratio between the maximum, λ_{max} , and the minimum, λ_{min} , eigenvalue of a matrix [25].

$$\chi = \frac{\lambda_{max}}{\lambda_{min}} \quad (2.47)$$

If $\lambda_i = 1 \forall i$, then the matrix is an identity matrix. A singular matrix has a $CN = \infty$, since $\lambda_{min} = 0$. That is, the higher the condition number, the more singular the channel correlation matrix gets and worse BER performance is to be expected. As will be shown in Chapter 3, increasing the antenna correlations leads to higher condition numbers and accordingly to deteriorating performance.

2.4.2 Diversity and Correlation Measures

Definition 1 For a flat fading channel, the diversity measure of $\underline{\underline{R}}_v$, $\Psi(\underline{\underline{R}}_v)$, was first introduced in [26, 27] and is given by

$$\Psi(\underline{\underline{R}}_v) = \left(\frac{\text{Tr}(\underline{\underline{R}}_v)}{\|\underline{\underline{R}}_v\|_F} \right)^2 = \left(\frac{\sum_{i=1}^M \lambda_i}{\sqrt{\sum_{i=1}^M \lambda_i^2}} \right)^2 \quad (2.48)$$

where $\underline{\underline{R}}_v = \mathcal{E}\{\underline{h}_v \underline{h}_v^H\}$ is an $M \times M$ ($M = n_T n_R$) matrix, $\underline{h}_v = \text{vec}\{\underline{h}\}$ and λ_i is the i th eigenvalue of $\underline{\underline{R}}_v$. $\Psi(\underline{\underline{R}}_v)$ is bound by [26]

$$1 \leq \Psi(\underline{\underline{R}}_v) \leq M \quad (2.49)$$

An important property of the diversity measure is, if the first n eigenvalues of $\underline{\underline{R}}_v$ are identical (i.e. $\lambda_1 = \lambda_2, \dots, = \lambda_n$) and the remaining eigenvalues are zeros, then $\Psi = n$. That is, the correlation measure becomes equal to the rank of $\underline{\underline{R}}_v$. If the nonzero eigenvalues of $\underline{\underline{R}}_v$ are not equal, $\Psi < n$ which can be interpreted as the number of dominant eigenvalues [27, 28]. That is, the diversity measure gives the number of dominant eigenvalues of $\underline{\underline{R}}_v$ and not its rank. The rank of $\underline{\underline{R}}_v$ is always equal to M as long as the correlation matrices have full rank. $\text{rank}(\underline{\underline{R}}_v) = 1$ if and only if the channel is fully correlated. For $\underline{\underline{R}}_v = \underline{\underline{1}}$ ($\underline{\underline{1}}$ is an all one matrix), the diversity measure is 1, while for $\underline{\underline{R}}_v = \underline{\underline{I}}$,

2 Theoretical Background

the diversity measure is M . Accordingly, the larger the spread the lower the diversity measure.

According to the correlation model given in Sec. 2.3, $\underline{\underline{R}}_v$ for a flat fading channel ($L = 1$) is given by

$$\underline{\underline{R}}_v = \underline{\underline{k}}_T \otimes \underline{\underline{k}}_R. \quad (2.50)$$

Consequently, from Appendix A.2.1, the $\text{rank } \underline{\underline{R}}_v = \text{rank } \underline{\underline{k}}_T \cdot \text{rank } \underline{\underline{k}}_R$, and $\underline{\underline{R}}_v$ remains full rank as long as both $\underline{\underline{k}}_T$ and $\underline{\underline{k}}_R$ are full rank. The diversity measure can accordingly also be expressed by [26]

$$\Psi(\underline{\underline{R}}_v) = \Psi(\underline{\underline{k}}_T) \Psi(\underline{\underline{k}}_R), \quad (2.51)$$

where $\Psi(\underline{\underline{k}}_T)$ and $\Psi(\underline{\underline{k}}_R)$ are the transmit and receive diversity measures respectively.

Definition 2 In [26, 27], the correlation measure was also defined, $\Phi(\underline{\underline{k}})$ for an $n \times n$ correlation matrix $\underline{\underline{k}}$

$$\Phi(\underline{\underline{k}}) = \sqrt{\frac{n/\Psi(\underline{\underline{k}}) - 1}{n - 1}} \quad (2.52)$$

which can be used as a measure for the amount of correlation present.

The correlation measure is especially useful if different correlation models are compared. $\Phi(\underline{\underline{k}})$ ranges between 0.0 (all equal eigenvalues: *uncorrelated*) and 1.0 (rank one correlation matrix: *full correlation*). It is also worth noting here, that the correlation measure for the constant correlation model is $\Phi(\underline{\underline{k}}) = \rho$.

In case of frequency selective channels, we look at the channel in the frequency domain. The diversity measure is the sum of the diversity measures of the uncorrelated subcarriers when using OFDM. Then the diversity measure can be calculated as follows,

$$\Psi(\underline{\underline{R}}_v) = \sum_{k=1}^{N_c} \Psi(\underline{\underline{R}}_{v_k})$$

where N_c is the number of coherence bandwidth, and $\Psi(\underline{\underline{R}}_{v_k})$ is the diversity measure calculated at each coherence bandwidth. If we assume the same antenna correlations at each frequency, and independent and uncorrelated fading of the channel taps of equal power delay profile then

$$\Psi(\underline{\underline{R}}_v) = L \Psi(\underline{\underline{R}}_{v_k}), \quad (2.53)$$

2.4 Effect of Antenna Correlations

where $\Psi(\underline{R}_{v_k})$ is the diversity measure at any frequency k , $\underline{R}_{v_k} = \mathcal{E}\{\text{vec}\{\underline{H}_k\} \text{vec}\{\underline{H}_k\}^H\}$, $\underline{H}_k$ is the $n_R \times n_T$ channel frequency response at frequency k and L is the number of channel taps. Thus, in case of a frequency selective channel, the diversity measure is bound by

$$L \leq \Psi(\underline{R}_v) \leq LM, \quad (2.54)$$

where $M = n_T n_R$.

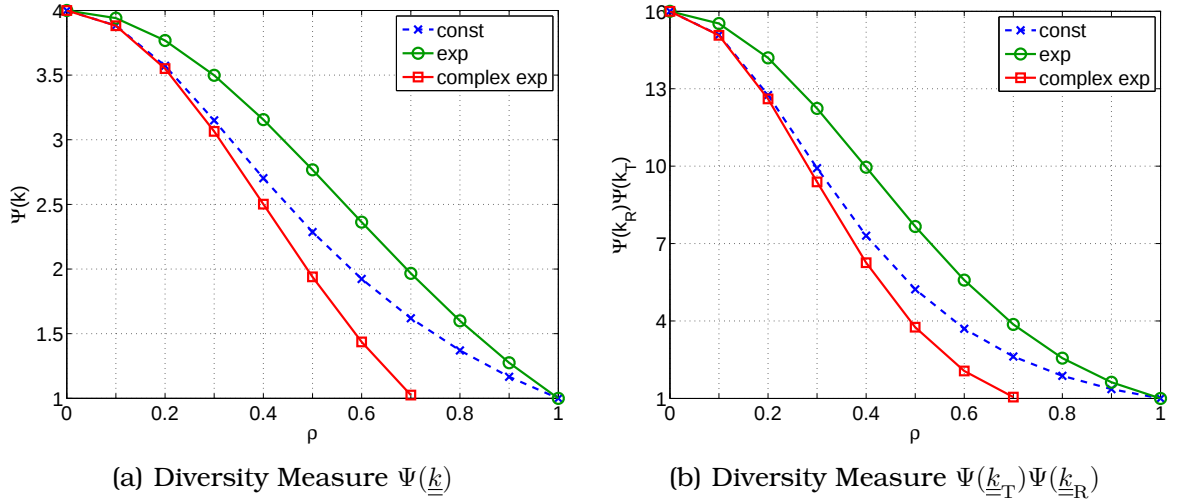


Figure 2.13: Diversity measure versus ρ for 4×4 constant, exponential and complex exponential correlation matrices.

Figures 2.13 and 2.14 show the diversity and correlation measures for the three previously described correlation matrices of size 4×4 . Figure 2.13(a) shows the diversity measure in presence of transmit correlations only and Fig. 2.13(b) in presence of both transmit and receive correlations. It is shown in Appendix 3.A.2, that the higher the antenna correlations, the wider the eigenvalue spread and accordingly, the lower the diversity measure. The diversity measure is lower if both transmit and receive correlations exist than if only one or the other is present since a larger eigenvalue spread is expected for the former case. The real exponential correlation matrix has the highest diversity measure and lowest correlation measure for a given ρ . The constant and complex exponential correlation matrices have comparable diversity and correlation measure values for $\rho \leq 0.4$. The complex exponential reaches its highest correlation measure (1.0) for $\rho \approx 0.7$ (i. e. $|c| = 1$). The diversity and correlation measures therefore give an indication of the expected system performance. The higher the correlation measure and the lower the diversity measure the worse the expected performance. Thus, for a given ρ , we expect

2 Theoretical Background

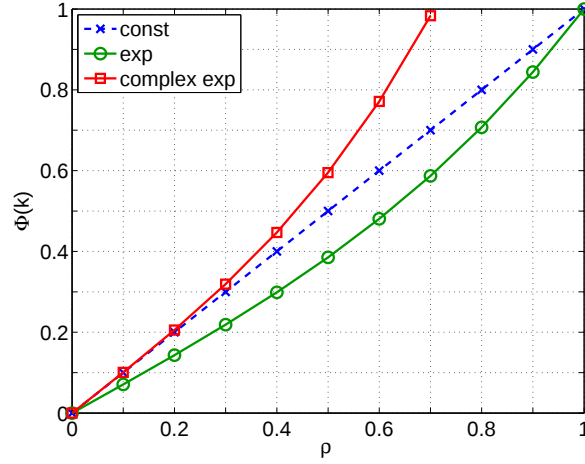


Figure 2.14: Correlation measure versus ρ for 4×4 constant, exponential and complex exponential correlation matrices.

a better performance in presence of the real exponential model than for the other two models. In addition, to allow for a fair comparison of the channel performance in presence of the different correlation models, ρ should be chosen such that the correlation measures for all models are equal.

2.5 Block Equalizers

Since we are dealing with block transmission schemes, in this section we present suitable equalization methods: *block equalization*. We start with hard decision non-iterative equalizers: the block linear equalizer (BLE) and the block decision feedback equalizer (BDFE) which were adapted for block transmission from the LE and DFE by Kaleh in [29]. We then look at soft decision iterative equalizers: the recurrent neural network equalizer (RNN) proposed by Teich et al. in [30] and the soft Cholesky equalizer (SCE) proposed by Egle et al. in [31]. Finally, we present iterative equalization and decoding for coded transmission using the RNN and SCE as proposed in [32, 33, 34, 35]. In all what follows, we assume that the channel to be fully known at the receiver.

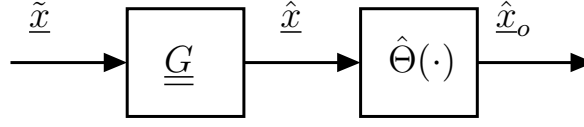


Figure 2.15: BLE estimating the transmit vector $\underline{x}$ based on matched filter output $\tilde{\underline{x}}$.

2.5.1 Hard Decision Equalizers

Block Linear Equalizer

Figure 2.15 shows the block diagram of a linear equalizer. In this section, we look at the most important linear equalizers: the zero forcing block linear equalizer (ZF-BLE) and the minimum mean square error BLE (MMSE-BLE). The ZF-BLE, also known as inverse filter, is given by [29]

$$\underline{\underline{G}} = \underline{\underline{R}}^{-1}, \quad (2.55)$$

provided that the channel correlation matrix $\underline{\underline{R}}$ is non-singular. The symbol estimates after the linear filter are thus given by

$$\hat{\underline{x}} = \underline{\underline{R}}^{-1} \tilde{\underline{x}} = \underline{x} + \tilde{\underline{n}}, \quad (2.56)$$

where $\tilde{\underline{n}}$ is the colored noise of covariance $\underline{\underline{\Phi}}_{\tilde{\underline{n}}\tilde{\underline{n}}} = \underline{\underline{G}}$. Hard decisions (denoted by $\hat{\Theta}(\cdot)$ in Fig. 2.15) are made on each symbol $\hat{x}_i$ in the symbol estimate vector $\hat{\underline{x}}$ independent of the other symbols. In case of noise-free transmission, the ZF-BLE leads to error-free performance. However, noise enhancement is a major problem of the ZF-BLE since the variance of the colored noise, g_{ii} , – given by the diagonal elements of $\underline{\underline{G}}$ – may become very large compared to the energy of the transmit symbols if $\underline{\underline{R}}$ is ill-conditioned. To overcome this noise enhancement problem, the minimum mean square error BLE (MMSE-BLE) is employed. The MMSE-BLE aims at minimizing the MSE between the estimated symbols $\hat{\underline{x}} = \underline{\underline{G}} \tilde{\underline{x}}$ and the transmit symbols $\underline{x}$. The solution is given by [29]

$$\underline{\underline{G}} = (\underline{\underline{R}} + \frac{\sigma_n^2}{\sigma_x^2} \underline{\underline{I}})^{-1}, \quad (2.57)$$

where σ_n^2 is the noise power and σ_x^2 is symbol variance. The MMSE-BLE outperforms the ZF-BLE at low E_b/N_o , but their performances converge at high E_b/N_o , since $\sigma_n^2/\sigma_x^2 \rightarrow 0$. In all what follows, we will therefore always employ the MMSE-BLE.

2 Theoretical Background

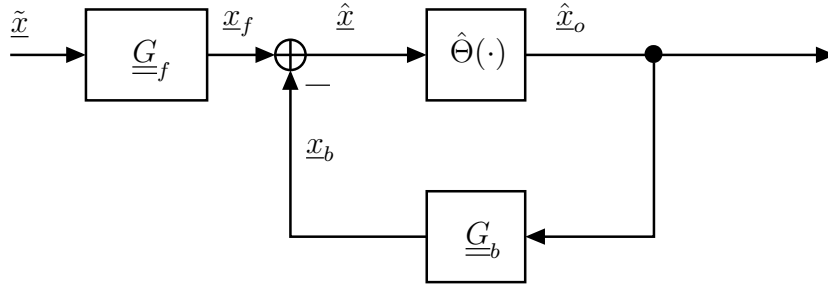


Figure 2.16: BDFE estimating the transmit vector $\underline{x}$ based on matched filter output $\tilde{\underline{x}}$.

Block Decision Feedback Equalizer

The BDFE is composed of two filters: a feedforward filter, $\underline{G}_f$, and a feedback filter, $\underline{G}_b$, as shown in Fig. 2.16. The feedback filter reconstructs the influence of already detected symbols – $\hat{\underline{x}}_o$ after the hard decision device, $\hat{\Theta}(\cdot)$ – which are then be subtracted from the output of the feedforward filter. The BDFE is designed assuming error free decision and thus error free feedback. The Cholesky decomposition provides an efficient method for calculating $\underline{G}_f$ [33]. Assuming $\underline{R}$ to be positive semidefinite, then it can be decomposed into a product of a lower diagonal matrix $\underline{F}$ and its Hermitian, i. e. $\underline{R} = \underline{F}^H \underline{F}$ [36, 37]. and the feedforward filter is accordingly given by [33],

$$\underline{G}_f = (\text{diag}(\underline{F}))^{-1}(\underline{F}^H)^{-1}, \quad (2.58)$$

where $\text{diag}(\underline{F})$ is a diagonal matrix containing the diagonal elements of $\underline{F}$. The feedback filter, $\underline{G}_b$, is given by

$$\underline{G}_b = (\text{diag}(\underline{F}))^{-1} \underline{F} - \underline{I}, \quad (2.59)$$

where $\underline{I}$ is the identity matrix of the same dimension as $\underline{R}$. Similar to the ZF-BLE, the ZF-BDFE suffers from noise enhancement especially if $\underline{R}$ becomes ill-conditioned. Thus, the MMSE-BDFE is designed based under the constrained of minimizing the MSE between $\hat{\underline{x}}_l$ and the corresponding transmit symbol $\underline{x}_l$ [29, 33]. Under this constraint, $\underline{G}_f$ and $\underline{G}_b$ are given by Eqns. 2.58 and 2.59. However, $\underline{R}$ is replaced by $\tilde{\underline{R}} = \underline{R} + \sigma_n^2/\sigma_x^2 \underline{I}$ and $\underline{F}$ by $\tilde{\underline{F}}$ obtained by the Cholesky decomposition of $\tilde{\underline{R}}$, $\tilde{\underline{R}} = \tilde{\underline{F}}^H \tilde{\underline{F}}$ [29, 33]. As with the BLE, since the ZF-BDFE and the MMSE-BDFE performances converge at high E_b/N_o values, from now on we will only employ the MMSE-BDFE.

2.5.2 Soft Decision Equalizers

The above described equalizers apply hard decisions on the symbol estimates. In case of the BLE, the hard decisions are made on each symbol independent of the others, while for the BDFE the already *hard* decided symbols are fed back through the feedback filter. In this section, we consider two iterative equalizers that use soft estimates of the detected symbols for the feedback: the recurrent neural network and the soft Cholesky equalizer.

Recurrent Neural Network Equalizer

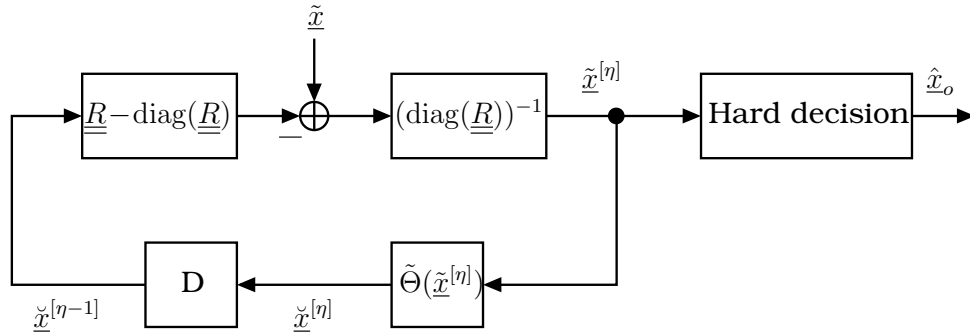


Figure 2.17: Block diagram of the RNN equalizer.

The recurrent neural network (RNN) equalizer is based on Hopfield neural networks used for pattern classification. The Hopfield network is a special type of recurrent neural network that has only one layer of neurons, and where the output of one neuron is fed back to all others after being multiplied by certain weights [30, 38, 9]. The received vector is thus treated as a classification problem, where the classes are all possible transmit vectors and the weights are given by the off diagonal elements of $\underline{R}$ [30, 38, 9].

Based on the transmission model in Fig. 2.12, any receive symbol $\tilde{x}_i$ can be written as

$$\tilde{x}_i = r_{ii}x_i + \sum_{j=1, i \neq j}^{Nn_T} r_{ij}x_j + n_{ci}, \quad (2.60)$$

where the second term of the above equation represents the interference and r_{ij} the off diagonal elements of $\underline{R}$. Figure 2.17 shows the block diagram for the RNN equalizer. The RNN equalizer as aforementioned has an iterative structure and employs a nonlinear decision function and operates as follows: after each iteration the interference, $(\underline{R} - \text{diag}(\underline{R}))\underline{\tilde{x}}^{[l-1]}$, is subtracted from the received vector $\tilde{\underline{x}}$, where l denotes the iteration number and $\underline{\tilde{x}}^{[l-1]}$ the soft output after the nonlinear *soft* decision function $\Theta(\cdot)$. After the last iteration,

2 Theoretical Background

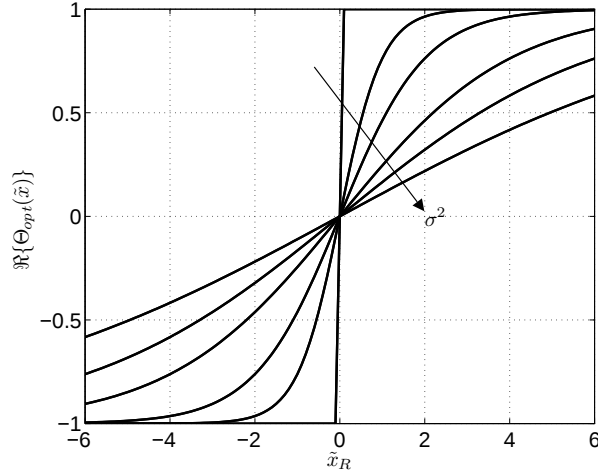


Figure 2.18: Nonlinear decision function for BPSK.

a hard decision is made to obtain the estimates $\hat{\underline{x}}_o$ for the transmit vector $\underline{x}$. The described cancellation scheme updates all symbols in parallel and thus is called parallel update. However, the scheme can also be done in serial and in this case $\tilde{x}_i^{[\eta]}$ for current iteration η is given by [30, 38, 9, 39]

$$\tilde{x}_i^{[\eta]} = x_i + \sum_{j=1}^{i-1} \frac{r_{ij}}{r_{ii}} \tilde{x}_j^{[\eta]} + \sum_{j=i+1}^{Nn_T} \frac{r_{ij}}{r_{ii}} \tilde{x}_j^{[\eta-1]} + \frac{n_{ci}}{r_{ii}}. \quad (2.61)$$

Since it was shown in [9, 38, 39, 40] that the parallel update leads to worse performance and a slower convergence than the serial update, from now on we shall only consider the latter. Now we focus our attention on the estimation device $\Theta(\cdot)$, whose task is to find an estimate $\check{x}_i$ of x_i such that the mean square error $J = \mathcal{E}\{|x_i - \check{x}_i^{[\eta]}|^2 | \tilde{x}_i^{[\eta]}\}$ is minimized [9], where the expectation $\mathcal{E}\{|x_i - \check{x}_i^{[\eta]}|^2 | \tilde{x}_i^{[\eta]}\}$ denotes the residual interference after deciding for $\check{x}_i$. The absolute minimum of J can only be reached if $\check{x}_i$ is a soft value not restricted to the transmit alphabet [41, 9]. The minimization problem can be treated as a parameter estimation problem [42], whose solution is [43]

$$\check{x}_i^{[\eta]} = \tilde{\Theta}_{opt}(\tilde{x}_i^{[\eta]}) = \mathcal{E}\{x_i | \tilde{x}_i^{[\eta]}\} = \sum_{m=0}^{M-1} a_m P(x_i = a_m | \tilde{x}_i^{[\eta]}). \quad (2.62)$$

Assuming that the symbol x is disturbed by a complex AWGN with variance σ^2 for each noise component, then, for any complex QAM signal with M equiprobable signals $a_i = a_{Ri} + ja_{Li}$, $i = 1, \dots, M$, $\tilde{\Theta}_{opt}$ in Eqn.2.62 can be

written as follows [9, 38, 40]

$$\check{x} = \tilde{\Theta}_{opt}(\tilde{x}) = \frac{\sum_{i=1}^M a_i \exp\left(\frac{-|a_i|^2}{2\sigma^2} + \frac{\Re\{a_i\}\Re\{\tilde{x}\}}{\sigma^2} + \frac{\Im\{a_i\}\Im\{\tilde{x}\}}{\sigma^2}\right)}{\sum_{i=1}^M \exp\left(\frac{-|a_i|^2}{2\sigma^2} + \frac{\Re\{a_i\}\Re\{\tilde{x}\}}{\sigma^2} + \frac{\Im\{a_i\}\Im\{\tilde{x}\}}{\sigma^2}\right)}. \quad (2.63)$$

For the special case of BPSK, Eqn. 2.63 simplifies to $\Re\{\tilde{\Theta}_{opt}(\tilde{x})\} = \tanh(\tilde{x}_R/\sigma^2)$ and $\Im\{\tilde{\Theta}_{opt}(\tilde{x})\} = 0$. The nonlinear decision function for increasing values of σ^2 and BPSK is shown in Fig. 2.18. As can be seen, the decision function becomes harder as σ^2 decreases.

In order to calculate $\check{x}$, the noise variance, σ^2 , should be known. The noise is assumed to be a sum of two statistically independent random variables: the colored noise of the channel, $\sigma_{n,i}^2 = \sigma_n^2/|r_{ii}|^2$, and the residual interference power which is assumed to be a kind of additive noise with variance $\sigma_{I,i}^2$ given as follows [9, 38]

$$\sigma_{I,i}^2 = \sum_{j=1}^{i-1} \frac{|r_{ij}|^2}{|r_{ii}|^2} \sigma_{res,j}^2 + \sum_{j=i+1}^{Nn_T} \frac{|r_{ij}|^2}{|r_{ii}|^2} \sigma_{res,j}^2. \quad (2.64)$$

It is clear from the previous equation that $\sigma_{I,i}^2$ is dependent on the iteration number since the residual interference after each iteration is dependent on the quality of the estimate of $\check{x}$. The individual interference powers $\sigma_{res,j}^2$ are given by [9, 38]

$$\sigma_{res,j}^2 = \mathcal{E}\{|x_j|^2 | \check{x}_j^{[\eta]}\} - |\check{x}_j^{[\eta]}|^2 = \frac{\sum_{i=1}^M |a_i|^2 \exp\left(\frac{-|a_i|^2}{2\sigma^2} + \frac{\Re\{a_i\}\Re\{\check{x}\}}{\sigma^2} + \frac{\Im\{a_i\}\Im\{\check{x}\}}{\sigma^2}\right)}{\sum_{i=1}^M \exp\left(\frac{-|a_i|^2}{2\sigma^2} + \frac{\Re\{a_i\}\Re\{\check{x}\}}{\sigma^2} + \frac{\Im\{a_i\}\Im\{\check{x}\}}{\sigma^2}\right)} - |\check{x}_j^{[\eta]}|^2, \quad (2.65)$$

where $\sigma^2 = \sigma_{n,i}^2 + \sigma_{I,i}^2$, since the two random variables are assumed to be independent. In case of MPSK modulation, Eqn. 2.65 reduces to $\sigma_{res,j}^2 = 1 - |\check{x}_j^{[\eta]}|^2$. Clearly, the noise variance decreases with increasing number of iterations (i. e. harder decisions).

Soft Cholesky Equalizer

The SCE is derived from the ZF-BDFE and employs the same filters, however, like the RNN equalizer, it is iterative and employs a soft decision device instead of the hard one for feedback. Like the ZF-BDFE, the feedforward filter $\underline{\underline{G}}_f = (\underline{\underline{F}}^H)^{-1}$ (note that the normalization by $(\text{diag}(\underline{\underline{F}}))^{-1}$ has been dropped here). Thus the output of the feedforward filter is given by

$$\underline{\zeta} = \underline{\underline{F}} \underline{x} + \underline{n}, \quad (2.66)$$

2 Theoretical Background

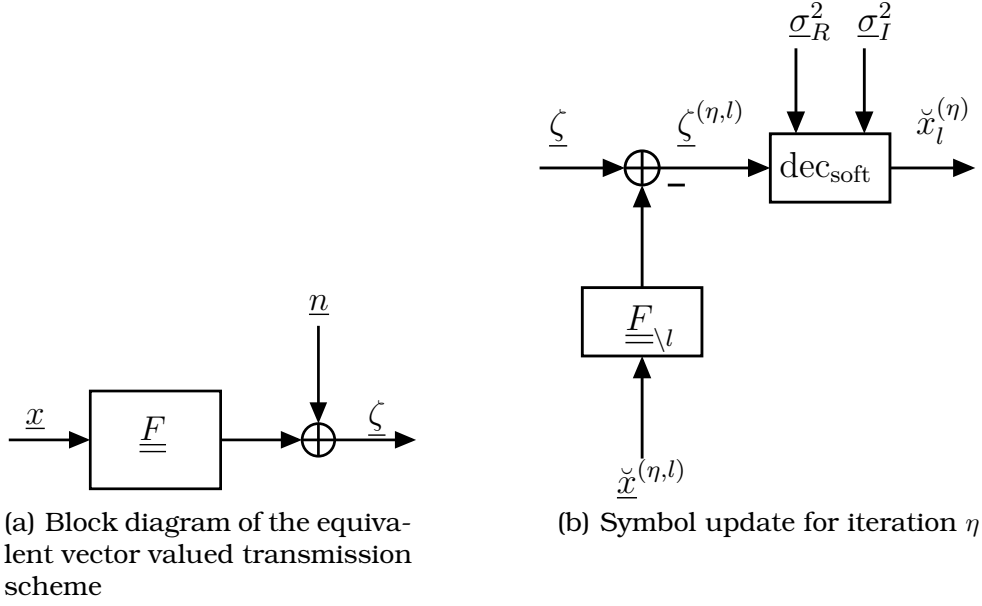


Figure 2.19: SCE equivalent vector valued transmission and symbol update.

where the noise $\underline{n}$ is now white due to the whitening filter $(\underline{F}^H)^{-1}$. Figure 2.19(a) gives the equivalent matrix vector transmission scheme after the feedforward filter. Figure 2.19(b) shows the updating model used by the SCE to obtain the estimate $\check{x}_l^{(\eta)}$ of the x_l at the η -th iteration. The estimate can also be expressed as follows [31, 33]

$$\check{x}_l^{(\eta)} = \text{dec}_{\text{soft}} \left(\underline{\zeta}^{(\eta,l)}, \underline{\sigma}_R^2, \underline{\sigma}_I^2 \right) \quad \text{where} \quad (2.67)$$

$$\underline{\zeta}^{(\eta,l)} = \underline{\zeta} - \underline{F}_{\setminus l} \underline{\check{x}}^{(\eta,l)} \quad \text{and} \quad (2.68)$$

$$\underline{\check{x}}^{(\eta,l)} = \left[\check{x}_1^{(\eta-1)}, \dots, \check{x}_l^{(\eta-1)}, \check{x}_{l+1}^{(\eta)}, \dots, \check{x}_{N_{n_T}}^{(\eta)} \right]. \quad (2.69)$$

The matrix $\underline{F}_{\setminus l}$ denotes the matrix $\underline{F}$ with the l -th column set to all-zero vector and accordingly $\underline{\zeta}^{(\eta,l)}$ can be written as follows,

$$\underline{\zeta}^{(\eta,l)} = \underline{f}_l x_l + \underline{i}^{(\eta,l)} + \underline{n} \quad (2.70)$$

where $\underline{f}_l$ is the l -th column of $\underline{F}$ and $\underline{i}^{(\eta,l)} = \underline{F}_{\setminus l} (\underline{x} - \underline{\check{x}}^{(\eta,l)})$ is the residual interference of the other symbols. The desired final state is when all interference has been subtracted, i. e. $\underline{\check{x}}^{(\eta,l)} = \underline{x}$ and consequently $\underline{\zeta}^{(\eta,l)} = \underline{f}_l x_l + \underline{n}$ [31]. Similar to the RNN equalizer, the SCE tries to find the soft decision function, dec_{soft} , that minimizes the MSE, $J = \{ |\check{x}_l^{[\eta]} - x_l|^2 | \underline{\zeta}^{(\eta,l)} \}$. The minimization leads

2.6 Iterative Equalization and Decoding

to the following soft decision function [31, 33]

$$\check{x}_l^{[\eta]} = \mathcal{E}\{x_l | \underline{\zeta}^{(\eta,l)}\} = \sum_{m=0}^{M-1} a_m P(x_l = a_m | \underline{\zeta}^{(\eta,l)}), \quad (2.71)$$

and by applying Bayes' rule, assuming equiprobable transmit symbols and white disturbance we get [31, 33]

$$\check{x}_l^{[\eta]} = \text{dec}_{\text{soft}} \left(\underline{\zeta}^{(\eta,l)}, \underline{\sigma}_R^2, \underline{\sigma}_I^2 \right) = \sum_{m=0}^{M-1} a_m \left[\frac{\prod_{j=0}^{Nn_T-1} p(\zeta_j^{(\eta,l)} | x_l = a_m)}{\sum_{k=0}^{M-1} \prod_{j=0}^{Nn_T-1} p(\zeta_j^{(\eta,l)} | x_k = a_m)} \right]. \quad (2.72)$$

Moreover, by assuming Gaussian interference the probability $p(\zeta_j^{(\eta,l)} | x_l = a_m)$ is given by [31, 33]

$$p(\zeta_j^{(\eta,l)} | x_l = a_m) = \frac{1}{2\pi\sigma_{Rj}^{(\eta,l)}\sigma_{Ij}^{(\eta,l)}} \exp\left(-\frac{(\Re\{\zeta_j^{(\eta,l)} - f_{jl}a_m\})^2}{2\sigma_{Rj}^{2(\eta,l)}}\right) \exp\left(-\frac{(\Im\{\zeta_j^{(\eta,l)} - f_{jl}a_m\})^2}{2\sigma_{Ij}^{2(\eta,l)}}\right) \quad (2.73)$$

and $\sigma_{Rj}^{2(\eta,l)}$ and $\sigma_{Ij}^{2(\eta,l)}$ are given by

$$\sigma_{Rj}^{(\eta,l)} = \sqrt{\sigma_n^2 + i_{Rj}^{2(\eta,l)}} \quad (2.74)$$

$$\sigma_{Ij}^{(\eta,l)} = \sqrt{\sigma_n^2 + i_{Ij}^{2(\eta,l)}}. \quad (2.75)$$

The decision function of the SCE thus collects all of the energy of the transmit symbol x_l spread over $\zeta_j^{(\eta,l)}$ by making the decision function dependent on the whole column $\underline{f}_l$. Accordingly, it does not suffer from the SNR-loss that is unavoidable for the BDFE, which can only make use of the energy on the main diagonal of $\underline{\underline{F}}$ [31]. We also would like to mention at this point, that the difference between the RNN detector and the SCE lies in the soft decision function and an additional preprocessing step of the SCE (feedforward whitening filter). The soft decision function of the RNN detector processes a single symbol, whereas the SCE as aforementioned makes a decision based on the whole symbol vector. Furthermore, since the SCE applies a whitening filter, it usually offers better performance. However, the computational complexity of the SCE is higher compared to that of the RNN detector.

2.6 Iterative Equalization and Decoding

In the coded case, we apply iterative equalization and decoding (also known as *turbo detection* or *iterative detection*). Figure 2.20 depicts the discrete-time

2 Theoretical Background

vector-matrix model, when additional coding is included. The source symbol vectors, $\underline{q}$, are encoded with a terminated convolutional code, optionally punctured, and permuted by a random interleaver, $\underline{\Pi}$. The code vectors are mapped on to symbols from an M -ary symbol alphabet. We assume that the resulting symbol vector is subdivided into blocks of length Nn_T , which are transmitted over the MIMO channel matrix $\underline{\underline{R}}$.

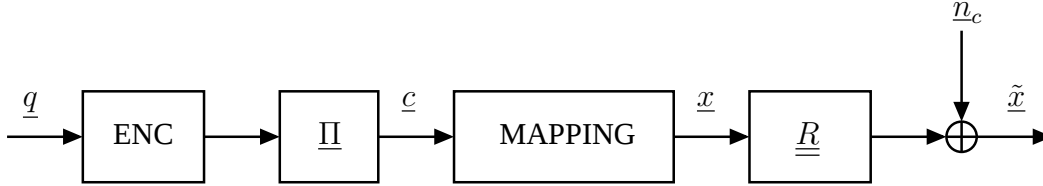


Figure 2.20: Matrix vector model of coded transmission.

The basic principle of turbo detection or iterative detection is depicted in Fig. 2.21. It was first introduced for intersymbol interference (ISI) channels in [44]. The detector (DET) and the decoder (DEC) benefit from each other during an iterative process by exchanging soft values of the code bits. In iterative schemes, the bit soft values are often given by their log-likelihood ratios (LLR) or for short L-values. The LLR of a bit q , $L(q)$, is given by [45, 46]

$$L(q) = \ln \frac{P(q=0)}{P(q=1)}, \quad (2.76)$$

where $P(q=0)$ is the probability that $q=0$ and $P(q=1)$ is the probability that $q=1$. Log-likelihood algebra, which gives convenient methods for calculating bit reliabilities, was introduced in [45]. The LLR can be extended to non-binary random values, for example symbols x chosen from an M -ary modulation. In this case, M L-values need to be calculated as well as M probabilities $P(x=a_m)$. These general LLR are defined as follows

$$L(x) = \ln \frac{P(x=a_m)}{P(x=a_M)}, \quad (2.77)$$

where, without loss of generality, $P(x=a_M)$ is chosen for the denominator. Thus instead of a scalar L-value, we have a vector of L-values. The symbol probabilities can also be obtained by reverse mapping Eqn. 2.77,

$$P(x=a_m) = \frac{\exp(L(x=a_m))}{1 + \sum_{m=1}^{M-1} \exp(L(x=a_m))}. \quad (2.78)$$

Now we turn our attention back to Fig. 2.21 and based on it we explain the concept of iterative equalization and decoding. We restrict our attention here

2.6 Iterative Equalization and Decoding

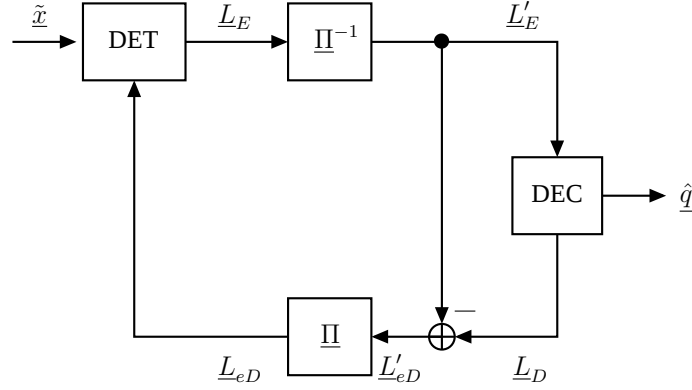


Figure 2.21: Matrix vector model of iterative equalization and decoding.

on equalizers with feedback, e. g. RNN equalizer and SCE. During each iteration η the equalizer (DET) calculates a vector of L-values, $\underline{L}_E$, based on the received vector $\tilde{x}$ and the additional information $\underline{L}_{eD}$ from the decoder. The information from the decoder, usually called *extrinsic* information, is based on the decoding results from the previous iteration $\eta - 1$ and provides information about the code restrictions and code bit reliabilities in order to improve the equalization [32, 33, 47]. The decoder is a symbol by symbol maximum a posteriori decoder (s/s MAP) implemented using the BCJR algorithm. Although the decoder computes a vector of code bit reliabilities $\underline{L}_D$ only the extrinsic information $\underline{L}'_{eD} = \underline{L}_D - \underline{L}'_E$ (after puncturing and interleaving) is passed to the equalizer and thereby preventing positive feedback [32, 33, 47].

We now consider the equalization in more details. In case of the feedback equalizers considered here, the only modification done is to the decision function of the equalizers [33]. As mentioned above, the detector now receives information about code restrictions as well as code bit reliabilities and consequently the decision function now takes decisions not only based on the received vector $\tilde{x}$ but also on $\underline{L}_{eD}$. The decision function, $\tilde{\Theta}_c(\cdot)$, for the coded case now includes an extra restriction, the code restriction $\mathcal{C}$, and can be approximated as follows [33]

$$\tilde{x}_l^{[\eta]} = \mathcal{E}\{x_l | \tilde{\chi}_l^{[\eta]}, \mathcal{C}\} = \sum_{m=1}^M a_m P(x_l = a_m | \tilde{\chi}_l^{[\eta]}, \mathcal{C}), \quad (2.79)$$

where $\chi_l^{[\eta]}$ stands for $\tilde{x}_l^{[\eta]}$ in case of RNN equalizer and $\zeta^{(\eta,l)}$ in case of the SCE. Assuming the statistical independence between $\chi_l^{[\eta]}$ and $\mathcal{C}$, which through the use of the random interleaver $\underline{\Pi}$ is ensured at least during the first iteration

2 Theoretical Background

[32, 48], and applying Bayes' rule we get

$$P(x_l = a_m | \tilde{\chi}_l^{[\eta]}, \mathcal{C}) = \frac{p(\chi_l^{[\eta]} | x_l = a_m) P(x_l = a_m | \mathcal{C})}{p(\chi_l^{[\eta]} | \mathcal{C})} = \frac{p(\chi_l^{[\eta]} | x_l = a_m) P(x_l = a_m | \mathcal{C})}{p(\chi_l^{[\eta]})}. \quad (2.80)$$

The probabilities $P(x_l = a_m | \mathcal{C})$ can be represented as a function of the decoder extrinsic values $\underline{L}_{eD}(c_{l,\nu} | \mathcal{C})$ [32, 48, 49],

$$P(x_l = a_m | \mathcal{C}) = \prod_{\nu=1}^{\log_2 M} \frac{\exp((1 - \text{bin}[a_m, \nu]) \underline{L}_{eD}(c_{l,\nu} | \mathcal{C}))}{1 + \exp((\underline{L}_{eD}(c_{l,\nu} | \mathcal{C})))}, \quad (2.81)$$

where $\text{bin}[a_m, \nu]$ denotes the value of the ν -th bit of symbol a_m . The channel reliability values of the code bits $c_{l,\nu}$, $\underline{L}_E(c_{l,\nu})$

$$L_E(\tilde{\chi}_l^{[\eta]} | c_{i,\nu}) = \ln \frac{\sum_{a_m \in \mathcal{A}_\nu^{[0]}} p(\chi_l^{[\eta]} | x_i = a_m)}{\sum_{a_m \in \mathcal{A}_\nu^{[1]}} p(\chi_l^{[\eta]} | x_i = a_m)}, \quad (2.82)$$

where $\mathcal{A}_\nu^{[b]}$ denotes the set of modulation symbols defined by a bit sequence b at the ν -th position, and $b = 0, 1$. The output of the detector, $\underline{L}_E$, is then deinterleaved $\underline{\Pi}^{-1}$ and depunctured and fed to the decoder.

Extrinsic Information Transfer Chart

The extrinsic information transfer chart or for short the EXIT chart is one of the most well known methods for visualizing the convergence behavior of iterative detection schemes. It is done by observing the exchange of mutual information between the partaking devices and was introduced by ten Brink in [50, 51, 52]. The mutual information between two random variables X and Ξ is given by [53, 54]

$$I(X, \Xi) = \int_X \int_\Xi p(\xi, x) \log_2 \left(\frac{p(\xi, x)}{p(\xi)p(x)} \right) d\xi dx = \int_X \int_\Xi p(\xi|x)p(x) \log_2 \left(\frac{p(\xi|x)}{p(\xi)} \right) d\xi dx, \quad (2.83)$$

where we made use of the relation $p(\xi, x) = p(\xi|x)p(x)$ to obtain the right-hand side of the above equation. We first consider the decoder and assume perfect interleaving i.e. we assume the input L-values, $\underline{L}_E(\tilde{\chi}_l^{[\eta]} | c_{i,\nu})$, to be independent and identically distributed random variables A_D . The probability density function (pdf) of A_D conditioned on the code bits $p(\zeta|c)$ is Gaussian with mean σ_A^2 and mean $\sigma_A^2/2(1 - 2b)$, where $b = 0, 1$ [51, 52],

$$p_{A_D}(\zeta|c = b) = \frac{1}{\sqrt{2\pi\sigma_A^2}} \exp \left(-\frac{\zeta - \frac{\sigma_A^2}{2}b}{2\sigma_A^2} \right). \quad (2.84)$$

2.6 Iterative Equalization and Decoding

Thus, the mutual information, $I_{A,\text{dec}}$, between the input L-values A_D and the unmapped code bits according to Eqn. 2.83, assuming equiprobable code bits, is [51, 52]

$$I_{A,\text{dec}} = \frac{1}{2} \sum_{b=-1,1} \int_{-\infty}^{\infty} p_{A_D}(\zeta|c=b) \log_2 \left(\frac{2p_{A_D}(\zeta|c=b)}{p_{A_D}(\zeta|c=-1) + p_{A_D}(\zeta|c=1)} \right) d\zeta. \quad (2.85)$$

The pdf of the extrinsic output L-values of the decoder, $L_{eD}(c_{i,\nu}|C)$, conditioned on the code bits, $p_{E_D}(\zeta|c=b)$, can not be calculated and are measured using Monte Carlo simulations. The mutual information at the output of the decoder, $I_{E,\text{dec}}$, between the code bits c and the random output L-values E_D are then calculated as in Eqn. 2.85. Since $I_{A,\text{dec}}$ is monotonically increasing in σ_A^2 and thus invertible, artificial L-values with mean $\sigma_A^2/2(1-2b)$, and variance σ_A^2 can be generated for a given value of $I_{A,\text{dec}}$. The generated L-values are then fed to the decoder and $p_{E_D}(\zeta|c=b)$ of E_D and in turn $I_{E,\text{dec}}$ are calculated.

The extrinsic information for the SCE and the RNN equalizer can not be obtained by the previously described straightforward method. The above derivation of the EXIT-chart assumes memory-less components, a condition that does not apply to feedback equalizers such as the SCE and RNN. These equalizers use decisions from previous iterations for interference cancellation and thus have memory. However, the condition of memory-less components can be relaxed so as to be able to find EXIT charts for the SCE and RNN [33]. This can be achieved by running those equalizers for several iterations using the same extrinsic information for all iterations and thereby reducing the memory effect [33, 35, 48]. In other words, extrinsic L-values of the decoder are artificially generated and fed to the equalizer which is then allowed to run for more than one iteration while keeping the L-values unaltered. As the number of iterations tends to infinity, an upperbound for the extrinsic transfer characteristics of the feedback equalizers can be obtained. Yet, it was shown in [33, 35, 48] that the extrinsic transfer curves for more than one iteration lie very close to each other. The curves for two or three iterations are thus enough to approximate the extrinsic transfer curves of the feedback equalizers.

Now, to analyze the convergence behavior of iterative equalization and decoding, the EXIT charts for both the equalizer and decoder are included in the same graph, where the input to the decoder is equal to the output of the equalizer ($I_{A,\text{dec}} = I_{E,\text{eq}}$ on the ordinate) and the output of the decoder is equal to the input of the equalizer ($I_{E,\text{dec}} = I_{A,\text{eq}}$ on the abscissa). Note that the decoder chart is now flipped. A schematic of the EXIT charts for the

2 Theoretical Background

equalizer and decoder is shown Fig. 2.22. The iterative process starts with the equalizer and ends with the decoder. The starting point is thus $I_{E,eq}$ for $I_{A,eq} = 0$ i. e. no extrinsic information available. After that, the equalizer output becomes the decoder input i. e. $I_{A,dec} = I_{E,eq}$. This is represented by a line parallel to the abscissa starting at the previous $I_{E,eq}$ point and ending at the intersection with the decoder chart at some new $I_{E,dec}$. This is the decoder output which in turn becomes the equalizer input $I_{E,dec} = I_{A,eq}$. Similarly, this is represented by a line parallel to the ordinate starting at the previous $I_{E,dec}$ and ending at the intersection with the equalizer chart at a new point $I_{E,eq}$. This is continued until equalizer and decoder EXIT characteristics intersect. This intersection point is important since the hard decision are made after the decoder in the last iteration i. e. a higher $I_{E,dec}$ corresponds to lower BER. Those above described lines are called the transient trajectory and, as can be seen in Fig. 2.22, they are upper bounded by the equalizer EXIT characteristics and lower bounded by the decoder EXIT characteristics. The convergence speed of the iterative process is thus dependent on the size and shape of the enclosed area. The larger the enclosed area the faster the convergence i. e. less iterations are required to achieve the best performance. It was shown through simulation results in [33, 18, 48, 19] that the starting point $I_{E,eq}$ of the equalizer chart is highly dependent on the amount of interference in the channel, where higher interference leads to lower $I_{E,eq}$ values. The final $I_{E,eq}$ on the other hand was found to be dependent on the matched filter bound (MFB), with better MFB leading to higher final $I_{E,eq}$ values. We shall look at this behavior again in more details in Chapter 4. It is also important to mention at this point, that those transient trajectories should only be seen as an approximation and that the actual trajectory may be significantly different from the above described theoretical trajectory. For example, the actual trajectory does not have to hit either of the EXIT characteristic curves.

2.7 Test Channels

To assess the effect of the total interference and the distribution of the individual interference values as well as the MFB on the performance of the equalizers described in Sec. 2.5, two standard channels are proposed in this section. These channels are then used for judging the behavior of the equalizers under different interference and matched filter bound (MFB) conditions.

K-Symmetric Channel

The K-symmetric channel is an $n \times n$ correlation matrix and was first defined in [55]. It was used to describe the cross-correlation matrix between K users,

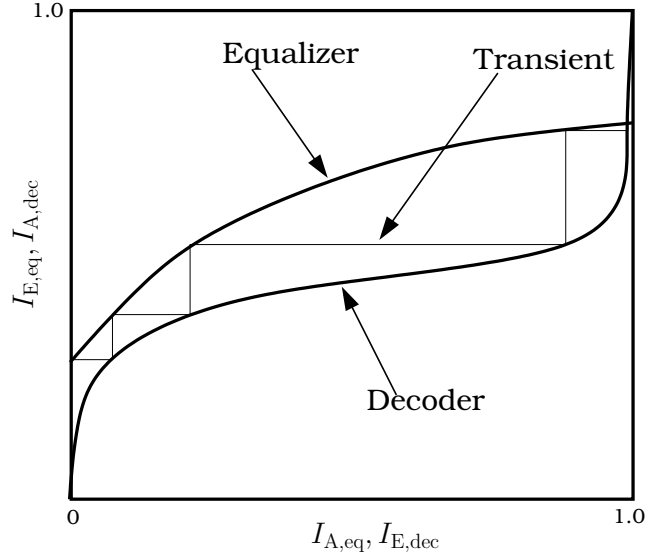


Figure 2.22: Example for EXIT chart.

where all users have the same cross-correlation with each other. It has equal off diagonal elements, i. e. $r_{ij} = r \ \forall i \neq j$ and $r \leq 1, r \in \mathbb{R}$.

$$\underline{\underline{R}}_{K,n} = \begin{pmatrix} 1 & r & r & \dots & r \\ r & 1 & r & \dots & \\ \vdots & \vdots & \ddots & \dots & \vdots \\ \vdots & \vdots & \dots & 1 & r \\ r & r & \dots & r & 1 \end{pmatrix} \quad (2.86)$$

From now on, we shall denote the K-symmetric channel by $\underline{\underline{R}}_{K,n}(r)$, where r is the value of the off-diagonal elements and n is the size of the square K-symmetric matrix.

K-Z-Symmetric Channel

We define another symmetric channel, the K-Z-symmetric channel. This channel is similar to the K-symmetric channel, except that some of the off diagonal elements are set to zero. The K-Z-symmetric channel of size $n \times n$ is defined as follows,

$$\underline{\underline{R}}_{KZ,n}(r_z) = \underline{\underline{R}}_{K,m}(r_z) \otimes \underline{\underline{I}}_z, \quad (2.87)$$

where $\underline{\underline{I}}_z$ is the identity matrix of size $z \times z$ and $\underline{\underline{R}}_{K,m}(r_z)$ is a K-symmetric channel (Eqn. 2.86) of size $m \times m$ and $m = n/z$. Note that m, n and z must be integer values and $z - 1$ defines the number of zero elements between any

2 Theoretical Background

Channel	Ψ	Φ	χ	$\sum_{i \neq j} r_{ij} $
K-Sym, $r = 0.2$	14.28	0.2	9.0	6.2
K-Z-Sym, $z = 2, r_z = 0.2875$	14.28	0.2	7.45	4.31
K-Z-Sym, $z = 4, r_z = 0.4209$	14.28	0.2	6.81	2.94
K-Z-Sym, $z = 8, r_z = 0.6429$	14.28	0.2	8.20	1.93

Table 2.1: K-symmetric ($r = 0.3$) and corresponding K-Z-symmetric channels for $z = 2, 4$ and 8 , $n = 32$, $\sum_{i \neq j} |r_{ij}|^2 = 1.24$.

two non-zero r_z values. From now on, we shall denote the K-Z-symmetric channel by $\underline{R}_{\text{KZ},n}(r, z)$.

BER for K- and K-Z-Symmetric Channels

In what follows, the performance of $n \times n$ K-symmetric and $n \times n$ K-Z-symmetric channels for various z values are compared. The value of z can be varied, but the following relation between the K-symmetric and K-Z-symmetric channels

$$(m - 1)r_z^2 = (n - 1)r^2 \quad (2.88)$$

must be satisfied to guarantee that $\|\underline{R}_{\text{KZ},n}(r_z)\|_F^2 = \|\underline{R}_{\text{K},n}(r)\|_F^2$. Since the diagonal elements of $\underline{R}_{\text{KZ},n}$ and $\underline{R}_{\text{K},n}$ are all ones, i. e. $\text{Tr}\{\underline{R}_{\text{K},n}\} = \text{Tr}\{\underline{R}_{\text{KZ},n}\}$, the sum of the square of the off-diagonal elements, $\sum_{i \neq j} |r_{ij}|^2$, remains the same for both matrices. Using these test channels, the effect of the amount of total interference (given by $\sum_{i \neq j} |r_{ij}|^2$ or $\sum_{i \neq j} |r_{ij}|$) and the distribution of the individual interference values on the used equalizer performances is tested. Note that although $\sum_{i \neq j} |r_{ij}|^2$ remains constant as long as Eqn. 2.88 is satisfied, the $\sum_{i \neq j} |r_{ij}|$ changes. In addition, the K- and K-Z-symmetric channels remain positive semidefinite for all $|r_{ij}| \leq 1$, $r_{ij} \in \mathbb{R}$.

Tables 2.1 and 2.2 give the diversity measure, Ψ , the correlation measure, Φ , the condition number, χ , and the sum of absolute values of r_{ij} for any row in $\underline{R}$. Since the Frobenius norm of the K-symmetric and K-Z-symmetric channels are equal, the diversity and correlation measures are the same for all channels. However, the sum $\sum_{i \neq j} |r_{ij}|$ increases as r decreases. The condition numbers for all K-Z-symmetric channels differ in value, yet they are of the same order of magnitude as that of the K-Symmetric channel for most r_z values. However, as $r_{ij} \rightarrow 1$, $\chi \rightarrow \infty$. Figure 2.23 shows the corresponding

2.7 Test Channels

Channel	Ψ	Φ	χ	$\sum_{i \neq j} r_{ij} $
K-Sym, $r = 0.3$	8.44	0.3	14.71	9.3
K-Z-Sym, $z = 2, r_z = 0.4313$	8.44	0.3	13.13	6.47
K-Z-Sym, $z = 4, r_z = 0.6313$	8.44	0.3	14.69	4.42
K-Z-Sym, $z = 8, r_z = 0.9644$	8.44	0.3	109	2.89

Table 2.2: K-symmetric ($r = 0.2$) and corresponding K-Z-symmetric channels for $z = 2, 4$ and 8 , $n = 32$, $\sum_{i \neq j} |r_{ij}|^2 = 2.79$.

K-symmetric channel with $r = 0.3$ and K-Z-symmetric channels with $z = 2, 4$ and 8 .

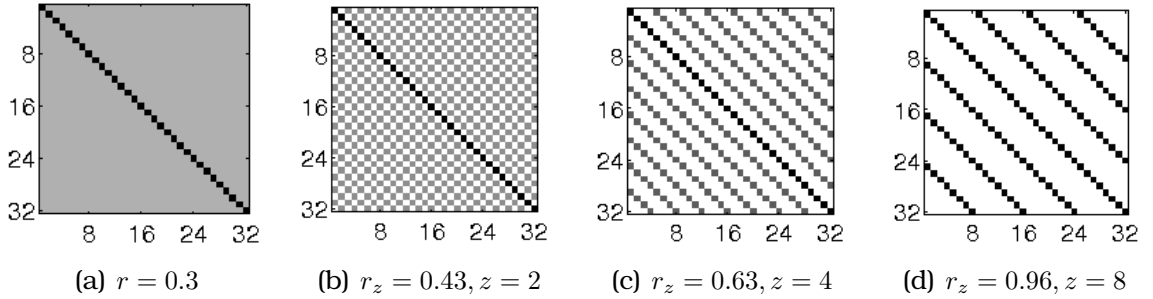


Figure 2.23: K-symmetric ($r = 0.3$) and corresponding K-Z-symmetric channels in Table 2.2.

Figure 2.24 shows the BER for the SCE, RNN, MMSE-BDFE and MMSE-BLE using 4PSK for the different K- and K-Z symmetric channels in Tables 2.1 and 2.2. Clearly, the BER for all equalizers deteriorates as r increases. In addition, for a given r , the BER also is shown to worsen as r_z increases although the corresponding $\sum_{i \neq j} |r_{ij}|$ decreases. Thus, as the BER curves show, for a given $\sum_{i \neq j} |r_{ij}|^2$ the equalizers are more sensitive to the individual values of the off-diagonal elements than to the total $\sum_{i \neq j} |r_{ij}|$. Note that the value of r_z should always be considered along with $\sum_{i \neq j} |r_{ij}|^2$. For instance, the K-Z-symmetric channel with $r_z = 0.6313$ in Fig. 2.24(b) has a total interference $\sum_{i \neq j} |r_{ij}|^2 = 2.79$ and $\sum_{i \neq j} |r_{ij}| = 4.42$ and the BER is worse than that of the K-Z-symmetric channel with $r_z = 0.6429$ (Fig. 2.24(a)), where $\sum_{i \neq j} |r_{ij}| = 2.79$ and $\sum_{i \neq j} |r_{ij}|^2 = 1.93$. Thus, the performance for the different r_z values should be compared for a given $\sum_{i \neq j} |r_{ij}|^2$. Furthermore, the simulation results show the performance of the RNN to be notably susceptible to high values of the off diagonal elements and to exhibit a rapidly deteriorating performance as either r or r_z increase.

2 Theoretical Background

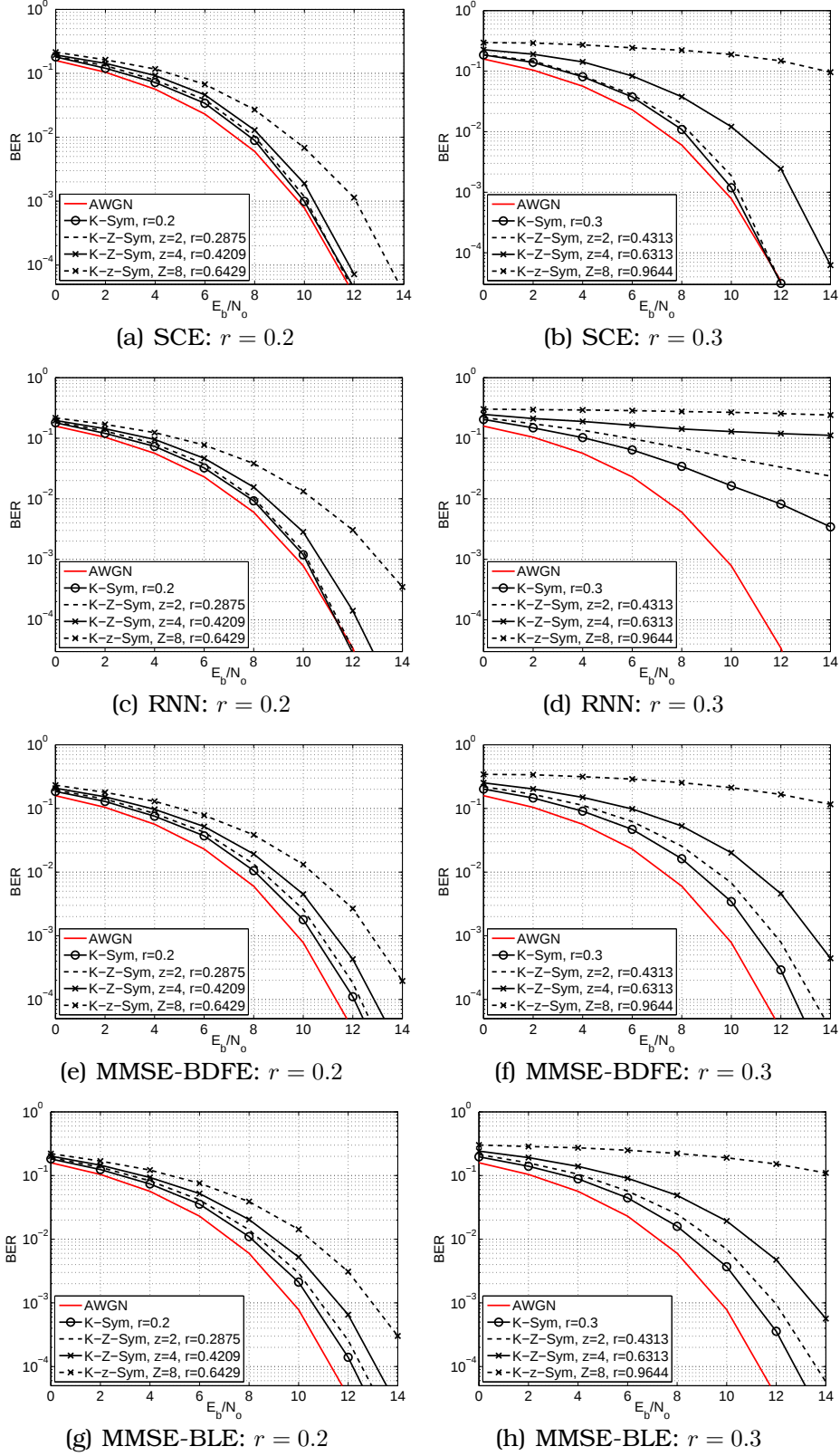


Figure 2.24: BER for $R_{K,32}$ for $r=0.2, 0.3$ and corresponding $R_{KZ,32}$ with $z=2, 4, 8$.

2.7 Test Channels

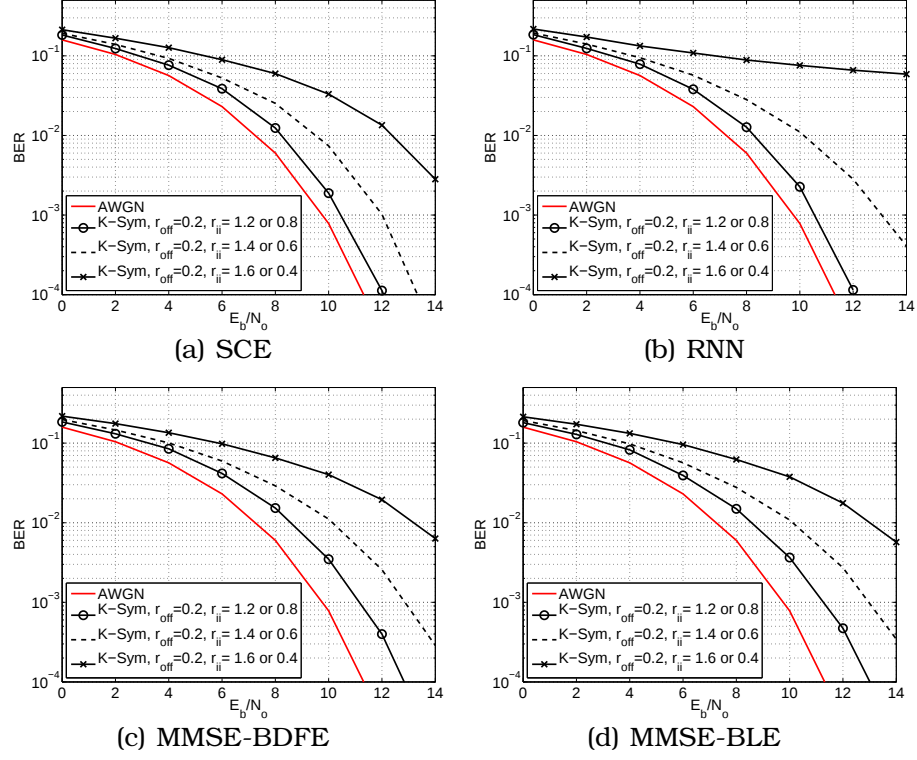


Figure 2.25: BER for $R_{K,32}(r = 0.2)$ and alternating unequal r_{ii} .

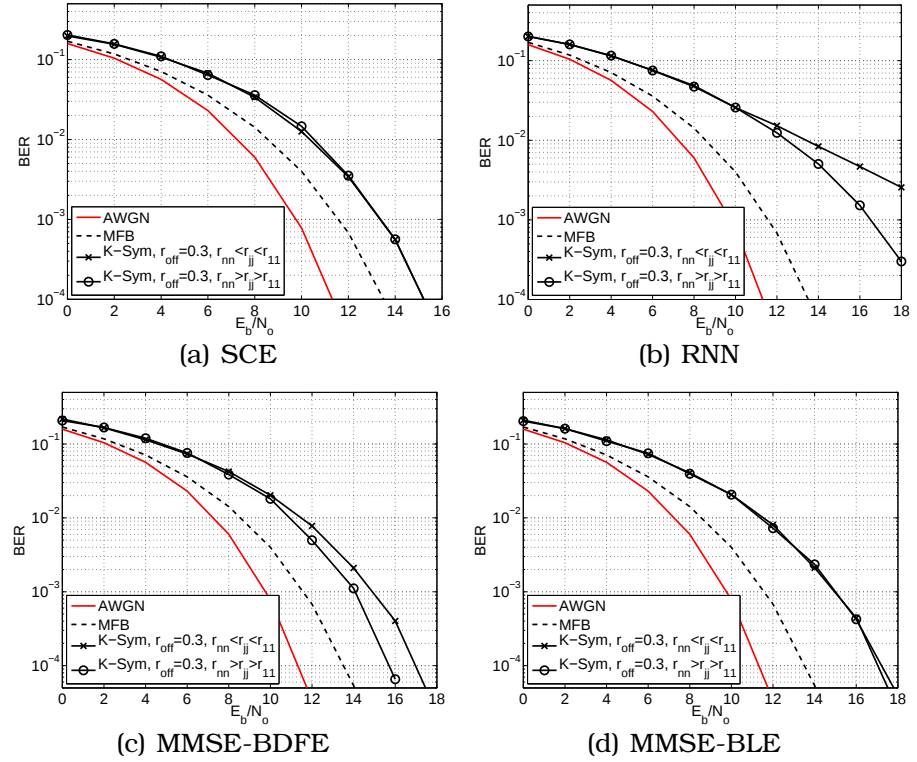


Figure 2.26: BER for $R_{K,16}(r = 0.3)$ and unequal r_{ii} sorted in increasing or decreasing order.

2 Theoretical Background

Effect of the MFB

Here, we modify the K-symmetric and K-Z-symmetric channels, such that the diagonal elements are not longer equal to one. The sum of the diagonal elements remains however the same. In Fig. 2.25, the diagonal elements, r_{ii} , alternately take on two different values, while in Fig. 2.26 the diagonal elements are sorted either in increasing or decreasing order with $r_{ii} = 0.87(1.1^{-l})$, and $l = -(n/2 - 1), \dots, n/2$. Note that the diagonal elements had to be carefully chosen so that the resulting channels remain positive definite. The off-diagonal elements are all equal and remain the same irrespective of the value of the diagonal elements. This way, we can test the effect of the MFB on the BER performance of the employed equalizers. Figures 2.25 and 2.26 show that the performance of all equalizers deteriorates as the MFB worsens, i.e. as the difference between the values of the diagonal elements increases. In addition, the RNN and MMSE-BDFE equalizers are sensitive to the order of sorting (increasing or decreasing) of the diagonal elements as can be seen in Figs. 2.26(b) and 2.26(c). They both seem to be more sensitive to propagation errors than the SCE. The MMSE-BLE, as expected, is not affected by the order since it does not employ any feedback filters.

The results of the test channels give a guideline as to how the different equalizers perform and how this performance is affected by interference, its distribution and MFB. They thus aid in predicting their operation for other channels as will be shown in the next chapters.

2.8 Summary

In this chapter, we described the matrix vector transmission model for SISO-OFDM and extended it to MIMO-OFDM systems and presented how spreading can be incorporated into that model. In addition, we showed how antenna correlations can be integrated in the MIMO-OFDM channel model. We described four different suboptimum equalizers (SCE, RNN, MMSE-BLE and MMSE-BDFE) that will be used throughout this work and presented how the RNN equalizer and SCE can be incorporated in iterative equalization and decoding schemes. We have also looked at the performance of those suboptimal equalizers for a set of test channels and showed that the performance is dependent on the MFB as well as on the amount of interference. The performance of the equalizers for those test channels will be used as a guideline to judge their performance in MIMO frequency selective channels.

Chapter 3

Spreading

Time, frequency and spatial diversity are available for signals transmitted over frequency selective, time varying MIMO channels. In order to capture those diversities, signals need to be coded. The codeword dimensions depend in general on the number of transmit and receive antennas as well as the codeword duration and bandwidth. In addition, the maximum diversity that can be captured by a codeword depends on the coherence time, coherence distance and coherence bandwidth. For instance, the maximum available time diversity a codeword of duration T can capture is T/T_c , where T_c is the coherence time. Similarly, for a bandwidth BW and coherence bandwidth BW_c , BW/BW_c represents the number of frequency diversity branches. In addition, the MIMO antennas must be at least separated by the coherence distance. Intuitively speaking, diversity gain can be seen as the number of independently fading paths that a symbol is transmitted through. In a flat fading channel with n_T transmit antennas and n_R receive antennas, the maximum achievable diversity is $n_T n_R$ for a codeword of duration $T \leq T_c$. In a frequency selective channel with L taps, the maximum achievable diversity would be $n_T n_R L$. In this Chapter, we consider spreading techniques for frequency selective MIMO channels assuming no channel knowledge at

3 Spreading

the transmitter. In Sec. 3.5, we introduce a family of spreading matrices for MIMO-OFDM that exploits the full diversity of frequency selective MIMO channels. The criteria behind the choice of those matrices will be given in Sec. 3.3. Spreading, although it increases diversity, also leads to increased interference. In addition, antenna correlations that often exist in MIMO systems also lead to higher interference. The effect of spreading and antenna correlations on interference will be discussed in Sec. 3.6.

3.1 Diversity

Wireless communication links often suffer from deep fades, i. e. large channel attenuation, leading to large errors at the receiver. Diversity is a method of providing the receiver with independently fading replicas of a transmitted signal. By increasing the number of independent links, the probability that all links are in fade at the same is reduced. From now on, we shall call those independent links diversity branches. These could be multiple coherence bandwidths to exploit frequency diversity or multiple antennas to exploit spatial diversity. Assuming flat fading across all diversity branches, Div, the received signal for each branch is

$$y_i = h_i x + n_i. \quad (3.1)$$

Assuming perfect channel knowledge at the receiver, maximum ratio combining (MRC) can be applied to obtain

$$\tilde{x}_i = \sum_{i=1}^{\text{Div}} h_i^* y_i = \sum_{i=1}^{\text{Div}} |h_i|^2 s + \tilde{n}, \quad (3.2)$$

where $\tilde{n} = \sum_{i=1}^{\text{Div}} h_i^* n_i$ and h_i is normalized such that $\mathcal{E}\{\sum_{i=1}^{\text{Div}} |h_i|^2\} = 1$. This normalization is also equivalent to dividing the transmit signal power equally over all diversity branches, assuming $\mathcal{E}\{|h_i|^2\} = 1$. Figure 3.1 shows the BER for bipolar (BP) transmission versus E_b/N_o for different number of parallel uncorrelated diversity branches. It is clear from the figure that as the diversity order increases ($\text{Div} \rightarrow \infty$), the BER approaches that of the AWGN channel (i. e. no fading). That is, in the presence of infinite diversity, fading can be completely alleviated [22]. Figure 3.1 shows that with diversity 32, the BER is only 0.5 dB from that of the AWGN channel at 10^{-4} .

Now, if those diversity branches become correlated, a coding loss, which manifests itself in a shift of BER curves to the right, is experienced by the

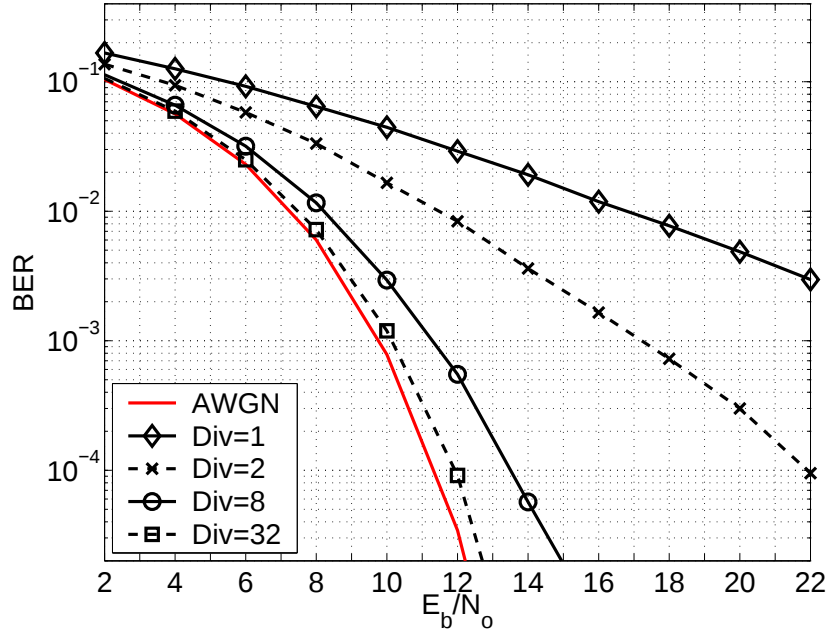


Figure 3.1: Effect of diversity on the BER performance in uncorrelated parallel fading channels.

BER curves. Correlation between the diversity branches can be modeled by the following equation,

$$\underline{h}_c = \underline{k}^{1/2} \underline{h}, \quad (3.3)$$

where $\underline{k}$ is the correlation matrix and is given by Eqn. 2.44. As can be seen in Fig. 3.2(a) where we have assumed $\rho_i = \rho \forall i$, the effect of correlations manifests itself at high E_b/N_o through a parallel shift of the BER curve to the right. The slope of the BER curves remains unchanged as long as $\underline{k}$ has full rank. Figure 3.2(b) shows the effect of correlations on a system with higher diversity, diversity=64. Although the BER performance is better, the same effects as for the system with diversity 4 can be observed. For both cases described here, we have assumed that all diversity branches are correlated. If this is not the case, the BER curves will change depending on the number of correlated branches and/or the degree of correlations present between the diversity branches. In addition, if the different diversity branches have different energies, the BER curves will change accordingly.

We have shown the effect of diversity and correlations on the BER. In this section, we have assumed parallel channels, i.e. no interference. The slope and shift of the BER in the case of channel with interference would not only be affected by correlations and diversity, but also by the equalizer employed as shall be shown later in this Chapter.

3 Spreading

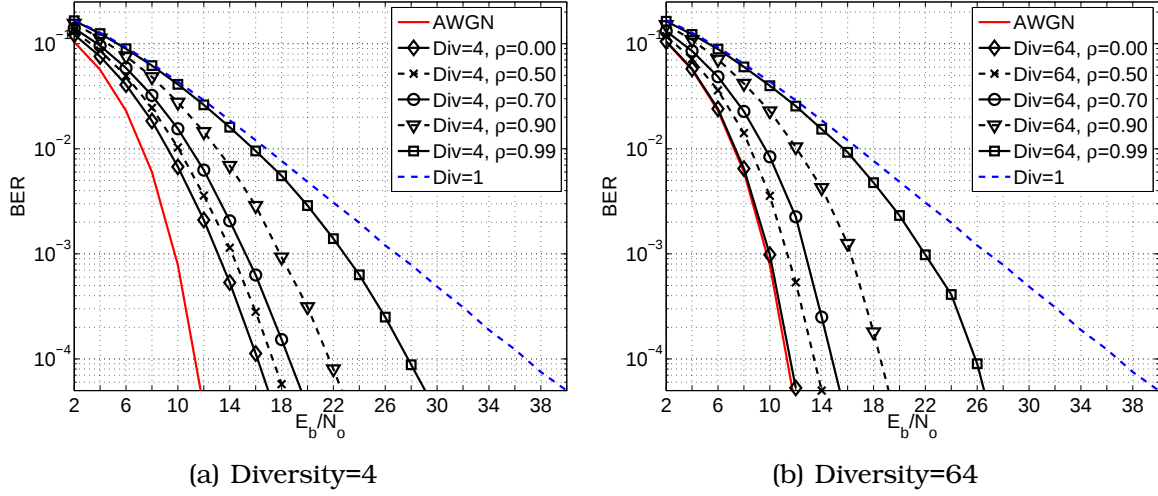


Figure 3.2: Effect of correlations on the BER performance in parallel fading channels.

3.2 Space Time and Space Frequency Codes

The most famous space time system is the Alamouti code introduced by Alamouti [56] in 1998 for a MISO 2×1 . In this technique, two symbols x_1 and x_2 are simultaneously transmitted from the first and second antennas respectively during the first signal period. At the second signal period, $-x_2^*$ and x_1^* are transmitted from the first and second antennas respectively. It is assumed that the channel does not change during those two signal periods, i.e. the coherence time is larger than twice the signal period. The channel is also assumed to be flat fading and perfectly known at the receiver. The received symbol vector, $\underline{y}$, can accordingly be written as follows,

$$\underline{y} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \underline{\underline{H}}_{eff} \underline{x} + \underline{n}, \quad (3.4)$$

where $\underline{n}$ is an AWGN vector and $\underline{\underline{H}}_{eff}$ is the effective channel matrix. Since $\underline{\underline{H}}_{eff}$ is orthogonal, that is $\underline{\underline{H}}_{eff}^H \underline{\underline{H}}_{eff} = ||h||_F^2 \underline{I}$, then

$$\tilde{\underline{x}} = \underline{\underline{H}}_{eff}^H \underline{y} = ||h||_F^2 \underline{x} + \tilde{\underline{n}}, \quad (3.5)$$

and the transmitted signals can be detected without interference. The Alamouti code achieves maximum diversity $n_T n_R = 2$ of the flat fading channel. The code rate is 1.0, since it takes two signal periods to transmit two signals. It is actually the only orthogonal STBC that has code rate 1.0. The

3.2 Space Time and Space Frequency Codes

above Alamouti scheme, although it requires only one receive antenna, can be used with multiple receive antennas which leads to improved BER and higher diversity. The design of STBC for larger number of transmit antennas can be found in [57]. In this work, the authors introduced two design criteria for ST code construction: the rank criterion and the determinant criterion. These criteria are based on the pairwise error probability (PEP) between any two codewords assuming maximum likelihood (ML) detection. For Rayleigh fading, the upperbound on the PEP between any two codewords $\underline{C}$ and $\underline{E}$ averaged over all channels for high SNR ($E_s/N_o \gg 1$) is

$$P(\underline{C} \rightarrow \underline{E}) \leq \frac{1}{\left(\prod_{k=1}^{r(\underline{Y})} \lambda_k(\underline{Y})\right)^{n_R}} \left(\frac{E_s}{4N_o}\right)^{-r(\underline{Y})n_R}, \quad (3.6)$$

where $\underline{Y} = \underline{G}\underline{G}^H$, $\underline{G} = \underline{C} - \underline{E}$, $\lambda_k(\underline{Y})$ are the non-zero eigenvalues of $\underline{G}$ and $r(\underline{Y})$ is the rank of $\underline{Y}$ (which is also equal to the rank of $\underline{G}$). The rank criterion optimizes the diversity gain while the determinant criterion optimizes the coding gain. To achieve maximum diversity $n_T n_R$, the matrix $\underline{G}$ must be of full rank for any two codeword pairs. To maximize the coding gain given a diversity target of $n_T n_R$, the minimum of the determinant of $\underline{Y}$ must be maximized for all pairs of codewords [57]. The coding gain is a measure of the gain due to coding compared to an uncoded system with the same diversity. At high SNR, the coding gain leads to a shift in the SER curve to the left while the diversity gain to an increase in its slope [22].

In [58], the authors showed that space-time codes designed to achieve full diversity in flat fading environment will in general not achieve the full space-frequency diversity, since they do not exploit the extra frequency diversity available. Similar to [57], they presented a design criteria for MIMO-OFDM also based on the pairwise error probability and maximum likelihood detection. The authors took the frequency diversity into account when deriving the design criteria. Assuming no spatial or tap correlations and a uniform power delay profile of the Rayleigh fading channels, the pairwise error probability, after averaging over all channel realizations, can be written as [58]

$$P(\underline{C} \rightarrow \underline{E}) = \prod_{i=0}^{r(\underline{Y})} \left(1 + \frac{E_s}{4N_o} \lambda_i(\underline{Y})\right)^{-n_R}. \quad (3.7)$$

The matrix $\underline{Y} = \underline{G}\underline{G}^H$ where

$$\underline{G} = [(\underline{C} - \underline{E})^T \quad \underline{D}(\underline{C} - \underline{E})^T \quad \cdots \quad \underline{D}^l(\underline{C} - \underline{E})^T \quad \cdots \quad \underline{D}^{L-1}(\underline{C} - \underline{E})^T], \quad (3.8)$$

$\underline{D} = \text{diag}\{e^{-j\frac{2\pi}{N}k}\}_{k=0}^{N-1}$ and λ_i are the eigenvalues of $\underline{Y}$ and $r(\underline{Y})$ is its rank. Again, similar to STBC, to achieve full diversity, the matrix $\underline{G}$ has to be full

3 Spreading

rank. That is, $\underline{\underline{C}} - \underline{\underline{E}}$ has to have full rank for all codeword pairs and each block $\underline{\underline{B}}_i = \underline{\underline{D}}^i(\underline{\underline{C}} - \underline{\underline{E}})$ should be linearly independent of the other blocks $\underline{\underline{B}}_l$ for all $i \neq l$ and $l = 0, \dots, L-1$. Ensuring that the space frequency code exploits the available frequency diversity, ensures the linear independence of the blocks $\underline{\underline{B}}_l$ [22, 58]. In [59], the authors provided a class of space-frequency codes which achieve full spatial and frequency diversity based on the criteria in [58]. As with ST codes, the design criteria in [59] impose limits on the proposed code rate. Another example of maximum diversity space time codes can be found for example in [60].

Space-time and space-frequency codes, although achieve high diversity and eliminate intersymbol interference at the receiver (i.e. no equalization is required), they offer a transmission rate which is only a fraction of that of a non-coded system. In the next Section, we look at spreading matrices for MIMO-OFDM that can achieve full diversity and full data rates (i.e. no data rate reduction compared to the non-coded system). In contrast to space time frequency codes, spreading matrices do not eliminate intersymbol interference at the receive and equalization is thus necessary.

3.3 Spreading Criteria for MIMO-OFDM

The aim of this Section is to introduce a family of spreading matrices that maintains the same transmission rate, and yet achieves the maximum possible diversity in a frequency selective channel. That is, we want to transmit every signal over all independently fading paths of the MIMO channel. We shall show the conditions under which this family of matrices achieves maximum diversity.

Before giving the criteria a spreading matrix needs to fulfill in order to achieve maximum diversity, we first take a closer look at the diagonal elements, $r_{i_T, k}$, of the channel matrix $\underline{\underline{R}}_{\text{MO}}$ before spreading. The indices of $r_{i_T, k}$ are separated by a comma since they only refer to the diagonal elements of $\underline{\underline{R}}_{\text{MO}}$ according to transmit antenna, i_T , and frequency, k . $r_{i_T, k}$ can be expressed as follows:

$$r_{i_T, k} = \frac{1}{n_R} \sum_{i_R=1}^{n_R} |H_{i_R i_T}(k)|^2, \quad (3.9)$$

which is the sum over all receive antennas, and represents the MRC technique at subcarrier k for symbols transmitted from antenna i_T . The transfer function between transmit antenna i_T and receive antenna i_R at frequency k is as follows

$$H_{i_R i_T}(k) = \sum_{m=1}^L h_{i_R i_T}(m) e^{-j2\pi(\frac{(k-1)(m-1)}{N})}. \quad (3.10)$$

3.3 Spreading Criteria for MIMO-OFDM

Equation 3.10 represents the k -th diagonal element of Eqn. 2.24. The absolute value squared of $|H_{i_R i_T}(k)|^2$ is thus given by

$$\begin{aligned} |H_{i_R i_T}(k)|^2 &= \sum_{m=1}^L h_{i_R i_T}(m) e^{-j2\pi(\frac{(k-1)m}{N})} \sum_{l=1}^L h_{i_R i_T}^*(l) e^{j2\pi(\frac{(k-1)l}{N})} \\ &= \sum_{m=1}^L |h_{i_R i_T}(m)|^2 + \sum_{m=1}^L h_{i_R i_T}(m) e^{-j2\pi(\frac{(k-1)m}{N})} \sum_{l \neq m}^L h_{i_R i_T}^*(l) e^{j2\pi(\frac{(k-1)l}{N})} \quad (3.11) \\ &= \sum_{m=1}^L |h_{i_R i_T}(m)|^2 + \sum_{m=1}^L \sum_{l > m}^L 2\text{Re}\{h_{i_R i_T}(m) h_{i_R i_T}^*(l) e^{-j2\pi(\frac{(k-1)(l-m)}{N})}\}. \end{aligned}$$

Equations 2.31, 3.9, and 3.11 give a guideline for selecting the appropriate spreading.

Criterion 1 To achieve maximum diversity, spreading should be applied such that the diagonal elements of $\underline{\underline{R}}_S$, ${}^s r_{i_T, k}$, satisfy the following equation

$${}^s r_{i_T, k} = \sum_{k \in \mathcal{S}_1, i_T \in \mathcal{S}_2} |w_{i_T, k}|^2 r_{i_T, k}, \quad (3.12)$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ are sets containing all frequencies and transmit antennas, i.e. $\mathcal{S}_1 = \{1, \dots, N\}$ and $\mathcal{S}_2 = \{1, \dots, n_T\}$. Since we are trying to achieve both spatial and frequency diversity, the subsets chosen from $\mathcal{S}_1$ and $\mathcal{S}_2$ can not be empty. The $|w_{i_T, k}|^2$ are weighting constants dependent on the subsets of $\mathcal{S}_1$ and $\mathcal{S}_2$ and should satisfy $\sum_{k \in \mathcal{S}_1, i_T \in \mathcal{S}_2} |w_{i_T, k}|^2 = 1$. They ensure that Eqn. 2.31 is satisfied after spreading. Equation 3.12 imposes the requirement that no signal should be transmitted from different antennas at the same frequency, i.e. no signal should interfere with itself after spreading. For the spreading matrices considered here, the absolute values for $w_{i_T, k}$ are the same for all i_T and k , i.e. $|w_{i_T, k}| = |w|$. That is, the signal energy is equally spread over all diversity branches.

Criterion 2 The subsets of $\mathcal{S}_1$ and $\mathcal{S}_2$ should be chosen such that

$$\sum_{k \in \mathcal{S}_1, i_T \in \mathcal{S}_2} |w_{i_T, k}|^2 \sum_{i_R=1}^{n_R} \sum_{m=1}^L \sum_{l > m}^L 2A = |w|^2 \sum_{k \in \mathcal{S}_1, i_T \in \mathcal{S}_2} \sum_{i_R=1}^{n_R} \sum_{m=1}^L \sum_{l > m}^L 2A = 0, \quad (3.13)$$

where

$$A = \text{Re}\{h_{i_R i_T}(m) h_{i_R i_T}^*(l) e^{-j2\pi(\frac{(k-1)(l-m)}{N})}\}. \quad (3.14)$$

Equation 3.13 is also satisfied, if only the following sum is zero,

$$\sum_{k \in \mathcal{S}_1} \sum_{m=1}^L \sum_{l > m}^L 2\text{Re}\{h_{i_R i_T}(m) h_{i_R i_T}^*(l) e^{-j2\pi(\frac{(k-1)(l-m)}{N})}\} = 0. \quad (3.15)$$

3 Spreading

Thus, if, through spreading, the subsets for i_T and k in Eqns. 3.12 and 3.13 (or 3.15) are adequately chosen, the diagonal elements of $\underline{\underline{R}}_S$ become

$$s_{r_{i_T,k}} = \frac{1}{n_R n_T} \sum_{i_R=1}^{n_R} \sum_{i_T=1}^{n_T} \sum_{l=1}^L |h_{i_R i_T}(l)|^2 = \frac{1}{n_R n_T} \|\underline{\underline{h}}\|_F^2, \quad (3.16)$$

for all i_T and k . The square norm $\|\underline{\underline{h}}\|_F^2$ is defined as

$$\|\underline{\underline{h}}\|_F^2 = \sum_{l=1}^L \|\underline{\underline{h}}(l)\|_F^2, \quad (3.17)$$

where $\|\underline{\underline{h}}(l)\|_F^2$ is the squared Frobenius norm of $\underline{\underline{h}}(l)$, and $\mathcal{E}\{\|\underline{\underline{h}}\|_F^2\} = n_T n_R$. That is, the diagonal elements of $\underline{\underline{R}}_S$ are equal, maximum diversity is achieved and for the ideal case, when all interference has been removed (MFB), the received OFDM symbol $\underline{\underline{x}}$ can be expressed by

$$\underline{\underline{x}} = \frac{1}{n_R n_T} \|\underline{\underline{h}}\|_F^2 \underline{\underline{I}}_{n_T N} \underline{\underline{x}} + \underline{\underline{n}}, \quad (3.18)$$

where $\underline{\underline{x}}$ is the transmitted OFDM symbol, $\underline{\underline{I}}_{n_T N}$ the identity matrix of size $n_T N$ and $\underline{\underline{n}}$ is the noise vector.

3.4 MC-Code Division Multiplexing

Multi-carrier coded division multiplexing (MC-CDM) spreads the energy of each symbol equally over the N subcarriers of its corresponding transmit antenna. No spreading is performed in the antenna direction. The spreading matrix for MC-CDM is given by [19, 61, 62, 18]:

$$\underline{\underline{U}}_{MC-CDM} = \begin{bmatrix} \underline{\underline{S}} & \underline{\underline{0}} & \underline{\underline{0}} & \dots & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{S}} & \underline{\underline{0}} & \dots & \underline{\underline{0}} \\ \vdots & \vdots & \ddots & \dots & \vdots \\ \underline{\underline{0}} & \underline{\underline{0}} & \dots & \underline{\underline{S}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \dots & \underline{\underline{S}} \end{bmatrix}, \quad (3.19)$$

where $\underline{\underline{S}}$ is an $N \times N$ orthogonal matrix, and $\underline{\underline{0}}$ are zero matrices of the same size. Alternatively, we can express the spreading matrix as follows

$$\underline{\underline{U}}_{MC-CDM} = \underline{\underline{I}}_{n_T} \otimes \underline{\underline{S}}, \quad (3.20)$$

where $\otimes$ is the Kronecker product and $\underline{\underline{I}}_{n_T}$ is the identity matrix of size $n_T \times n_T$. Note that although the MC-CDM spreading matrix satisfies Eqn. 3.15, the subset $i_T \subset \mathcal{S}_2$ in Eqn. 3.12 is empty (Criterion 2 is not satisfied) and thus full diversity can not be achieved. The maximum achievable diversity for MC-CDM spreading is thus $n_R L$.

3.5 MC-Cyclic Antenna Frequency Spreading

In this section, we describe a family of spreading matrices that utilize the frequency as well as the spatial dimensions offered by the MIMO system and were first proposed in [62, 61]. These spreading matrices spread each signal over all transmit antennas and over a set of frequencies at each transmit antenna. These frequency sets are different for each antenna and thus the name **MC-CAFS**: multi-carrier cyclic antenna frequency spreading. We shall show that this family of spreading matrices can satisfy Eqns. 3.12 through 3.16 under certain conditions and thus achieves maximum diversity. These conditions, as shall be shown later, are easily satisfied in practical OFDM systems. As mentioned earlier, we shall only consider orthogonal spreading matrices. The spreading matrix, $\underline{U}_{MC-CAFS}$, is defined by

$$\underline{U}_{MC-CAFS} = \begin{bmatrix} s_{11}\underline{I} & s_{12}\underline{I} & \cdots & \cdots & \cdots & s_{1(B_{n_T})}\underline{I} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{B1}\underline{I} & s_{B2}\underline{I} & \cdots & \cdots & \cdots & s_{B(B_{n_T})}\underline{I} \\ \hline s_{(B_1+1)1}\underline{I}_{p_1} & s_{(B_1+1)2}\underline{I}_{p_1} & \cdots & \cdots & \cdots & s_{(B_1+1)(B_{n_T})}\underline{I}_{p_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{(B_2)1}\underline{I}_{p_1} & s_{(B_2)2}\underline{I}_{p_1} & \cdots & \cdots & \cdots & s_{(B_2)(B_{n_T})}\underline{I}_{p_1} \\ \hline s_{(B_2+1)1}\underline{I}_{p_2} & s_{(B_2+1)2}\underline{I}_{p_2} & \cdots & \cdots & \cdots & s_{(B_2+1)(B_{n_T})}\underline{I}_{p_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{(B_3)1}\underline{I}_{p_2} & s_{(B_3)2}\underline{I}_{p_2} & \cdots & \cdots & \cdots & s_{(B_3)(B_{n_T})}\underline{I}_{p_2} \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline s_{(B_{n_T-1}+1)1}\underline{I}_{p_n} & \cdots & \cdots & \cdots & \cdots & s_{(B_{n_T-1}+1)(B_{n_T})}\underline{I}_{p_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{(B_{n_T})1}\underline{I}_{p_n} & \cdots & \cdots & \cdots & \cdots & s_{(B_{n_T})(B_{n_T})}\underline{I}_{p_n} \end{bmatrix} \quad (3.21)$$

where B is the number of frequency blocks per transmit antenna and represents the number of frequencies over which each signal is spread at each transmit antenna (the horizontal lines show the separation between the transmit antennas), $B_m = mB$, and s_{ij} are elements of the orthogonal matrix $\underline{S}$

3 Spreading

(i. e. $\underline{\underline{S}}^H \underline{\underline{S}} = \underline{\underline{I}}$) of size $(n_T B) \times (n_T B)$. $\underline{\underline{I}}$ is identity matrices of size $(N/B) \times (N/B)$ and $\underline{\underline{I}}_{\underline{\underline{p}}_i}$ is its i -th permutation. The permutations imply that each signal is spread over different frequencies at each transmit antenna and ensure that the first criterion (Eqn. 3.12) is satisfied [62]. Note that the number of allowed permutations, n , must satisfy $n \leq (n_T - 1)$ (i. e. no permutation should repeat), else $\underline{\underline{U}}_{MC-CAFS}$ will lose its orthogonality. This condition translates to $(N/B) \geq n_T$ and gives an **upper bound** for B :

$$B \leq N/n_T. \quad (3.22)$$

In addition, to insure that Eqn. 3.15 is satisfied, B must be **lower bounded** by:

$$B \geq L \quad (3.23)$$

This can be proven by closely looking at the exponential term of Eqn. 3.13. At any transmit antenna, this equation is satisfied if we spread over a subset of frequencies such that:

$$\begin{aligned} \sum_{k=\mathcal{S}_{i1} \subset \mathcal{S}_1} \cos(k \frac{2\pi}{N}(l-m)) &= 0, \\ \sum_{k=\mathcal{S}_{i1} \subset \mathcal{S}_1} \sin(k \frac{2\pi}{N}(l-m)) &= 0, \end{aligned} \quad (3.24)$$

where $\sum_i \mathcal{S}_{i1} = \mathcal{S}_1$. Note that Eqns 3.24 are readily satisfied for $k = \mathcal{S}_1$. However, choosing $k = \mathcal{S}_1$ contradicts with Criterion 1. Without loss of generality, we consider the case where N is an even integer. This is applicable to OFDM systems where the fast Fourier transform (FFT) and the inverse fast Fourier transform (IFFT) are employed. Accordingly, $\mathcal{S}_{i1}$ include the subset of frequencies such that

$$\Delta k \frac{2\pi}{N}(l-m) = \pi, \quad (3.25)$$

$$\Delta k = \begin{cases} \lceil \frac{N}{2(l-m)} \rceil & \text{if } (l-m) \text{ even} \\ \frac{N}{2} & \text{if } (l-m) \text{ odd} \end{cases}, \quad (3.26)$$

where $\lceil x \rceil$ rounds x towards infinity. Thus, to insure that Eqn. 3.15 is satisfied, any one symbol must be spread over an integer number of B frequencies satisfying

$$B = \max\left\{\frac{N}{\Delta k}\right\} \geq L. \quad (3.27)$$

3.5 MC-Cyclic Antenna Frequency Spreading

The **MC-CAFS** spreading would therefore achieve maximum diversity, if B satisfies the following criteria

$$L \leq B \leq \frac{N}{n_T}. \quad (3.28)$$

Note that for a given N , n_T and L , there is a set of B values (i.e. set of spreading matrices) that would satisfy eqns. 3.12 to 3.16. In addition, B must also be chosen such that N/B is an integer.

To summarize, it is clear that for Eqn. 3.13 to be satisfied, it is enough to insure that any symbol is spread over the adequate number of frequencies of its corresponding transmit antenna (Eqn. 3.15). The choice of the subset $i_T \subset S_2 = \{1, \dots, n_T\}$ does not affect the equality in Eqn. 3.13, but affects validity of Eqn. 3.16. By choosing $i_T = S_2$, Eqn. 3.16 is satisfied. That is, although the symbols need not be spread over all frequencies, they must be spread over all antennas. Through the use of the permutations, MC-CAFS satisfies this requirement without violating the first criterion.

Example 1

For example, consider a $n_R \times n_T$ MIMO-OFDM system with $N = 8$ and for the cases, $L = 2$, $L = 3$ and $L = 4$. For $L = 2$, the maximum possible value for $(l - m) = 1$, for $L = 3$ and $L = 4$, they are $(l - m) = 2$ and 3 respectively. Fig. 3.3 shows the values of $E(k) = e^{-j2\pi(k-1)\frac{(l-m)}{N}}$ for $(l - m) = 1, 2$ and 3. For every point on the circle, there is another point satisfying $E(k + \Delta k) = -E(k)$ (separated by an angle of π). For $L = 2$, $\Delta k = 4$, while for $L = 3$ and $(l - m) = 2$, $\Delta k = 2$. Thus, for $L = 2$, it would be enough to spread over $B = 2$ frequencies separated by $\Delta k = 4$ and over $B = 4$ frequencies separated by $\Delta k = 2$ for $L = 3$. For $L = 4$ and $(l - m) = 3$, $\Delta k = 4$ and accordingly $B = 2$ would be sufficient. However, since the sum in Eqn. 3.15 also includes the sum for $(l - m) = 2$, thus $\Delta k = 2$ must be chosen. Also note that in the case of $L = 3$, the condition for $(l - m) = 1$ is automatically satisfied if $\Delta k = 2$, that is $B = 4$.

Example 2

Consider a MIMO-OFDM channel with $n_T = n_R = 2$, $L = 2$ and $N = 4$. The diagonal elements of $\underline{\underline{R}}$ before spreading for the first antenna at $k = 1$ and 3

3 Spreading

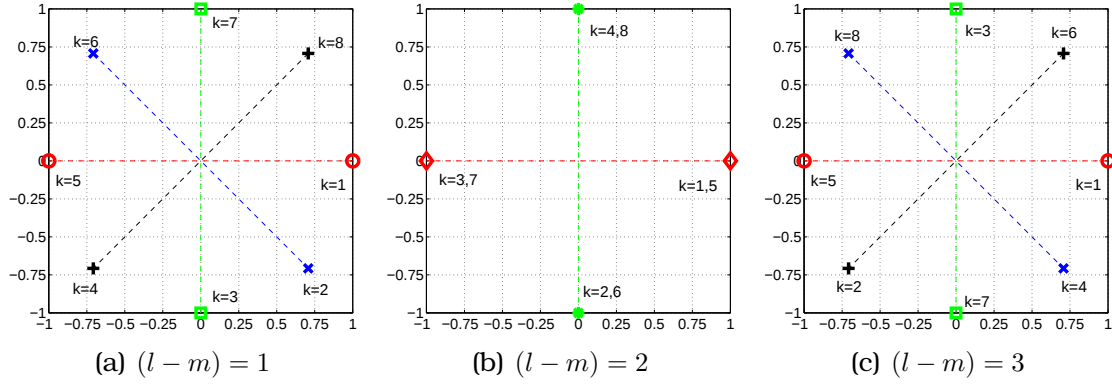


Figure 3.3: $E(k) = e^{-j2\pi(k-1)\frac{(l-m)}{N}}$, $N = 8$, $k = 1, \dots, N$.

are,

$$r_{1,1} = \sum_{l=1}^{L=2} |h_{11}(l)|^2 + \sum_{l=1}^{L=2} |h_{21}(l)|^2 + 2\text{Re}\{h_{11}(1)h_{11}^*(2)\} + 2\text{Re}\{h_{21}(1)h_{21}^*(2)\}$$

$$r_{1,3} = \sum_{l=1}^{L=2} |h_{11}(l)|^2 + \sum_{l=1}^{L=2} |h_{21}(l)|^2 - 2\text{Re}\{h_{11}(1)h_{11}^*(2)\} - 2\text{Re}\{h_{21}(1)h_{21}^*(2)\}$$

and for the second antenna at $k = 1$ and 4 are,

$$r_{2,2} = \sum_{l=1}^{L=2} |h_{22}(l)|^2 + \sum_{l=1}^{L=2} |h_{12}(l)|^2 + 2\text{Re}\{jh_{22}(1)h_{22}^*(2)\} + 2\text{Re}\{jh_{12}(1)h_{12}^*(2)\}$$

$$r_{2,4} = \sum_{l=1}^{L=2} |h_{22}(l)|^2 + \sum_{l=1}^{L=2} |h_{12}(l)|^2 - 2\text{Re}\{jh_{22}(1)h_{22}^*(2)\} - 2\text{Re}\{jh_{12}(1)h_{12}^*(2)\}.$$

If the signals are spread over the two transmit antennas at the above given frequencies (MC-CAFS spreading with $B = 2$), then the diagonal elements of $\underline{\underline{R}}_s$ are

$${}^s r_{i_T, k} = \sum_{l=1}^{L=2} |h_{11}(l)|^2 + \sum_{l=1}^{L=2} |h_{21}(l)|^2 + \sum_{l=1}^{L=2} |h_{12}(l)|^2 + \sum_{l=1}^{L=2} |h_{22}(l)|^2 = \sum_{l=1}^{L=2} \|\underline{h}(l)\|_F^2$$

which is in accordance with Eqn. 3.18.

BER Comparison for MIMO-OFDM, MC-CDM and MC-CAFS

Figures 3.4 and 3.5 compare the BER for MIMO-OFDM, MC-CDM and MC-CAFS, $B = 8$ for a 4×4 block fading MIMO channel with $L = 2$ and $L = 4$

3.5 MC-Cyclic Antenna Frequency Spreading

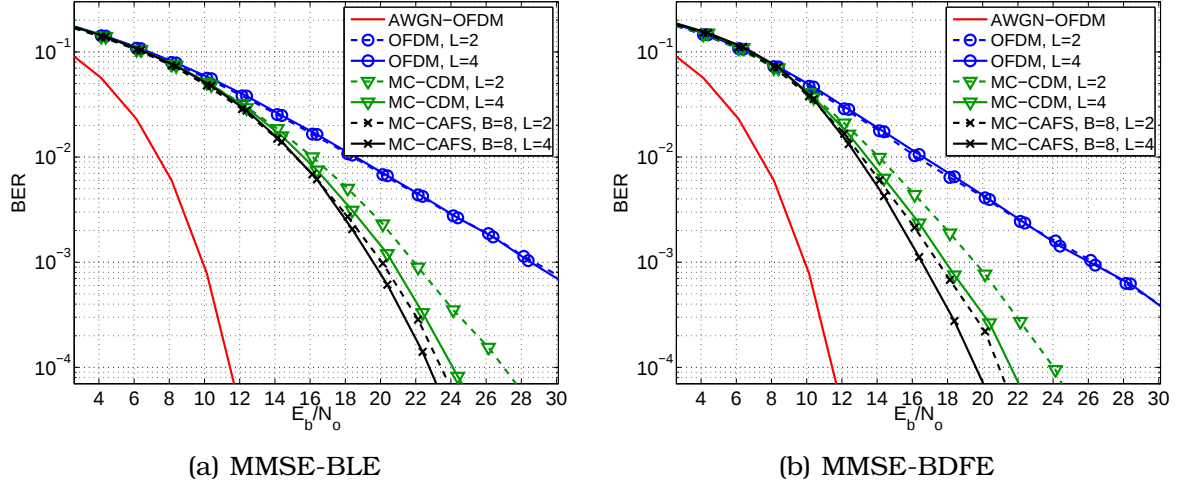


Figure 3.4: BER versus E_b/N_0 for MIMO-OFDM, MC-CDM and MC-CAFS, $n_T = n_R = 4$.

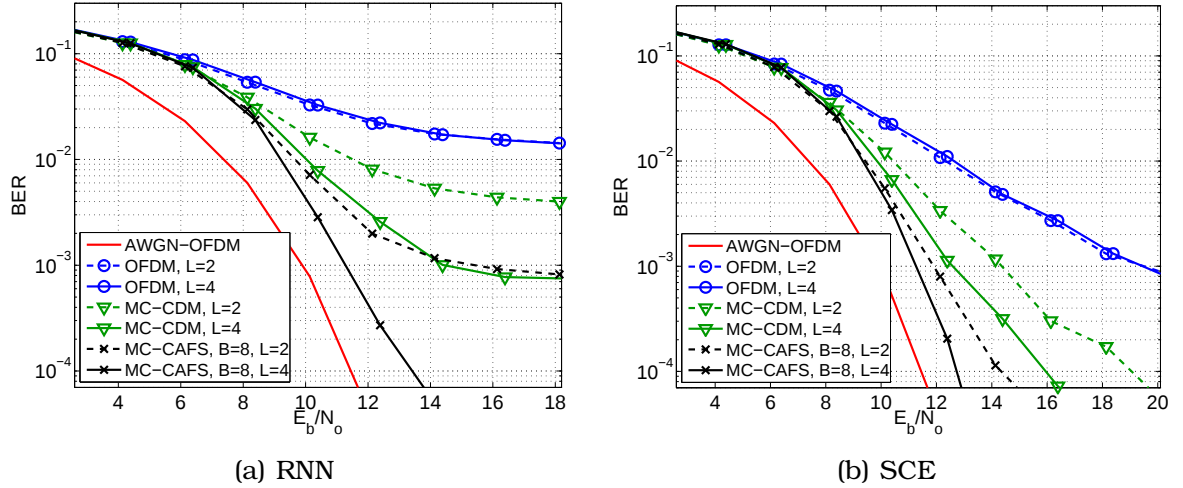


Figure 3.5: BER versus E_b/N_0 for MIMO-OFDM, MC-CDM and MC-CAFS, $n_T = n_R = 4$.

and $N = 32$ using MMSE-BLE, MMSE-BDFE, RNN and SCE. The transmitted symbols are chosen from a 4 PSK alphabet. The BER using MC-CAFS is lowest for all equalizers, outperforming MIMO-OFDM by at least 8 dB at a BER of 10^{-3} . At 10^{-4} MC-CAFS also outperforms MC-CDM with a minimum gain of 1 dB using MMSE-BLE for $L = 4$ and with a maximum of more than 5 dB using the SCE for $L = 2$. The BER for MIMO-OFDM is not affected by L , since the transmitted signals only experience receive diversity (n_R). The

3 Spreading

small shift between the BER curve for $L = 2$ and $L = 4$ is only due to the cyclic prefix. The diversity gain (steeper BER curve), for all L , using MC-CAFS compared to MC-CDM is most obvious when the SCE or RNN equalizers are employed (see Fig. 3.5). In addition, the BER curve for MC-CAFS for $L = 4$ is steeper than that for $L = 2$ due to increased diversity. This result is not as obvious for MMSE-BLE and MMSE-BDFE for $L = 4$. However, at higher E_b/N_o , the difference in the BER curve slopes might become more noticeable. The RNN equalizer is sensitive to interference and can suffer from an error floor (cf. Chap. 2). Here, the error floor is observed to occur at around 14 dB. Increasing the E_b/N_o beyond 14 dB does not lead to lower BER. The BER performance for the RNN equalizer benefits the most from MC-CAFS spreading. The error floor drops by at least one decade for $L = 2$ compared to OFDM. For $L = 4$ the error floor drops even further. Due to the high interference present, the performance of the MMSE-BLE and the MMSE-BDFE is in general worse than that of the RNN and SCE. Even with MC-CAFS spreading, the BER curves are at least 8 dB from the AWGN-OFDM curve at a BER of 10^{-4} .

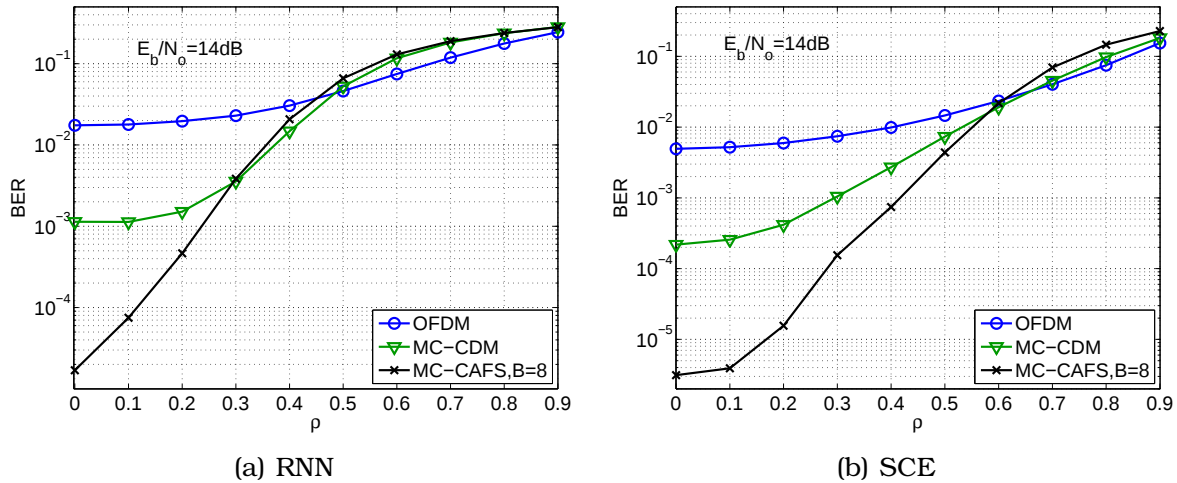


Figure 3.6: BER versus $\rho_{Tx,exp} = \rho_{Rx,exp} = \rho$ at $E_b/N_o = 14$ dB for MIMO-OFDM, MC-CDM and MC-CAFS, $n_T = n_R = 4$, $L = 4$.

Figure 3.6 shows the BER versus ρ ($\rho_{Tx,exp} = \rho_{Rx,exp} = \rho$) assuming an exponential correlation model at $E_b/N_o = 14$ dB for RNN and SCE using 4 PSK. The BER for MC-CAFS is the lowest for low to moderate correlation values ($\rho \leq 0.6$ for SCE and $\rho \leq 0.3$ for RNN). In case of the RNN equalizer, both MC-CDM and MC-CAFS have comparable performance for $\rho \geq 0.3$. For $\rho > 0.5$, the system performance for MC-CAFS, MC-CDM and OFDM is poor and the BER curves lie close together with OFDM leading to slightly lower BERs. Similarly

3.5 MC-Cyclic Antenna Frequency Spreading

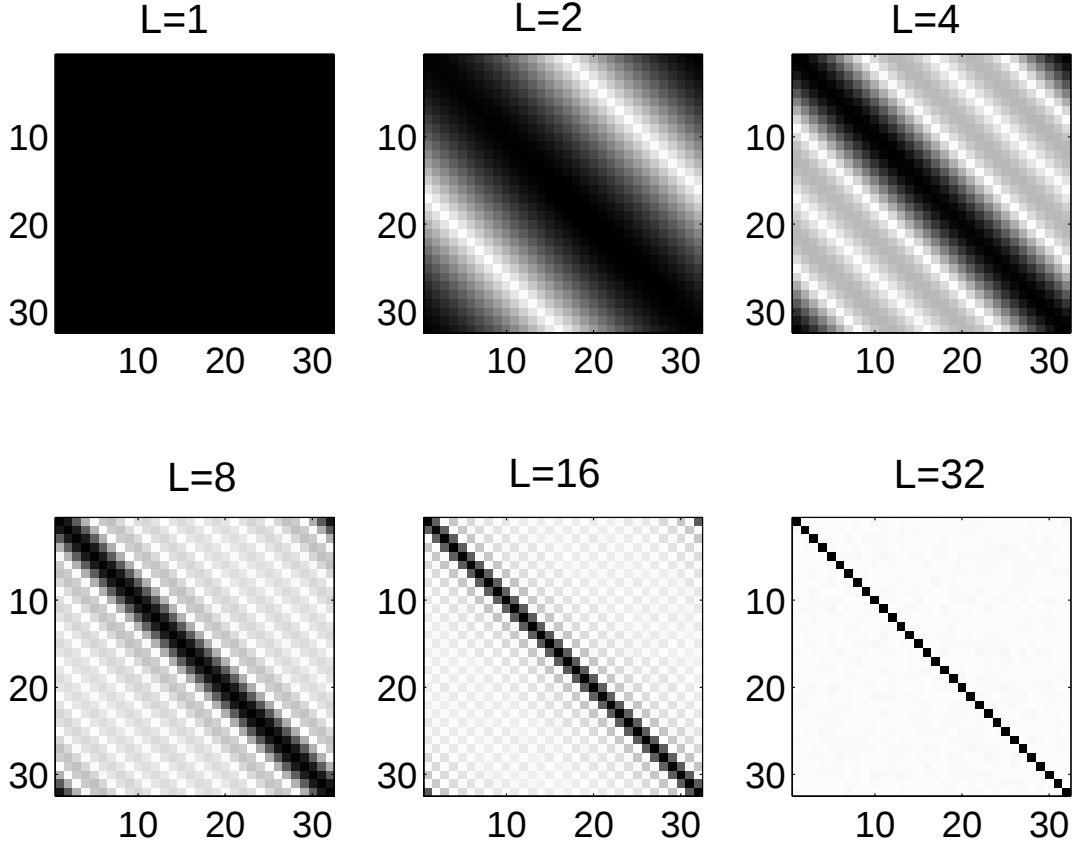


Figure 3.7: Effect of the number of taps, L , of a channel in the time domain on the auto-correlation between the channel transfer function at any two frequencies ($N = 32$).

for the SCE, MC-CAFS outperforms both the MC-CDM and OFDM for low to moderate correlation values ($\rho < 0.6$). As the correlations increase, MC-CDM or OFDM lead to slightly lower BERs. It is important to mention here, that the intersection point between the different BER curves shifts to the right as the E_b/N_o increases. As we shall see in the coming sections, increased correlations are associated with increased interference, which explains the above BER behavior.

Coherence Bandwidth

The choice of B or Δk for MC-CAFS can also be understood by considering the auto-correlation function between any two subcarriers, k_1 and k_2 , $k_1 \neq k_2$. The channel transfer function between any pair of transmit antennas at any

3 Spreading

subcarrier k_1 can be written as follows

$$H_{ij}(k_1) = \sum_{l=1}^L h_{ij}(l) e^{-j \frac{2\pi}{N} k_1 (l-1)}. \quad (3.29)$$

The auto-correlation function between any two subcarriers, assuming independent fading paths, can thus be given by

$$\mathcal{E}\{H_{ij}(k_1)H_{ij}^*(k_2)\} = \sum_{l=1}^L \mathcal{E}\{|h_{ij}(l)|^2\} e^{-j \frac{2\pi}{N} \Delta k (l-1)}, \quad (3.30)$$

where $\Delta k = k_2 - k_1$. Assuming a uniform power profile, i. e. $\mathcal{E}\{|h_{ij}(l)|^2\} = \sigma^2, \forall l$, then

$$\mathcal{E}\{H_{ij}(k_1)H_{ij}^*(k_2)\} = \sigma^2 \sum_{l=1}^L e^{-j \frac{2\pi}{N} \Delta k (l-1)}. \quad (3.31)$$

Now we find Δk for which Eqn. 3.31 is equal to zero. In other words, the correlations are zero, if

$$\begin{aligned} \sum_{l=1}^L \cos \frac{2\pi}{N} \Delta k (l-1) &= 0, \\ \sum_{l=1}^L \sin \frac{2\pi}{N} \Delta k (l-1) &= 0, \end{aligned} \quad (3.32)$$

which is satisfied if

$$\Delta k = \frac{N}{L}. \quad (3.33)$$

The value of Δk in Eqn. 3.33 can also be interpreted as the coherence bandwidth if we assume uncorrelated scattering (US) of the channel taps with equal PDP. That is, the delay spread in the channel have independent fading [22]. The subcarriers with spacing larger than the coherence bandwidth experience independent fading. To achieve maximum diversity it is thus enough to spread over $N/B_c = N/\Delta k$ frequencies separated by the coherence bandwidth, i. e. to spread over all coherence bandwidths. Figure 3.7 shows the auto-correlation between any two subcarriers for different number of channel taps, L . As the number of channel taps increases, the coherence Bandwidth decreases. For $N = L$, the coherence Bandwidth is zero.

Even for non-equal PDP, Eqns. 3.32 are still satisfied for $\Delta k = N/L$. However, in this case Δk is not necessarily equal to the coherence bandwidth. In case of non-equal PDP, where some of the taps have more power than others,

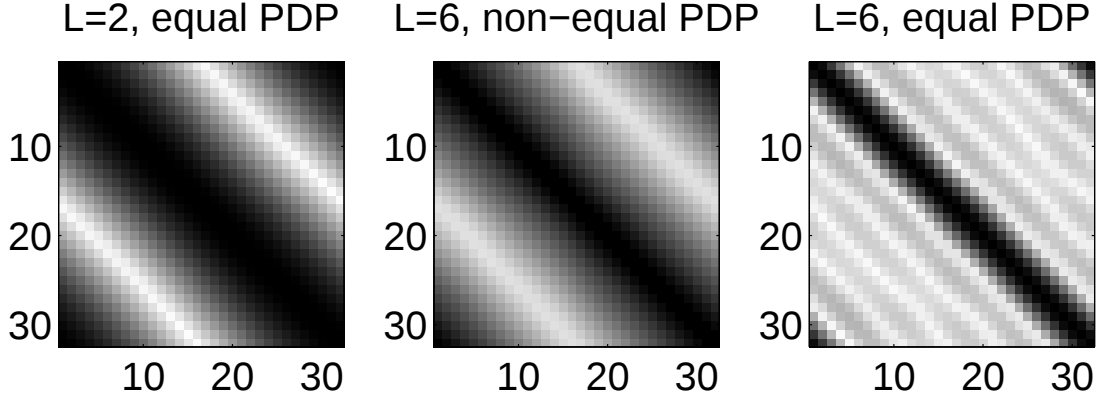


Figure 3.8: Effect of the power delay profile on the coherence bandwidth.

the coherence bandwidth can be smaller than L [63]. For instance consider the $L = 6$ channel, where the first 2 taps have most of the channel energy. The PDP is, $\sigma_l^2 = [0.48, 0.42, 0.053, 0.027, 0.015, 0.005]$, for $l = 1 \dots 6$, where the first 2 taps have 90% of the channel power. As can be seen in Fig. 3.8, the coherence bandwidth of the above described $L = 6$ channel with non-equal PDP is almost the same as that of a two path channel with equal PDP. The correlation matrix for a six tap channel with equal PDP is also shown for comparison. Thus, the coherence bandwidth is dependent not only on the number of channel taps, but also on the average power in each of these taps. The coherence bandwidth is minimum if the powers of all taps are equal [63].

Thus, B in MC-CAFS should be chosen such that, B is at least

$$B = \max(L, B_W/B_c),$$

where B_W is the total bandwidth.

BER Performance Comparison for different PDP

We now investigate the effect of the PDP and spreading factor B on the BER performance. Again, we consider a 4×4 MIMO system with $N = 32$ and transmission using 4 PSK. Figures 3.9 and 3.10 show the BER curves using MC-CAFS spreading with $B = 2$ or 8 for three channels. The channels have different power delay profiles and/or number of taps. EPDP stand for equal PDP and NPDP stands for non-equal PDP. In case of the NPDP channel, a six path MIMO channel is assumed with the following power delay profile: $\sigma_l^2 = [0.48, 0.42, 0.053, 0.027, 0.015, 0.005]$. The other two channels are a two path and a six path channel both with EPDP. The length of the cyclic prefix

3 Spreading

was chosen to be $L_{cp} = 5$ for all channels. The figures show that the BER performance of the NPDP channel lies between that of the two path and six path channel. The BER curve of the six path EPDP channel using the SCE even reaches the MFB performance, i. e. AWGN-OFDM BER curve (Fig. 3.10(b)). The NPDP channel, although it has 6 taps, can not achieve the same performance as the six path EPDP channel. Clearly, the amount of power present in each tap dictates how much power is allocated to the corresponding diversity branch. Thus the diversity effect is more evident if all taps have the same PDP. At high E_b/N_o , the BER curves for the six path EPDP and NPDP channels are expected run parallel to each other with the BER curve of the NPDP shifted to the right. The shift is dependent on the power delay profile. It is important to mention at this point, that the improvement in the BER for EPDP and $L = 2$ as B increases is not due to increased diversity, since that is already achieved with $B = 2$. This improvement is solely due to the distribution of the interference after spreading as shall be shown in the next section. As mentioned in Chapter 2, the performance of the equalizers is not only dependent on the total interference, but also on its distribution.

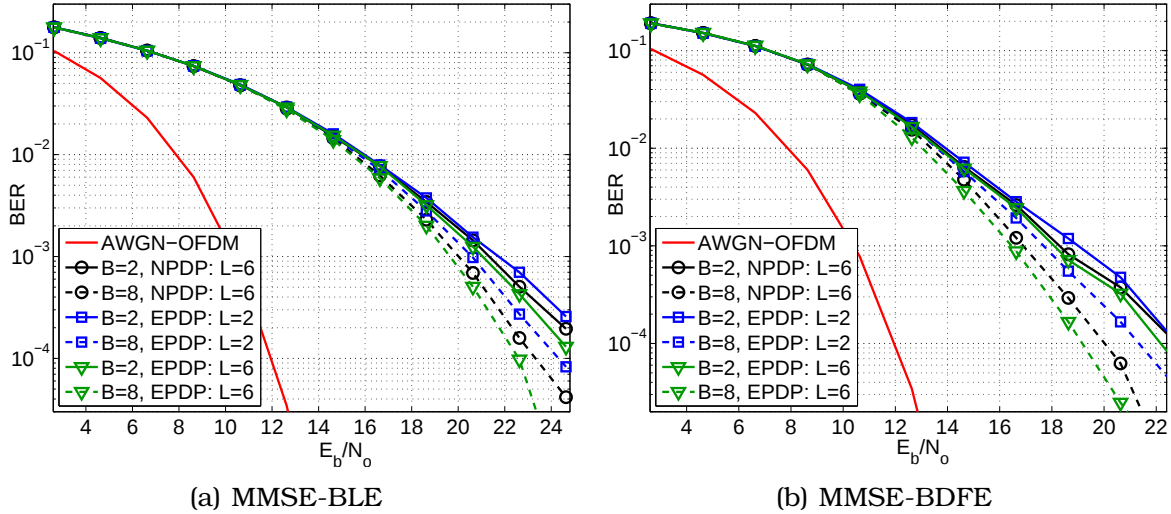


Figure 3.9: MMSE-BDFE and MMSE-BLE: Comparison of BER using MC-CAFS spreading for equal and non-equal PDP, $n_T = n_R = 4$, $N = 32$.

3.6 Spreading, Matched Filter Bound and Interference

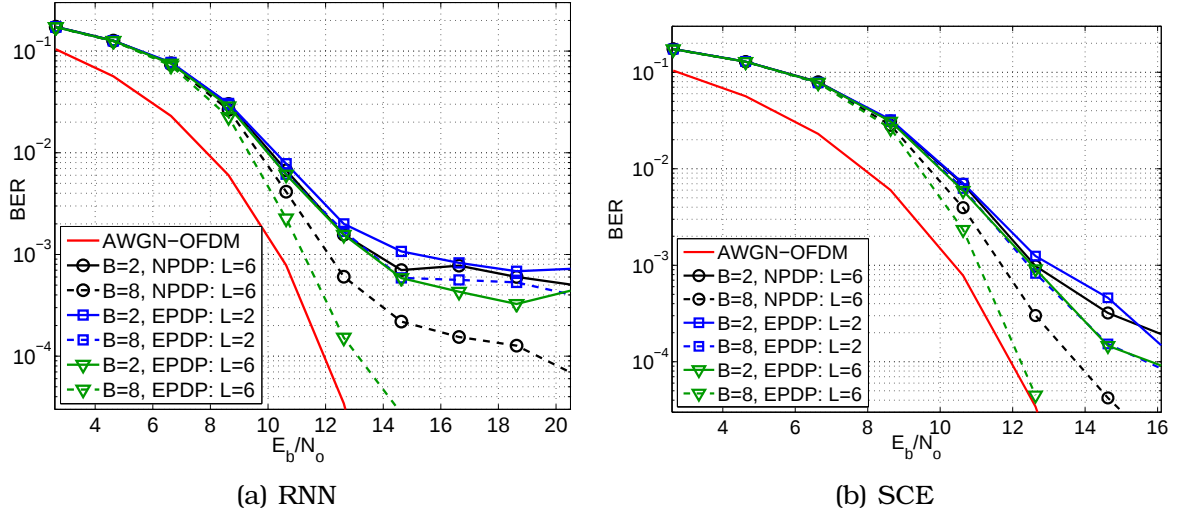


Figure 3.10: SCE and RNN: Comparison of BER using MC-CAFS spreading for equal and non-equal PDP, $n_T = n_R = 4$, $N = 32$.

3.6 Spreading, Matched Filter Bound and Interference

In this Section, we investigate the effect of the transmit and receive antenna correlations on the matched filter bound (MFB) and interference. The MFB describes the BER performance of the system if all interference is perfectly removed. In all what follows, we assume that all channel taps experience the same transmit and receive correlations (Eqn. 2.42) and based on the Kronecker correlation model given by Eqn. 2.39, the channel matrix $\underline{\underline{H}}^c$ in the frequency domain can be modeled in the same fashion as that in the time domain as shown below,

$$\underline{\underline{H}}^c = \underline{\underline{K}}_R^{1/2} \underline{\underline{H}} \underline{\underline{K}}_T^{1/2}, \quad (3.34)$$

where $\underline{\underline{H}}$ is given by Eqn. 2.26. The matrices $\underline{\underline{K}}_T$ and $\underline{\underline{K}}_R$ are given by

$$\underline{\underline{K}}_T = \underline{\underline{k}}_T \otimes \underline{\underline{I}}_N \quad (3.35)$$

$$\underline{\underline{K}}_R = \underline{\underline{k}}_R \otimes \underline{\underline{I}}_N, \quad (3.36)$$

where $\otimes$ is the Kronecker product and $\underline{\underline{I}}_N$ is the identity matrix of size $N \times N$. The off-diagonal elements, r_{ij} , of $\underline{\underline{R}}_{MO}$ represent the effect of the interference occurring between signals transmitted at the same frequency from different antennas. One measure for the total interference, β , experienced by each

3 Spreading

transmit symbol is defined as follows [64, 65],

$$\beta = \frac{1}{n_T N} \sum_{i \neq j} |r_{ij}|^2 \geq 0, \quad (3.37)$$

where r_{ij} are the off-diagonal elements of $\underline{\underline{R}}$, and $\underline{\underline{R}}$ being either $\underline{\underline{R}}_{\text{MO}}$ or $\underline{\underline{R}}_{\text{S}}$. Another measure for the total interference would be

$$\beta_{1/2} = \frac{1}{n_T N} \sum_{i \neq j} |r_{ij}| \geq 0, \quad (3.38)$$

which is simply the sum of the absolute values of the off-diagonal elements. In what follows, we will concentrate on β , but we shall also consider $\beta_{1/2}$ later on. The variance of the subchannel power, α , from the mean $\bar{r}$, is given by [64, 65]

$$\alpha = \frac{1}{n_T N} \sum_i (r_{ii} - \bar{r})^2 \geq 0, \quad (3.39)$$

where r_{ii} are the diagonal elements of $\underline{\underline{R}}$ and

$$\bar{r} = \frac{1}{N n_T} \sum_{i=1}^{N n_T} r_{ii}. \quad (3.40)$$

α gives a measure for the MFB bound, with a larger value of α corresponding to a worse MFB and vice versa. If $\alpha = 0$, then all signals undergo the same amount of fading. In this case, the MFB is governed by the amount of diversity and correlations present in the channel as was discussed in Sec. 3.1.

From the above definitions of α and β , we can write

$$\alpha + \beta = \frac{\|\underline{\underline{R}}\|_{\text{F}}^2}{n_T N} - \bar{r}^2, \quad (3.41)$$

where $\|\underline{\underline{R}}\|_{\text{F}}^2$ is the Frobenius norm of $\underline{\underline{R}}$. Since, we only consider orthogonal spreading matrices, neither $\|\underline{\underline{R}}\|_{\text{F}}^2$ nor $\bar{r}$ change after spreading. Thus, the individual values of α and β can change, but their sum remains constant. As mentioned above, a large α value corresponds to a poor MFB and a large β value to high interference. Yet, since α and β are both positive, it is obvious from Eqn. 3.41 that if α increases, then β decreases and vice versa. The sum $\alpha + \beta = 0$, if and only if $\underline{\underline{R}}$ is a scaled identity matrix, which is the ideal case. MC-CAFS spreading leads to $\alpha = 0$ and β thus reaches its maximum value ($\beta = \frac{\|\underline{\underline{R}}\|_{\text{F}}^2}{n_T N} - \bar{r}^2$). However, as we have seen in the previous Sections, the BER performance of MC-CAFS is still better than for other spreading matrices due to increased diversity (improved MFB). On the other hand, if

3.6 Spreading, Matched Filter Bound and Interference

perfect channel knowledge is present at the transmitter, we can use the eigenvectors of the channel correlation matrix, $\underline{\underline{R}}_{\text{MO}}$, as precoding matrices and thus diagonalizing the channel matrix. In this case, $\beta = 0$, and α reaches its maximum value. Precoding will be discussed in more detail in the next Chapter.

Equation 3.41 gives the values of $\alpha + \beta$ for one channel realization. The expected value of the sum in Eqn. 3.41 is thus,

$$\mathcal{E}\{\alpha + \beta\} = \mathcal{E}\{\alpha\} + \mathcal{E}\{\beta\} = \frac{\mathcal{E}\{\|\underline{\underline{R}}\|_{\text{F}}^2\}}{n_{\text{T}}N} - (1 + \text{var}(\bar{r})), \quad (3.42)$$

where $\mathcal{E}\{\bar{r}^2\} = (\mathcal{E}\{\bar{r}\})^2 + \text{var}(\bar{r}) = 1 + \text{var}(\bar{r})$. We have shown in Appendix 3.A.2, that $\text{var}(\bar{r}) = 1/\Psi(\underline{\underline{R}}_v)$, where $\Psi(\underline{\underline{R}}_v)$ is given by Eqn. 2.51 and $\underline{\underline{R}}_v$ by Eqn. 2.53.

Accordingly, based on derivations in Appendix 3.A.2, we obtain

$$\mathcal{E}\{\alpha\} + \mathcal{E}\{\beta\} = \frac{n_{\text{T}}}{n_{\text{R}}} + \frac{2n_{\text{T}}}{n_{\text{R}}^2} \sum_{i=1}^{n_{\text{R}}-1} (n_{\text{R}} - i) |\rho_{\text{Rx}i}|^2 + \frac{2}{n_{\text{T}}} \sum_{j=1}^{n_{\text{T}}-1} (n_{\text{T}} - j) |\rho_{\text{Tx}j}|^2 - \text{var}(\bar{r}), \quad (3.43)$$

where $\rho_{\text{Rx}i}$ and $\rho_{\text{Tx}j}$ are the elements of $\underline{\underline{k}}_{\text{R}}$ and $\underline{\underline{k}}_{\text{T}}$ respectively. For the unspread MIMO-OFDM, by expanding Eqn. 3.39, we get

$$\alpha = \frac{1}{n_{\text{T}}N} \sum_{i=1}^{n_{\text{T}}N} (r_{ii} - \bar{r})^2 = \frac{1}{n_{\text{T}}N} \sum_{i=1}^{n_{\text{T}}N} (r_{ii}^2 + \bar{r}^2 - 2\bar{r}r_{ii}) = \frac{1}{n_{\text{T}}N} \sum_{i=1}^{n_{\text{T}}N} r_{ii}^2 - \bar{r}^2 \quad (3.44)$$

Thus,

$$\mathcal{E}\{\alpha\} = \frac{1}{n_{\text{R}}} + \frac{2}{n_{\text{R}}^2} \sum_{i=1}^{n_{\text{R}}-1} (n_{\text{R}} - i) |\rho_{\text{Rx}i}|^2 - \text{var}(\bar{r}). \quad (3.45)$$

Note that the $\text{var}(\bar{r})$ represents the effect of the different fading occurring in the different MIMO channels, for example, frequency selectivity.

The expected value of β is

$$\mathcal{E}\{\beta\} = (n_{\text{T}} - 1) \left(\frac{1}{n_{\text{R}}} + \frac{2}{n_{\text{R}}^2} \sum_{i=1}^{n_{\text{R}}-1} (n_{\text{R}} - i) |\rho_{\text{Rx}i}|^2 \right) + \frac{2}{n_{\text{T}}} \sum_{j=1}^{n_{\text{T}}-1} (n_{\text{T}} - j) |\rho_{\text{Tx}j}|^2. \quad (3.46)$$

For equal correlation coefficients ($\rho_m = \rho, \forall m$), Eqn. 3.43 reduces to

$$\mathcal{E}\{\alpha + \beta\} = \frac{n_{\text{T}}}{n_{\text{R}}} + \frac{n_{\text{T}}}{n_{\text{R}}} (n_{\text{R}} - 1) |\rho_{\text{Rx}}|^2 + (n_{\text{T}} - 1) |\rho_{\text{Tx}}|^2 - \text{var}(\bar{r}), \quad (3.47)$$

3 Spreading

where $\sum_{j=1}^{n-1} j = n(n-1)/2$ was applied.

As mention earlier, in case of MC-CAFS spreading, $\alpha = 0$ (all symbols experience the same fading), and thus according to Eqn. 3.41, β reaches its maximum value. In this case, the expected value of β is directly obtained from Eqn. 3.43, by setting $\mathcal{E}\{\alpha + \beta\} = \mathcal{E}\{\beta\}$.

3.6.1 Effect of Antenna Correlations on Interference and MFB

Equation 3.45 predicts that α and thus the MFB worsens with increasing receive antenna correlations. Figures 3.11(a) and 3.11(b) show α and β as a function of the antenna correlations, $\rho_{\text{Rx}} = \rho_{\text{Tx}} = \rho$, assuming constant correlation model, for various n_{R} and L , respectively. Figure 3.11(a) shows α for various L and thus make apparent the effect of $\text{var}(\bar{r})$ on α . Unlike the receive correlations, a larger $\text{var}(\bar{r})$ reduces α , i.e. improves the MFB even without any kind of spreading. Yet, it is important to mention here that this reduction is not a positive effect. As shown in Appendix 3.A.2, $\text{var}(\bar{r})$ increases with increase of antenna correlations or with decreasing number of transmit or receive antennas or channel taps (i.e. reduced diversity). In other words, the reduction in α is due to reduced diversity. For e.g consider a 2×2 MIMO system with $L = 1$ and full transmit correlations ($\underline{H}$ is rank one with identical columns) and the resulting channel correlation matrix is an all ones matrix ($\underline{R} = \underline{1}$). In this case, all diagonal elements are equal, yet the channel is rank deficient and the MIMO channel acts as a SIMO one. β , on the other hand, is not affect by L , and as expected it increases with increased antenna correlations.

Figure 3.12 shows the effect of either the transmit or receive antenna correlations on α and β for an unspread MIMO-OFDM system for $n_{\text{T}} = 4$ and various n_{R} . The receive correlations lead to lower total interference than the transmit correlations (Fig. 3.12(b)). This is especially obvious as the amount of antenna correlations increases. On the other hand, the presence of receive correlations worsen the MFB in contrast to transmit correlations (Fig. 3.12(a)). Thus, for the unspread MIMO-OFDM system, the receive correlations worsen the MFB, yet they lead to less interference compared to transmit antenna correlations. In case of MC-CAFS ($\alpha = 0$), assuming $n_{\text{T}} = n_{\text{R}}$, the presence of either transmit or receive correlations leads to same total interference β .

Now we look at how the above predictions affect the BER performance of the employed equalizers. Figure 3.13 shows the BER versus ρ for $n_{\text{T}} = n_{\text{R}} = 4$, $L = 4$, $N = 32$ and 4PSK at $E_b/N_o = 20$ dB and $E_b/N_o = 14$ dB respectively for different equalizers. We have chosen a higher E_b/N_o for MMSE-BLE and

3.6 Spreading, Matched Filter Bound and Interference

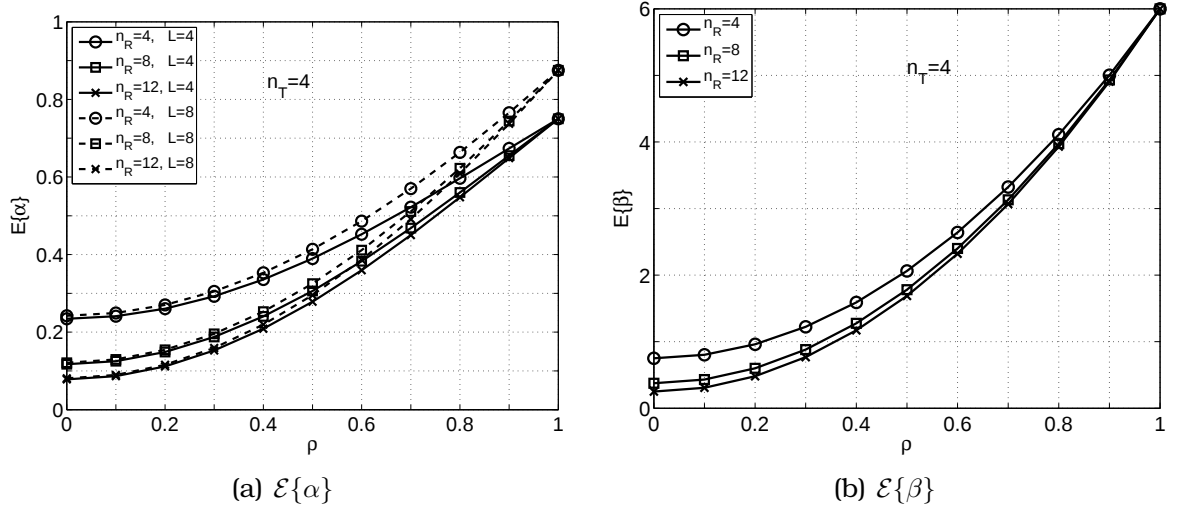


Figure 3.11: $E\{\alpha\}$ and $E\{\beta\}$ versus $\rho_{Tx} = \rho_{Rx} = \rho$, $c_{ij} = \rho$ for various n_R and L and $n_T = 4$.

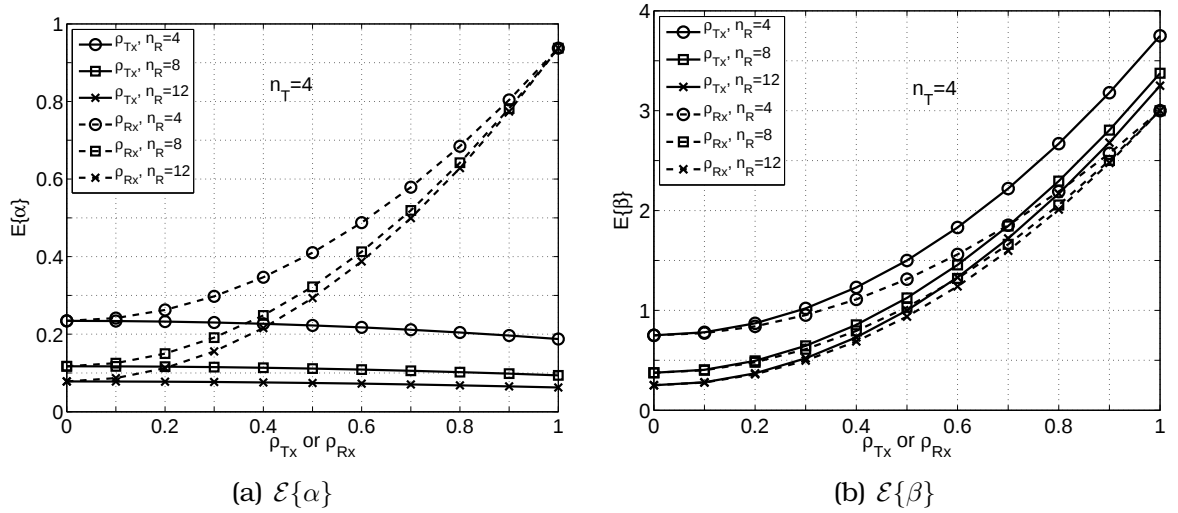


Figure 3.12: $E\{\alpha\}$ and $E\{\beta\}$ versus ρ_{Tx} (assuming $\rho_{Rx} = 0$) or ρ_{Rx} (assuming $\rho_{Tx} = 0$) for various n_R , $L = 4$, $n_T = 4$.

MMSE-BDFE in order to better distinguish between the different BER curves. We assume the exponential correlation model, where ρ represents either the transmit or receive correlations or both. The BER in presence of both transmit and receive correlations, $\rho_{Tx,exp} = \rho_{Rx,exp} = \rho$, is as expected highest. The BER in presence of receive correlations only ($\rho_{Rx,exp} = \rho$) is lowest for both the RNN and SCE. For the MMSE-BDFE, the lowest BER is for $\rho_{Tx,exp} = \rho$.

3 Spreading

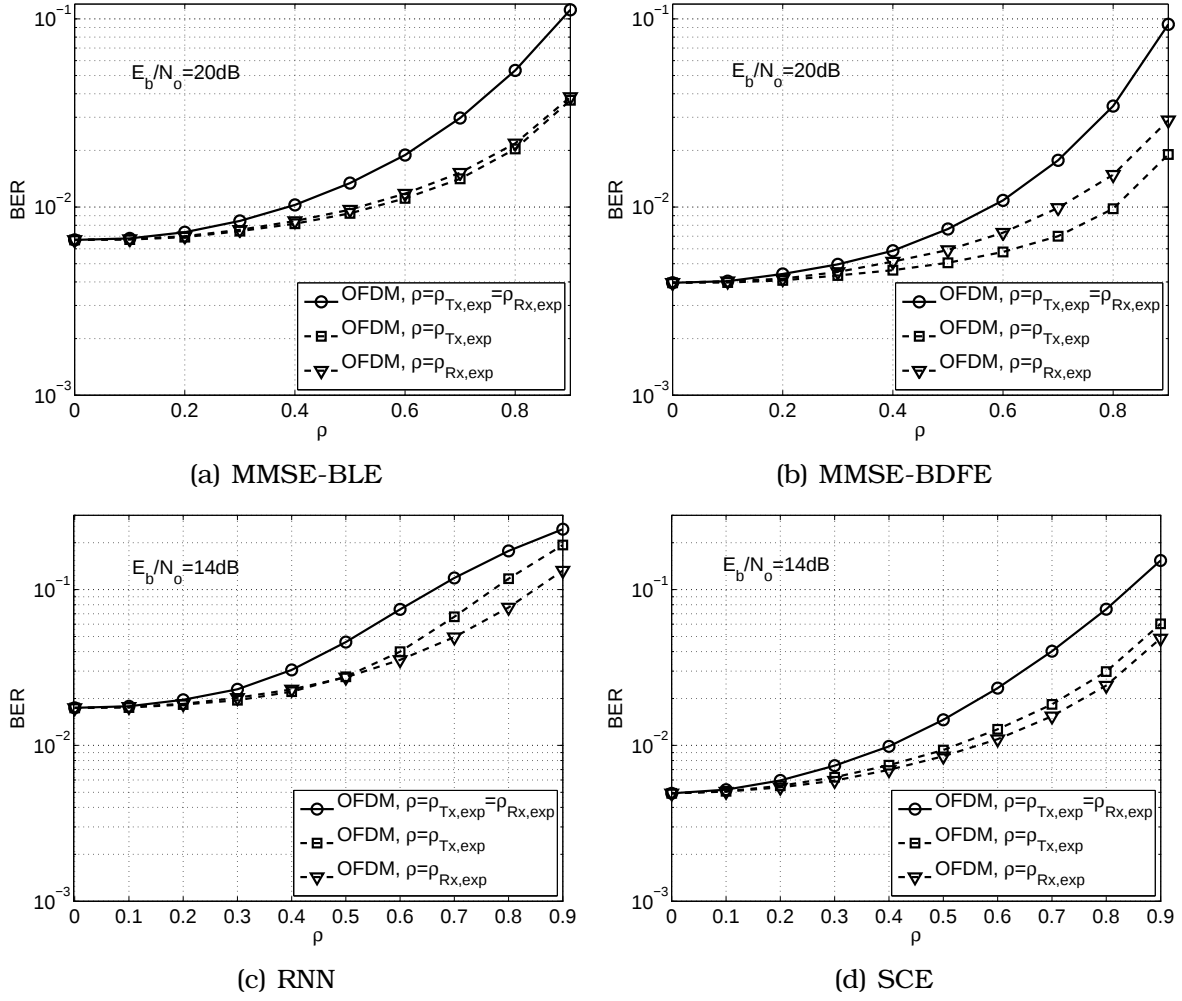


Figure 3.13: MIMO-OFDM: Comparison of BER in presence of either transmit or receive correlations or both for $n_T = n_R = 4$, $L = 4$ and $N = 32$.

In presence of either transmit or receive correlations, the MMSE-BLE BER curves are almost identical for both cases. As afore mentioned, receive correlations, although worsen the MFB, leads to lower interference, which seems to play a more important factor in the system performance at high antenna correlations for the RNN and SCE. For the MMSE-BDFE, the MFB appears to play the more important role. This may be due to the fact the MMSE-BDFE is prone to propagation errors.

Figure 3.14 shows $\mathcal{E}\{\alpha + \beta\}$ for different correlation models: constant, exponential and complex exponential correlation model. We assume the presence of antenna correlations at both the transmitter and receiver with $\rho_{Rx} = \rho_{Tx} = \rho$. Based on the definition of the correlation measure in Sec. 2.4.2,

3.6 Spreading, Matched Filter Bound and Interference

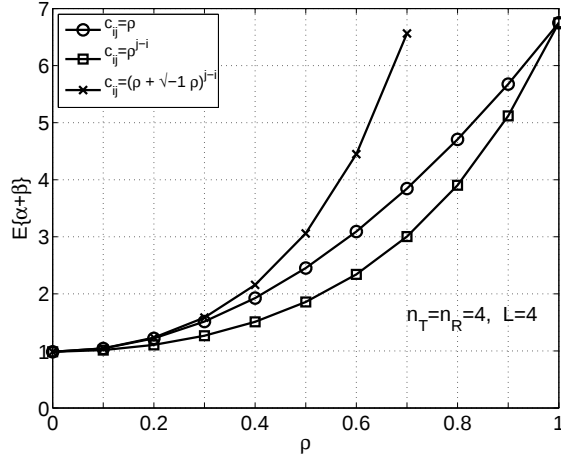


Figure 3.14: $\mathcal{E}\{\alpha + \beta\}$ versus $\rho_{Tx} = \rho_{Rx} = \rho$ for various correlations models.

and for any given ρ , the complex exponential correlation model possesses the highest correlation measure, while the exponential model the lowest. This effect can be observed in Figure 3.14 where the different correlation models lead to different $\mathcal{E}\{\alpha + \beta\}$. The complex exponential correlation model, which has the highest correlation measure, leads to the highest $\mathcal{E}\{\alpha + \beta\}$. The exponential model, on the other hand to the lowest $\mathcal{E}\{\alpha + \beta\}$. By choosing ρ for each model, to obtain the same correlation measure, the corresponding $\mathcal{E}\{\alpha + \beta\}$ obtained are also equal. For example, by choosing $\rho = 0.5, 0.6, 0.7$ for the complex exponential, constant and exponential models respectively, we obtain the same correlation measure $\Phi = 0.6$ for all models, and from Fig. 3.14 we can see that $\mathcal{E}\{\alpha + \beta\} = 3$ at those ρ values. The results therefore seem to agree with the correlation measures of the corresponding correlation model. We show later on that the BER performance for the different correlation models is also in accordance with the results presented here.

Finally, we look at the MFB in presence of antenna correlations. Figure 3.15 shows the MFB for MIMO-OFDM and MC-CAFS in presence of antenna correlations. We assume a 4×4 MIMO channel with $L = 4$, constant correlation model and transmission using 4 PSK. The MFB for OFDM in presence of transmit correlations coincides with the MFB for zero antenna correlations. The reason is that the signals transmitted over a MIMO-OFDM channel experience receive diversity only (diversity= n_R) and thus only receive correlations affect the MFB. Following from that, the MFB in presence of receive correlation coincides with the MFB for a channel with transmit and receive correlations. In case of the MC-CAFS, both the transmit and receive antenna correlations affect the MFB, since all transmitted signals have full diversity (diversity= $n_T n_R L$). The MFB is worst when both transmit and receive

3 Spreading

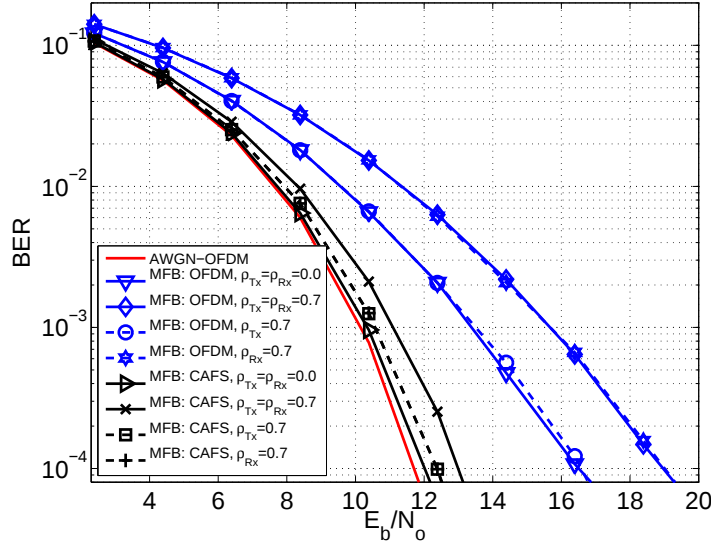


Figure 3.15: MFB for 4×4 MIMO channel with $L = 4$, constant correlation model.

correlations are present. The MFB in presence of either transmit or receive correlations coincide with each other. Thus, the MFB is not only affected by the amount of correlations, but also by the number of correlated diversity branches. In all cases, the MFB of MC-CAFS clearly outperforms that for MIMO-OFDM.

3.6.2 Asymptotic Behavior of α and β

For the uncorrelated case, the expected values of α and β are

$$\mathcal{E}\{\alpha\} = \frac{1}{n_R} - \frac{1}{\text{var}(\bar{r})} = \frac{1}{n_R} - \frac{1}{n_T n_R L}, \quad (3.48)$$

and

$$\mathcal{E}\{\beta\} = \frac{n_T - 1}{n_R}. \quad (3.49)$$

Thus $\alpha \rightarrow 0$ and $\beta \rightarrow 0$ as $n_R \rightarrow \infty$. In other words, $\underline{R} \rightarrow \underline{I}$. However, in the correlated case, $\alpha \rightarrow |\rho_{RX}|^2$ and $\beta \rightarrow (n_T - 1)(|\rho_{RX}|^2 + |\rho_{TX}|^2)$ for nonzero ρ_{RX} and ρ_{TX} . That is, increasing the number of receive antennas indefinitely would not lead to a better MFB nor to considerably lower interference if high antenna correlations are present. Figure 3.16 shows the $\mathcal{E}\{\alpha\}$ and $\mathcal{E}\{\beta\}$ versus n_R for various ρ values for a MIMO channel with $n_T = 4$ and $L = 4$. The largest drop in $\mathcal{E}\{\alpha\}$ and $\mathcal{E}\{\beta\}$ is observed for $n_R \leq 8$. The drop is more significant

3.6 Spreading, Matched Filter Bound and Interference

for low or zero correlations. As the correlations increase, the rate of drop decreases.

Figure 3.17 shows the expected value of the condition number of $\underline{\underline{R}}$, $\mathcal{E}\{\chi\}$,

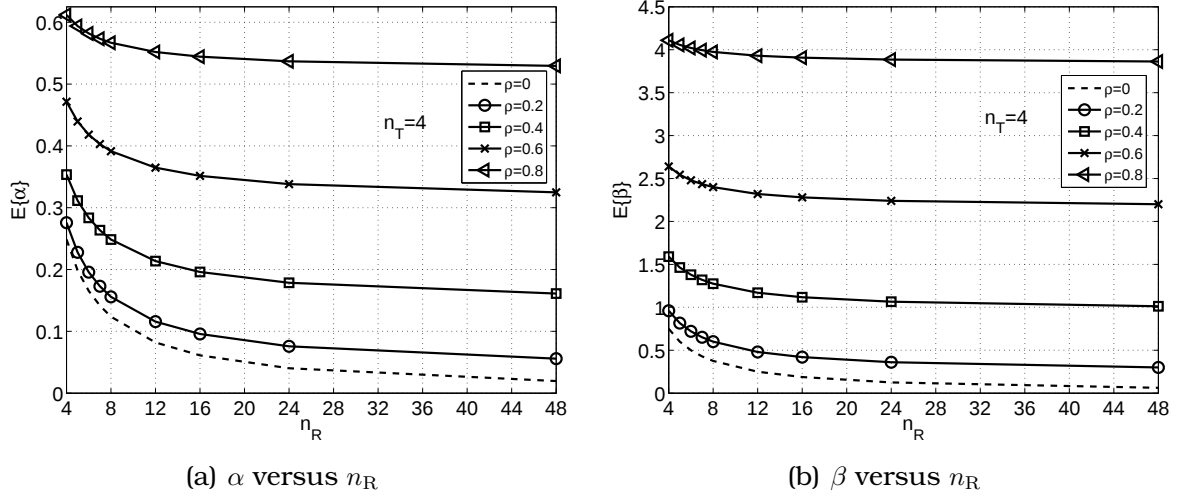


Figure 3.16: α and β versus n_R for $\rho_{Tx} = \rho_{Rx} = \rho$, $c_{ij} = \rho$, $n_T = 4$ and $L = 4$.

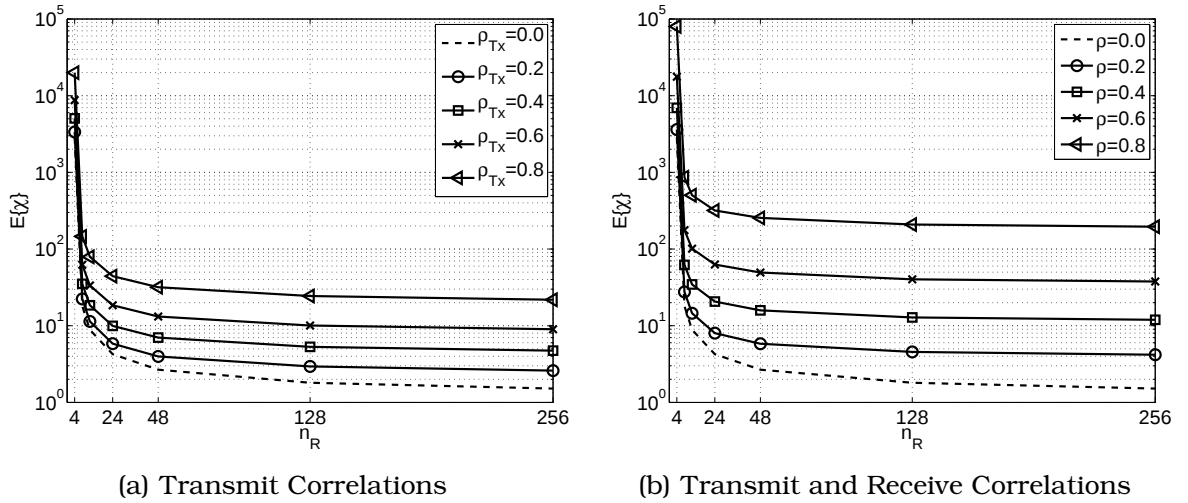


Figure 3.17: Expected value of the condition number of the channel correlation matrix with $n_T = 4$ and $L = 4$, versus the number of receive antennas and for different antenna correlation values, $c_{ij} = \rho$.

versus the number of receive antennas for $n_T = 4$. As expected, the condition number increases with the increase in antenna correlations, being highest when both transmit and receive correlations are present. Increasing

3 Spreading

the number of receive antennas lowers the condition number for all values of antenna correlations. The largest drop however occurs by increasing n_R from 4 to 12. As the antenna correlations increase, the condition number, although is reduced by increasing n_R , does not tend to 1.0 as in the case of zero correlations.

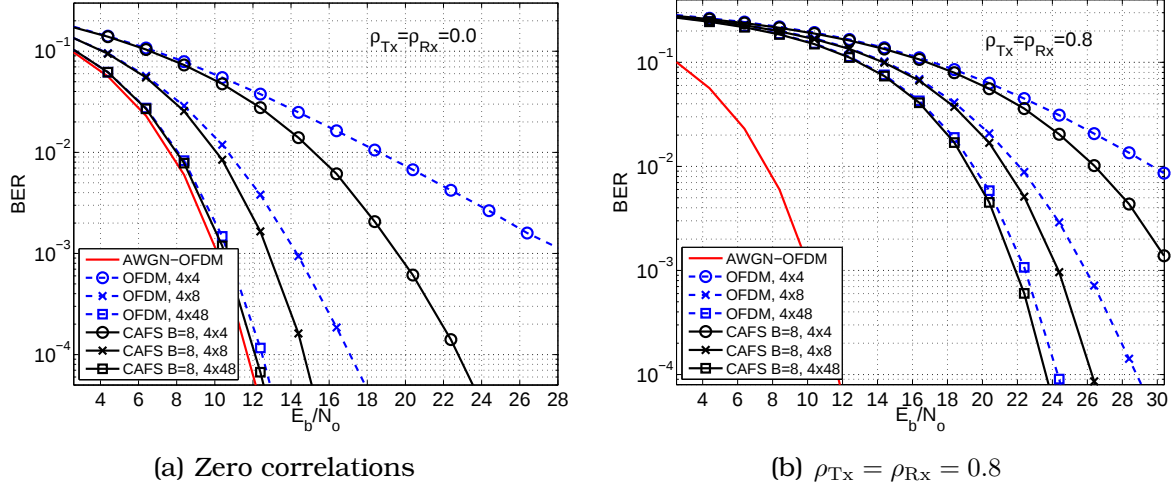


Figure 3.18: MMSE-BLE: BER performance for various n_R and antenna correlations, $n_T = 4, L = 4$.

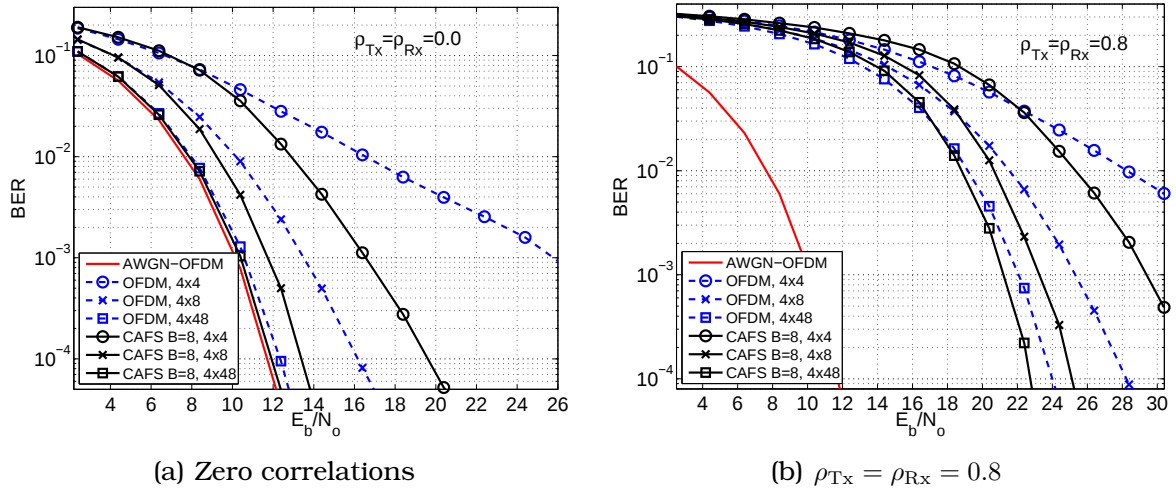


Figure 3.19: MMSE-BDFE: BER performance for various n_R and antenna correlations, $n_T = 4, L = 4$.

3.6 Spreading, Matched Filter Bound and Interference

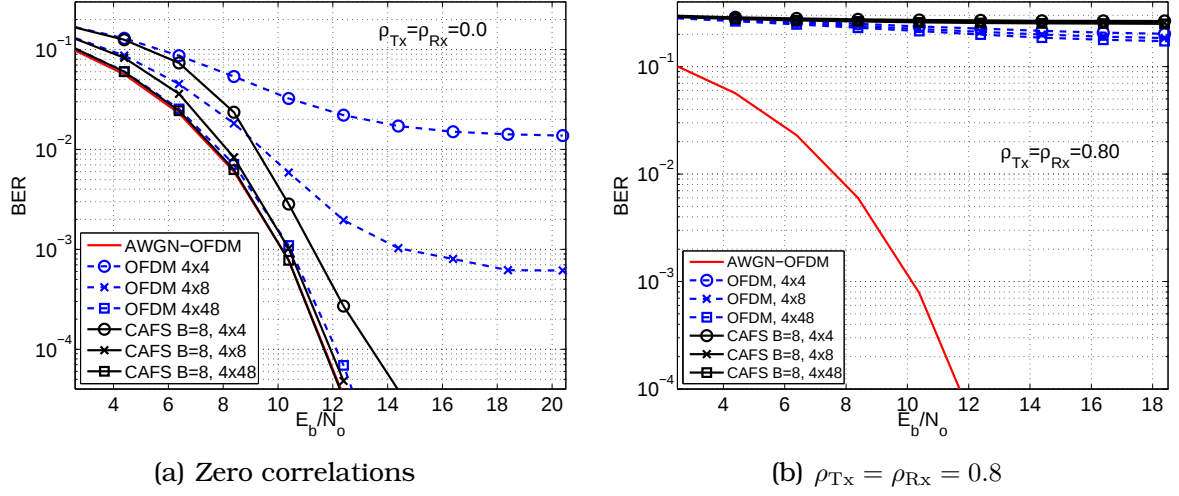


Figure 3.20: RNN: BER performance for various n_R and antenna correlations, $n_T = 4, L = 4, N = 32$.

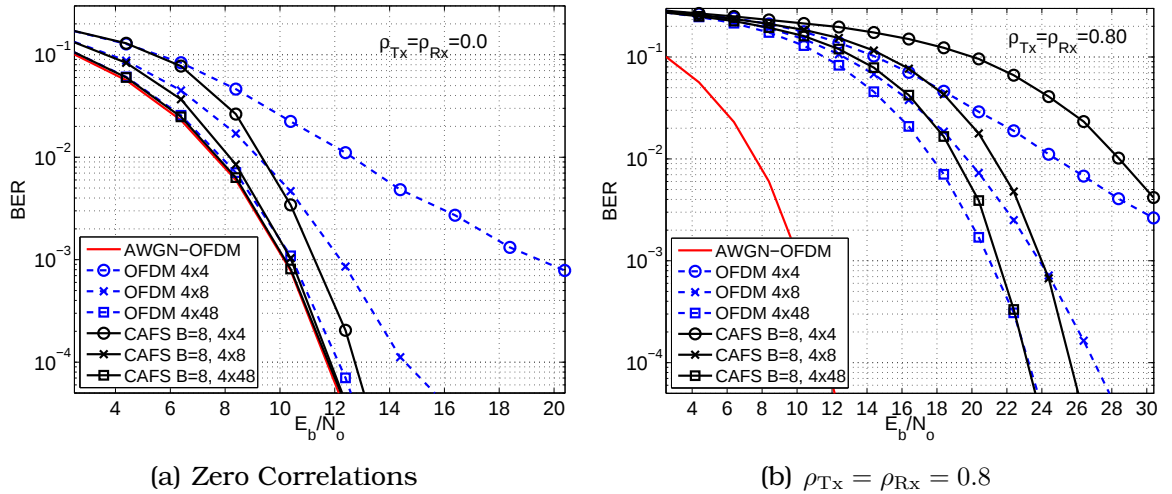


Figure 3.21: SCE: BER performance for various n_R and antenna correlations, $n_T = 4, L = 4, N = 32$.

Figures 3.18 to 3.21 show the effect of increasing n_R on the BER for the MMSE-BLE, MMSE-BDFE, RNN and SCE for the uncorrelated case and for $\rho_{Tx} = \rho_{Rx} = 0.8$, assuming the constant correlation model at both the transmitter and receiver for $n_T = 4, N = 32$ using 4PSK modulation. Increasing the number of receive antennas, has the followings effects on the system: it increases the diversity, reduces the interference and improves the MFB. How-

3 Spreading

ever, as mentioned above, the reduction of interference and improvement of the MFB is limited by the amount of antenna correlations. This effect is can be clearly seen in Figs. 3.18 to 3.21. For zero correlations, the BER tends to that of AWGN as n_R increases for all equalizers. At high E_b/N_o , both the RNN and SCE equalizers achieve the AWGN performance for $n_R = 8$ using MC-CAFS spreading. As predicted from the above analysis of the asymptotic behavior of α and β at high antenna correlations, the BER does not approach the AWGN curve, not even for $n_R = 48$. This applies to all four equalizers employed here. The RNN equalizer does not even converge and the SCE performs worse than the MMSE-BDFE in case of MC-CAFS spreading. The SCE assumes that the residual interference has a Gaussian distribution. This assumption may not be fulfilled in case of high antenna correlations and accordingly the decision function becomes suboptimum [33], leading to a slightly worse BER than the MMSE-BDFE.

For all correlation values, the largest drop in the BER is observed when n_R is increased from 4 to 8. Take for example the MMSE-BDFE, in the uncorrelated case, Fig. 3.19(a), a gain of 6 dB at 10^{-4} is attained on increasing n_R from 4 to 8 using MC-CAFS. Increasing the number of receive antennas to 48 leads to a further gain of less than 2 dB for the same BER. This is in accordance with the drop in α , β and χ in Figs. 3.16 and 3.17.

3.6.3 Effect of Interference Distribution on the BER

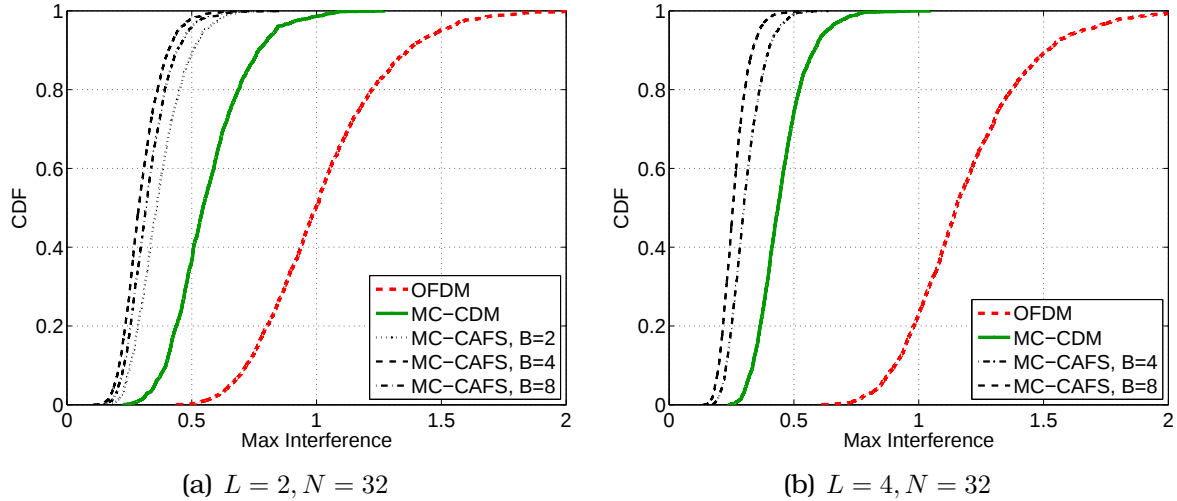


Figure 3.22: CDF of maximum interference ($\max(|r_{ij}|)$) for MIMO 4×4 .

As mentioned above, in case of MC-CAFS spreading, several B values are possible depending on the system parameters: L , N , and n_T . Although, for

3.6 Spreading, Matched Filter Bound and Interference

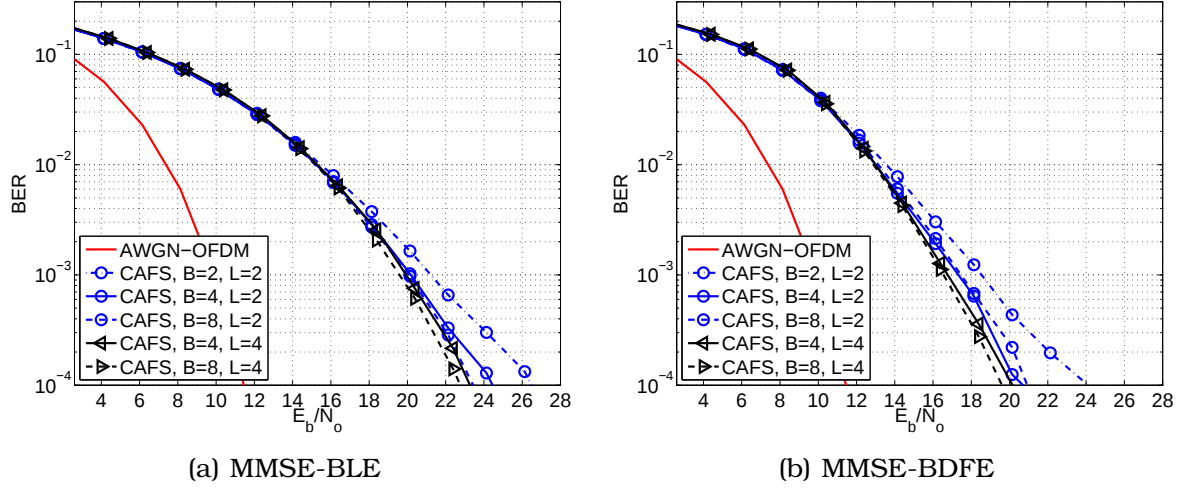


Figure 3.23: BER for MIMO 4×4 using MC-CAFS spreading for various B and L for $\rho = 0$ and $N = 32$.

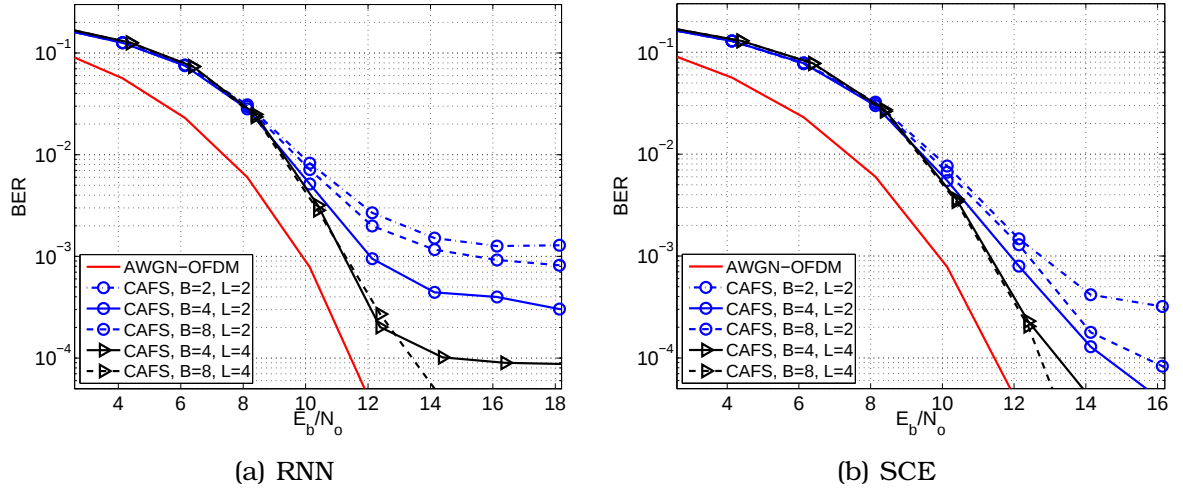


Figure 3.24: BER for MIMO 4×4 using MC-CAFS spreading for various B and L for $\rho = 0$ and $N = 32$.

all possible B values, the same β is obtain, the sum of the absolute values of r_{ij} ($\beta_{1/2}$), changes as well as the distribution of the individual values of $|r_{ij}|$. In this section we look at the effect of B on the distribution of the absolute value of the interference. As discussed in Chap.2, the equalizers used are sensitive not only to the sum of the total interference, but also to its distribution. The BERs for standard channels using different equalizers show that the lower the value of r_{ij} for a given β , the lower the BER. That is,

3 Spreading

for a given β , the distribution seems to be more important than the absolute sum of the interference, $\beta_{1/2}$.

Figure 3.22 compares the cdf for the maximum value of $|r_{ij}|$ ($\max(|r_{ij}|)$) for zero correlations for different number of channel taps, L , and different spreading matrices. Both the OFDM and MC-CDM systems, lead to higher $\max(|r_{ij}|)$ than MC-CAFS, even though $\beta_{\text{CAFS}} > \beta_{\text{CDM}} > \beta_{\text{OFDM}}$. Figures 3.22 also shows that different B values of MC-CAFS lead to different interference distributions, and thus most likely to different BERs. For example, for $L = 2$, $B = 4$ leads to lowest $\max(|r_{ij}|)$ while for $L = 4$, $B = 8$ leads to the lowest $\max(|r_{ij}|)$. Figures 3.23 to 3.24, show the BER for the above mentioned equalizers using different B values for MC-CAFS. The BER results correspond to the cdf of $\max(|r_{ij}|)$, where the lower $\max(|r_{ij}|)$ the lower the BER for all equalizers.

The results obtained in this Section are thus clearly in accordance with the predictions of the test channels in Chapter 2. Therefore, although all possible B values achieve full diversity, they lead to different interference distributions, an important factor that needs to be considered when designing a communications system.

3.7 Rotated MC-CAFS

In addition to the previously mentioned types of diversity (frequency, space and time), signal space diversity (also known as modulation diversity) – achieved through symbol rotation – is another method for improving the system performance. The aim of symbol rotation is to maximize the number of distinct components between any two constellation points [66, 67, 68]. For example, in SISO-OFDM, the aim of rotations is to guarantee that the signal constellations after spreading at the input of the each OFDM subchannel have M^N distinct points [7], where M is the alphabet size and N the number of subchannels. Thus if all but one subchannel fade completely, detection of the transmitted block is still possible [7] assuming ML detection. However, simulation results with non-optimum receivers, still showed BER improvement when rotations are employed [7].

Following are two possible rotations types for MC-CAFS that were proposed in [69]. These rotated spreading matrices are obtained by rotating the columns of the original spreading matrix by a certain angle. For the first type of rotated spreading matrix, every consecutive set of N/B columns are rotated by the same angle, while for the second type, each and every column is rotated by a different angle.

3.7.1 Rotations Type I

To obtain $\underline{U}_{\text{MC-CAFS}_{\text{RotI}}}$, the orthogonal matrix, $\underline{S}$, in Sec. 3.5 used for calculating the spreading matrix $\underline{U}_{\text{MC-CAFS}}$ is replaced by $\underline{S}_{\text{Rot}}$ which is given as follows:

$$\underline{S}_{\text{Rot}} = \underline{S} \underline{D}_n, \quad (3.50)$$

where $\underline{D}_n$ is diagonal matrix of size $n \times n$. The diagonal elements, d_{ii} , of $\underline{D}_n$ for M -PSK are given by [7, 70]

$$d_{ii} = \exp \left(j \frac{2\pi}{M} \frac{(i-1)}{n} \right), \quad (3.51)$$

where $i = 1, \dots, n$ and for rotations of Type I, $n = n_T B$, the size of $\underline{S}$. Thus, each column of $\underline{S}$ is rotated by a different angle or equivalently every N/B consecutive columns of $\underline{U}_{\text{MC-CAFS}}$ are rotated by the same angle. The rotated spreading matrix, $\underline{U}_{\text{MC-CAFS}_{\text{RotI}}}$, can alternatively be obtained as follows,

$$\underline{U}_{\text{MC-CAFS}_{\text{RotI}}} = \underline{U}_{\text{MC-CAFS}} \underline{D}_{\text{kronI}} \quad (3.52)$$

where

$$\underline{D}_{\text{kronI}} = \underline{D}_{n_T B} \otimes \underline{I}_{N/B}, \quad (3.53)$$

and $\otimes$ is the Kronecker product, $\underline{I}_{N/B}$ is the identity matrix of size $N/B \times N/B$ and $\underline{D}_{n_T B}$ is again given by Eqn. 3.51 with $n = n_T B$.

3.7.2 Rotations Type II

To obtain $\underline{U}_{\text{MC-CAFS}_{\text{RotII}}}$, the spreading matrix $\underline{U}_{\text{MC-CAFS}}$ is postmultiplied by the diagonal rotation matrix $\underline{D}_n$ as follows:

$$\underline{U}_{\text{MC-CAFS}_{\text{RotII}}} = \underline{U}_{\text{MC-CAFS}} \underline{D}_n \quad (3.54)$$

Again, $\underline{D}_n$ is a diagonal matrix whose diagonal elements are given by Eqn. 3.51. However, $n = n_T N$ in this case. Thus, for rotations type II, all antennas and frequencies are rotated by different angles.

Assessment of Rotated MC-CAFS

To assess how rotations can affect the system performance, we shall look at the following three aspects:

- the minimum Euclidean distance after the channel,

3 Spreading

- the rank of $\underline{\underline{G}}$ (which affects the PEP given in Eqn. 3.8),
- the number of distinct constellation points after spreading.

For a given channel realization, $\underline{\underline{H}}$, the probability that the receiver erroneously decides for $\underline{\underline{E}}$ assuming codeword $\underline{\underline{C}}$ was transmitted, is given by [58]

$$P(\underline{\underline{C}} \rightarrow \underline{\underline{E}} | \underline{\underline{H}}) = Q\left(\sqrt{\frac{E_s}{2N_o}} \Delta^2\right)$$

$$P(\underline{\underline{C}} \rightarrow \underline{\underline{E}} | \underline{\underline{H}}) \leq e^{-\frac{E_s}{4N_o} \Delta^2} \quad (3.55)$$

where $\Delta^2 = \|\underline{\underline{H}}(\underline{\underline{C}} - \underline{\underline{E}})\|^2$ is the Euclidean distance between any two codewords, $\underline{\underline{C}}$ and $\underline{\underline{E}}$, and E_s is the average symbol energy. Δ^2 can also be given by

$$\Delta^2 = Tr\left(\underline{\underline{H}}(\underline{\underline{C}} - \underline{\underline{E}})(\underline{\underline{C}} - \underline{\underline{E}})^H \underline{\underline{H}}^H\right). \quad (3.56)$$

The minimum Euclidean distance after the channel is important for the asymptotic behavior (high E_b/N_o) of the system. The larger the minimum distance, the better is the system performance asymptotically. Spreading can be considered as a type of coding, where the codewords are given by

$$\underline{\underline{C}} = \underline{\underline{U}} \underline{\underline{x}}. \quad (3.57)$$

The spreading matrix $\underline{\underline{U}}$ can be chosen to be any of the spreading matrices discussed in this work. In case of no spreading, $\underline{\underline{U}} = \underline{\underline{I}}$.

In order to use Eqn. 3.8 to calculate the rank of $\underline{\underline{G}}$, the codeword vectors $\underline{\underline{C}}$ of size $Nn_T \times 1$ are simply reshaped to $\underline{\underline{C}}$ of size $n_T \times N$, which can be directly substituted into Eqn. 3.8. Let $c_i(k, i_T)$ be the i th element of the $\underline{\underline{C}}$ or $\underline{\underline{C}}$ at frequency k and transmit antenna i_T . If all elements of the codewords $\underline{\underline{C}}$ and $\underline{\underline{E}}$ are zeros except for $c_i(1, i_T)$ and $e_i(1, i_T)$, or are all equal except for the first row, then $\underline{\underline{G}}$ would have rank one. This leads to both diversity and gain losses and accordingly to degraded system performance. Thus, it is important to find codewords that differ at all positions if possible.

The constellation points after spreading represent the set of all possible values $c_i(k, i_T)$ can take after spreading for all i_T and k . For MIMO-OFDM, ideally, this set should contain M^{Nn_T} distinct points. That is, for any transmit vectors $\underline{\underline{C}}$ and $\underline{\underline{E}}$, $c_i(k_1, i_{T1}) \neq e_i(k_2, i_{T2})$ for $k_1 = k_2$ and $i_{T1} = i_{T2}$. This is, however, not possible using MC-CAFS, since we do not spread over all Nn_T subchannels, but only over n_TB . However, the larger the number of distinct points, the better is the expected BER performance. Following, we look at a MIMO 2×2 channel, with $N = 4$ and $L = 2$. Through that example, we show how the rotations can affect the BER performance based on the above given assessment aspects.

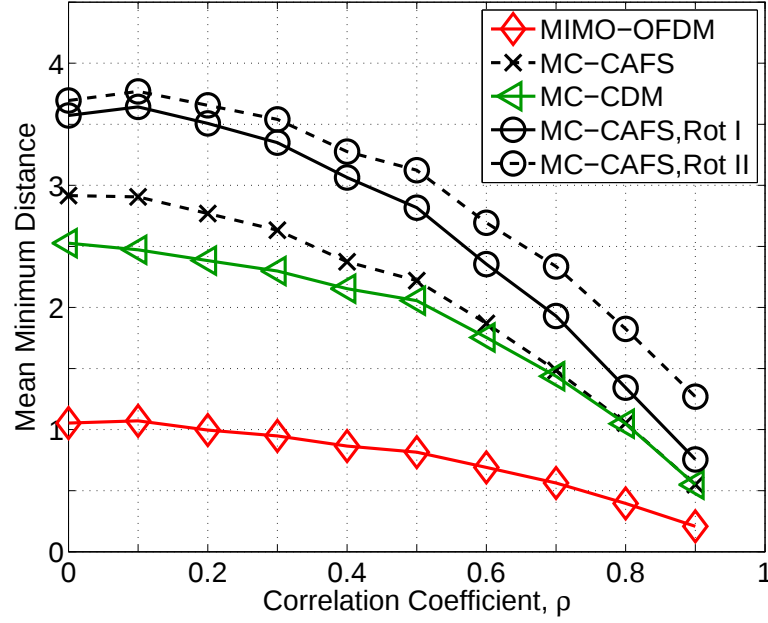


Figure 3.25: Mean Minimum Distance versus Correlation Coefficient, 2 PSK, MIMO 2×2 , $N = 4$, $L = 2$.

Example: MIMO 2×2 , $N = 4$, $L = 2$

We consider a transmit vector $\underline{x}$ consisting of symbols chosen from an alphabet $A = [1 \ -1]$. The $\underline{h}_{ij}$ were assumed to be 2 path with each path being $CN(0, \sigma^2)$. Fig. 3.25 shows the mean minimum distance between all possible receive vectors for a time varying block fading 2×2 MIMO-OFDM channel with $L = 2$, $N = 4$ using BPSK ($M = 2$) alphabet versus the correlation coefficient ρ . We have assumed a constant correlation model, $\rho_{ij} = \rho$ for $i \neq j$. We also assume that the transmit and receive correlations are the same, $\rho_{Tx} = \rho_{Rx} = \rho$. It is clear that spreading in general increases the minimum distance after the channel irrespective of the channel correlation. Rotated MC-CAFS, $B = 2$, leads to the largest minimum distance. Rotations of type II outperforms the rotations type I. However, it should be noted here that the minimum distance is not the only factor affecting the system performance. Other factors such as the distribution of the distance after the channel, and as mentioned in previous Sections, the MFB, the interference distribution, and the channel condition number [71] play an important role as well. In addition, the curves for the mean minimum distance of the spread systems are expected to move together for a larger L , n_T or n_R (i.e. higher diversity) and/or for higher modulation alphabet [7]. However, Fig. 3.25 still shows how spreading and rotations can affect the system performance.

3 Spreading

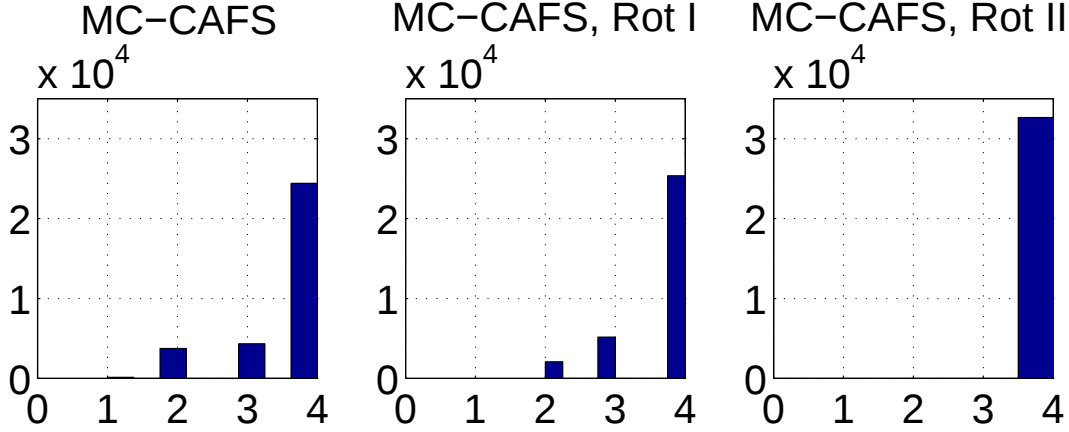


Figure 3.26: Histogram for the rank of $\underline{\underline{G}}$ for MC-CAFS with and without rotations for 2 PSK, $n_T = 2$, $L = 2$ and $N = 4$.

As shown in Fig. 3.26, $\underline{\underline{G}}$ is always full rank with MC-CAFS, Rot II. Fig. 3.27 also shows that type II rotations leads to the "largest" number of distinct constellation points possible with MC-CAFS spreading. The constellation points for a rotated Hadamard matrix (WH+Rot) of size $n_T N$ (i.e. spreading over all subchannels) and MC-CAFS without rotations are shown for comparison. Figures 3.26 and 3.27 show that, although MC-CAFS, Rot II, does not lead to the maximum possible number of distinct point, the $\underline{\underline{G}}$ still has full rank. The results for the rank of $\underline{\underline{G}}$ were obtained through simulations. An analytical proof could be a topic for future work. In what follows, we investigate the BER performance using the MMSE-BLE, MMSE-BDFE, RNN and SCE. We consider a 4 PSK modulation, and examine the effect of B , antenna correlations, and diversity (n_T , n_R and L) values on the BER performance using rotated MC-CAFS.

Effect of B

As shown in the previous section, the larger the chosen B value, the more distinct points there are after rotations. The rank of $\underline{\underline{G}}$ was also shown to be full rank for rotations type II. However, the above discussion concentrated on 2 PSK and short block length. Since similar results for 4 PSK and/or larger block sizes are computationally impractical to obtain, we only look at the BER for rotated MC-CAFS types I and II with different B values for different equalizers. We assume a 4×4 MIMO channel with $N = 32$ and $L = 4$ using 4 PSK. Figures 3.28 show the BER for MMSE-BLE, MMSE-BDFE for zero antenna correlations. Only the MMSE-BDFE seems to benefit from

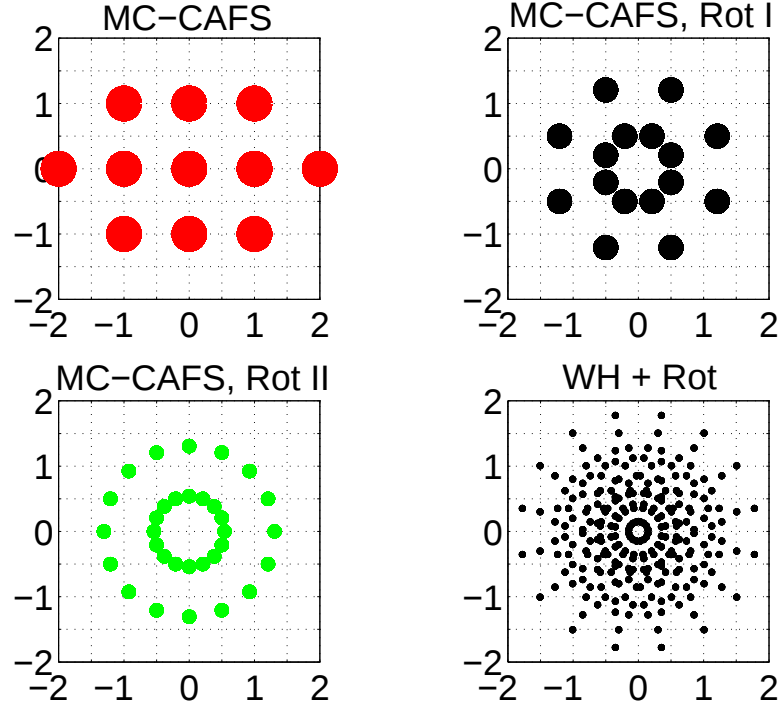


Figure 3.27: Number of distinct constellation points for different spreading matrices with $n_T = 2$ and $N = 4$.

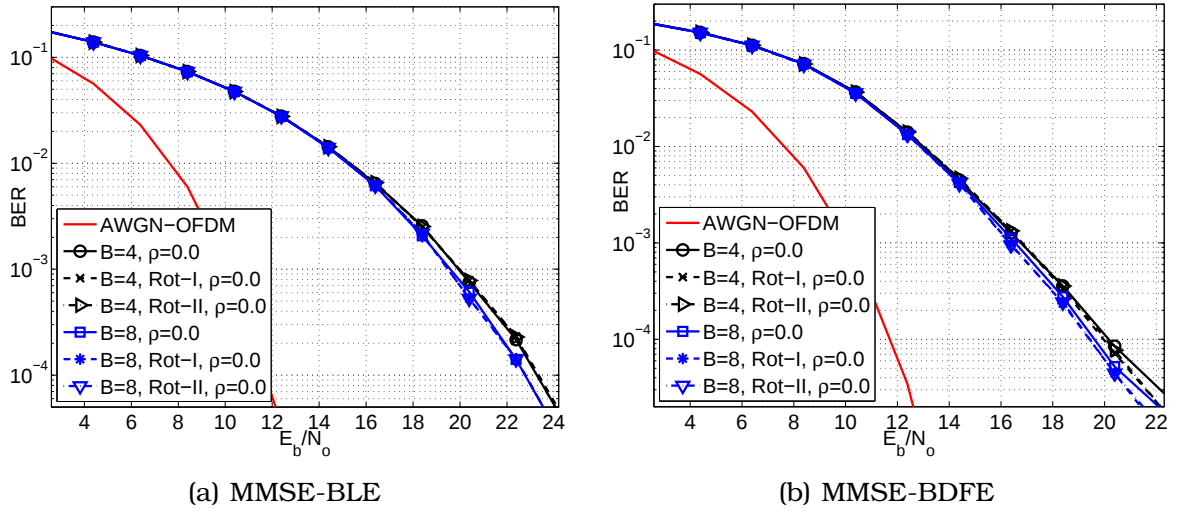


Figure 3.28: BER for Rot-I and Rot-II and $\rho_{Tx} = \rho_{Rx} = \rho = 0$, $L = 4$, $N = 32$ and $n_T = n_R = 4$.

3 Spreading

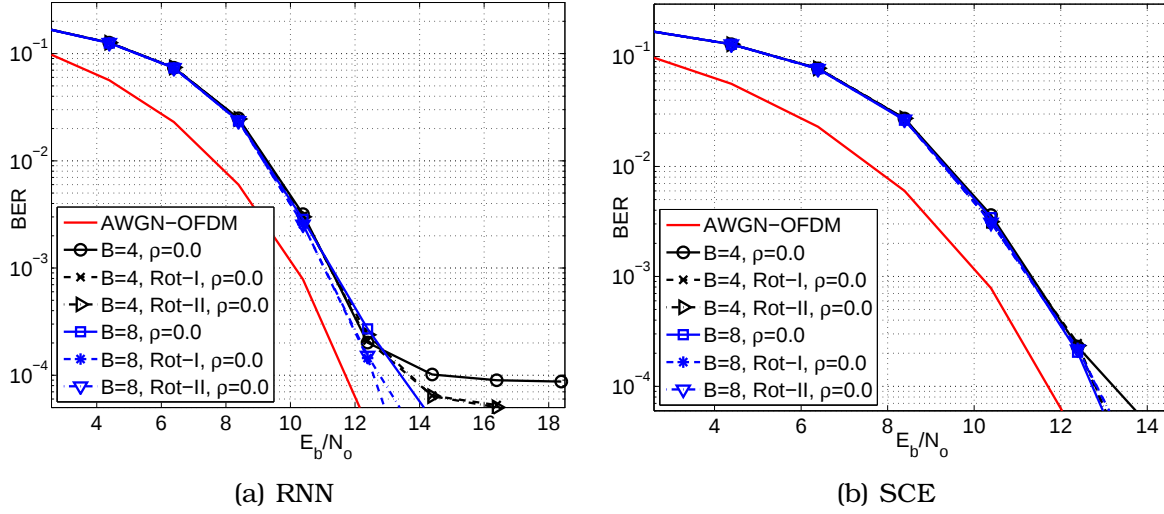


Figure 3.29: BER for Rot-I and Rot-II and $\rho_{Tx} = \rho_{Rx} = \rho = 0$, $L = 4$, $N = 32$ and $n_T = n_R = 4$.

rotations. However, the gains are very small and occur at very high E_b/N_0 . The MMSE-BLE BER is not affected by rotations, only by B . Using MC-CAFS, Rot I or Rot II, the BER for RNN, Fig.3.29(a), improves by almost 1 dB at 10^{-4} for $B = 8$ and for $B = 4$ the error floor shifts to lower BER compared to MC-CAFS without rotations. The SCE also benefits from rotations, but only for $B = 4$. A gain of almost 0.5 dB can be obtain at 10^{-4} . For all equalizers, both rotation types lead to almost the same BER. Thus, from now on, only rotations of type II will be considered, since as shown above, it leads to more distinct constellation points.

Effect of Antenna Correlations

Figures 3.30 and 3.31 show the BER for different antenna correlation values and $L = 4$. The exponential correlation model is assumed. Again, the MMSE-BLE and MMSE-BDFE do not seem to benefit from the rotations. The RNN, due to higher correlations and accordingly higher interference, reaches an error floor even for low correlation values. The RNN seems to benefit slightly from rotations, but only for very low antenna correlations. The SCE on the other hand, benefits from the rotations at higher correlation values, e. g. MC-CAFS, Rot-II outperforms MC-CAFS by about 1.5 dB at $3 \cdot 10^{-4}$ for $\rho = 0.5$. It is possible that for higher E_b/N_0 , that rotated MC-CAFS would lead to lower BERs at lower correlation values.

If only transmit correlations are present, i. e. $\rho_{Rx} = 0$, the gains through

3.7 Rotated MC-CAFS

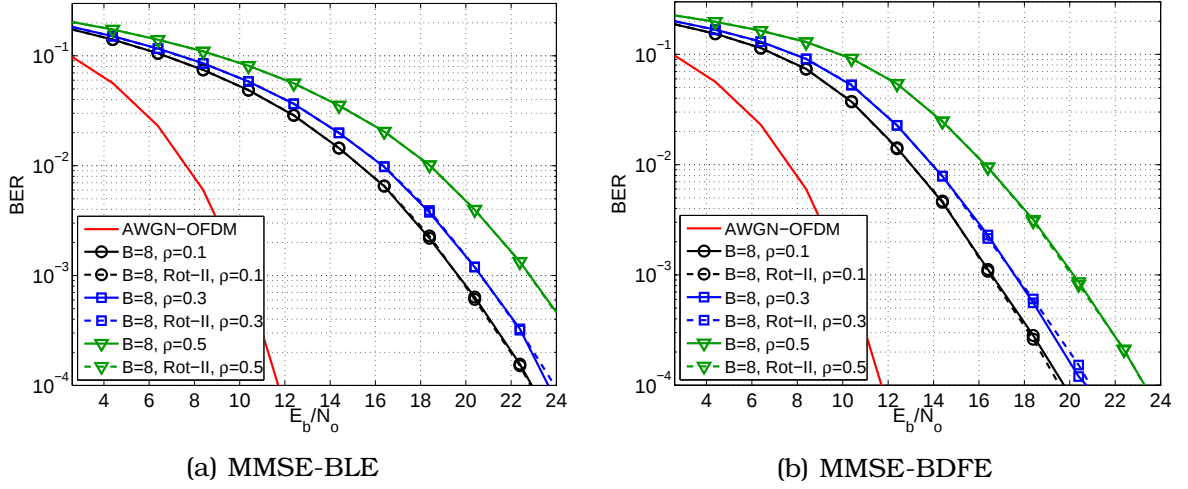


Figure 3.30: BER comparison for MC-CAFS and MC-CAFS Rot-II and various $\rho_{T_x, \text{exp}} = \rho_{R_x, \text{exp}} = \rho$, $L = 4$, $N = 32$ and $n_T = n_R = 4$.

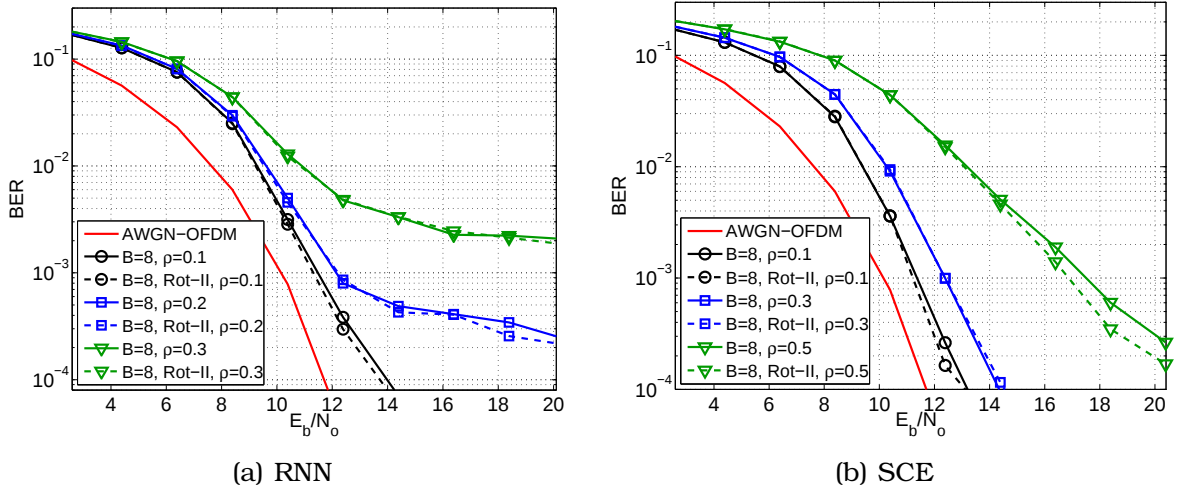


Figure 3.31: BER comparison for MC-CAFS and MC-CAFS Rot-II and various $\rho_{T_x, \text{exp}} = \rho_{R_x, \text{exp}} = \rho$, $L = 4$, $N = 32$ and $n_T = n_R = 4$.

rotations become even more significant for the SCE. Figure 3.32(b) shows the BER for SCE for different transmit correlation values. An improvement of almost 2 dB for $\rho_{T_x} = 0.5$ at 10^{-4} can be achieved only through rotations. The RNN again shows slight benefits from rotations for low correlation values.

3 Spreading

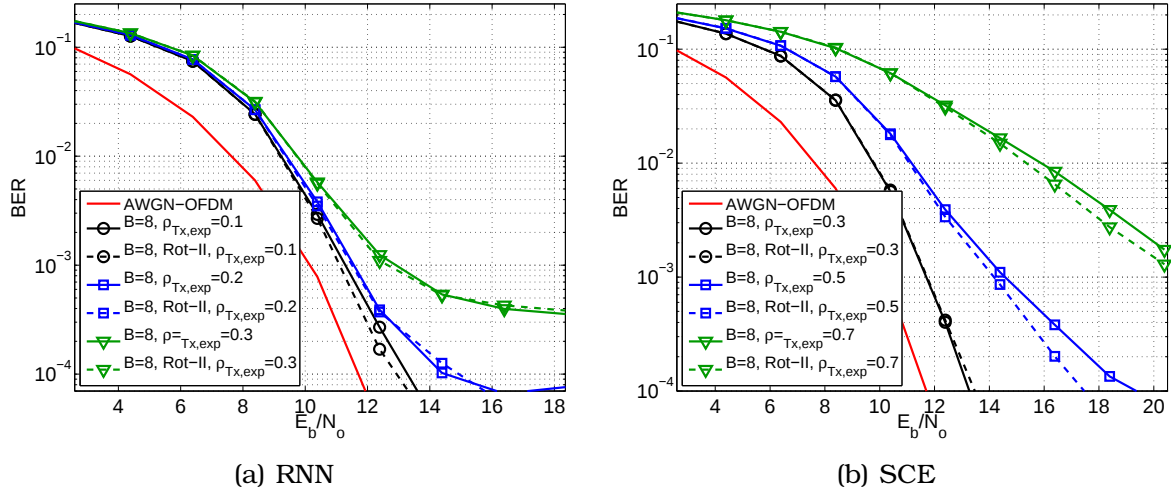


Figure 3.32: BER comparison for MC-CAFS and MC-CAFS Rot-II and various $\rho_{Tx,exp} = \rho$, $L = 4$, $N = 32$ and $n_T = n_R = 4$.

Effect of Diversity

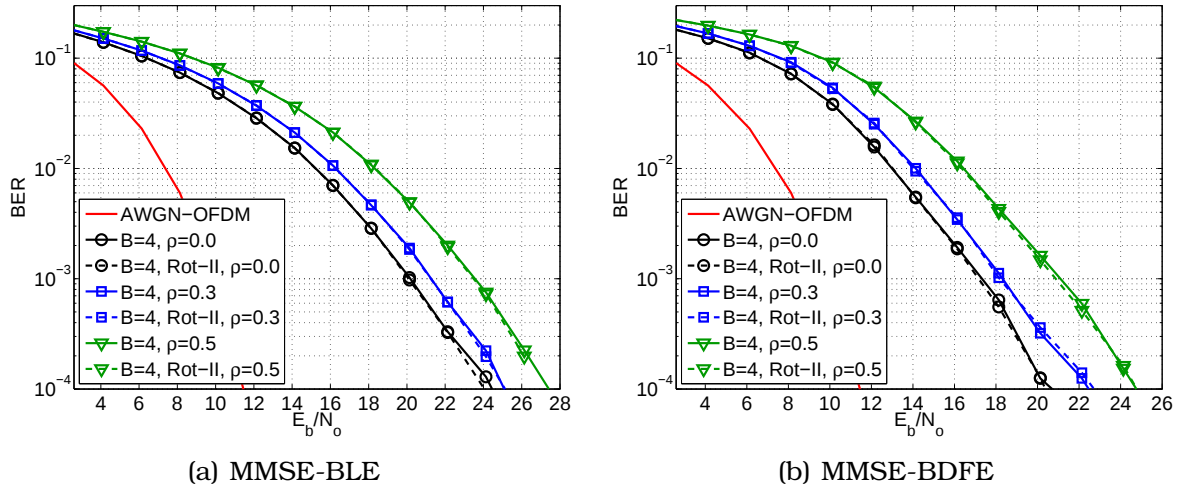


Figure 3.33: BER for MC-CAFS Rot-II and various $\rho_{Tx,exp} = \rho_{Rx,exp} = \rho$, $L = 2$, $N = 32$ and $n_T = n_R = 4$.

Figures 3.33 and 3.34 show the BER for different antenna correlation values, $n_T = n_R = 4$ and $L = 2$, also assuming the exponential correlation model. A smaller number of taps is equivalent to lower frequency diversity and the MIMO channel has thus diversity 32 compared to 64 in the previous exam-

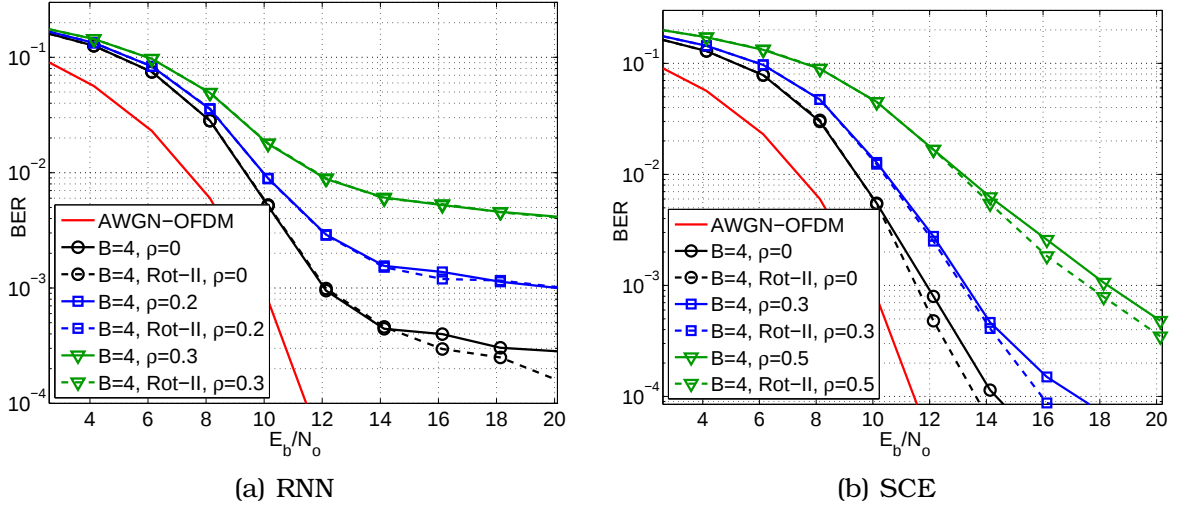


Figure 3.34: BER for MC-CAFS Rot-II and various $\rho_{Tx,exp} = \rho_{Rx,exp} = \rho$, $L = 2$, $N = 32$ and $n_T = n_R = 4$.

ples. Similar to the above, the MMSE-BLE and MMSE-BDFE do not benefit from rotations. The SCE on the other hand seems to benefit the most from rotations as L decreases. Reducing the number of antennas also leads to reduced diversity. Figures 3.35 shows the BER for a 2×2 MIMO channel with $L = 4$. This MIMO channel has thus diversity 16. Figure 3.35 clearly shows that the BER improvement through rotation is quite significant for both the RNN and SCE. The gains are even observable at low E_b/N_0 . A gain of 1 dB at 10^{-3} for $B = 8$ for SCE and a fair drop in the error floor for the RNN is attained only through rotations i. e. at no extra cost.

3.8 Coded Transmission

In all the previous simulations results, we have only considered uncoded transmission. In this section, we look at iterative equalization and decoding using the RNN and SCE equalizers. We consider time varying block fading 4×4 MIMO channels with $L = 4$, where the channel is assumed constant during the transmission of one codeword and changes randomly from one codeword to the next. As explained in Chap. 2, the RNN or the SCE equalizers exchange L-values with the decoder for several iteration steps. We shall consider convolutional codes with rate $1/2$ and $3/4$. For the rate $1/2$ code, we consider the convolutional code with memory 2, and generator polynomial $[7, 5]_8$. Rate $3/4$ code is obtained by puncturing above mother code, since by doing so, very good distance properties can be obtained [72]. The rows of the

3 Spreading

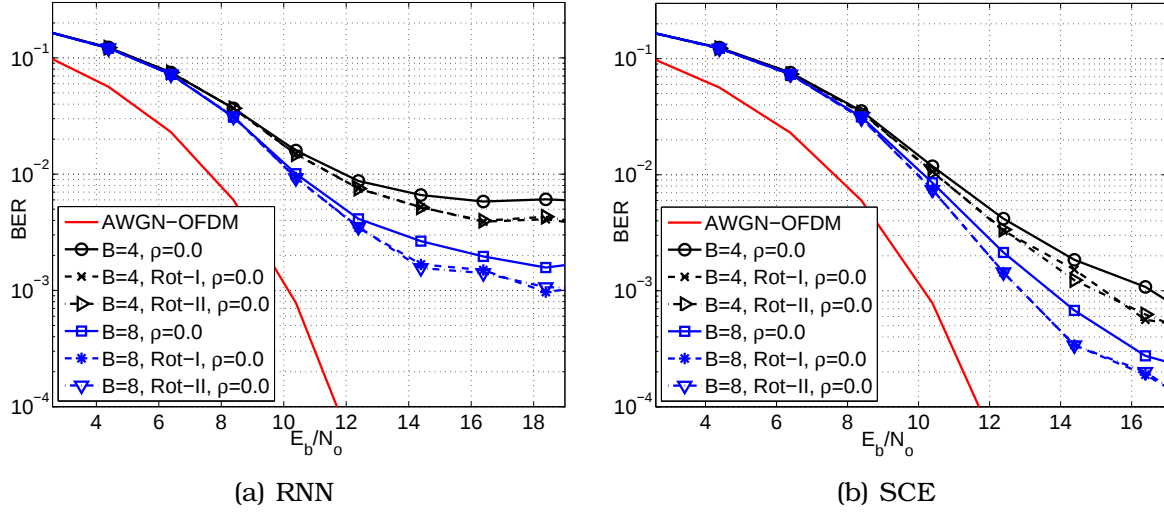


Figure 3.35: MIMO 2×2 : BER for Rot-I and Rot-II, $\rho_{T_x, \text{exp}} = \rho_{R_x, \text{exp}} = \rho = 0$, $L = 4$ and $N = 32$.

puncturing array are $P1 = [1 \ 0 \ 1]$ and $P2 = [1 \ 1 \ 0]$ [73]. An interleaver with length equal to one codeword is applied. The length of one codeword is adapted such that it is an integer multiple of Nn_T . The codeword length is in the order of 10^4 bits. In case of the RNN, 10 iterations are used (10 equalization and 10 decoding steps, one iteration is one equalization followed by one decoding step), while for the SCE only 5 iterations are employed.

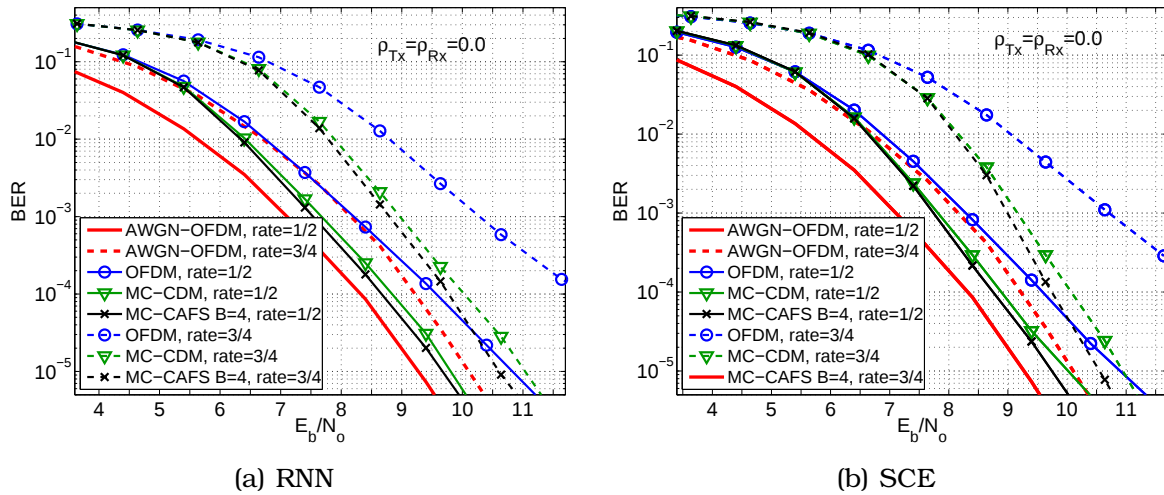


Figure 3.36: Coded transmission, convolutional code, memory 2, zero antenna correlations, $n_T = n_R = 4$, $N = 32$ and $L = 4$.

3.8 Coded Transmission

Figure 3.36 shows the BER for the RNN and SCE for zero antenna correlations and two code rates, 1/2 and 3/4. The transmitted symbols are chosen from a 4 PSK alphabet. The MC-CAFS spreading outperforms the MC-CDM and OFDM using either of the two equalizers. The gain through spreading increases as the code rate increases. For example, using the RNN equalizer, MC-CAFS outperforms OFDM by about 1.5 dB at 10^{-3} for code rate 1/2. At the same BER, MC-CAFS outperforms OFDM by 3 dB for code rate 3/4. The BER reduction of MC-CAFS compared to MC-CDM also increases with increase in the code rate. Both the RNN and SCE exhibit almost the same BER performance although the SCE only undergoes 5 iterations compared to 10 for RNN. Figure 3.37(a) shows the BER for RNN for $\rho_{Tx} = \rho_{Rx} = 0.3$ assuming exponential correlation model and again for two code rates, 1/2 and 3/4. For the rate 1/2 code, similar to the uncorrelated case, MC-CAFS leads to the lowest BER followed by MC-CDM and OFDM. In case of the punctured rate 3/4 code, the BER significantly deteriorates with OFDM leading to the lowest BER at high E_b/N_o .

As for the SCE (Fig. 3.37(b)), MC-CAFS leads to the lowest BER for all E_b/N_o ranges and for both code rates. Again, as with the uncorrelated case, we see here that the improvement in the BER through spreading is more evident as the code rate increases. The SCE clearly outperforms the RNN for high code rates. We shall look at coded transmission in more details in the next Chapter.

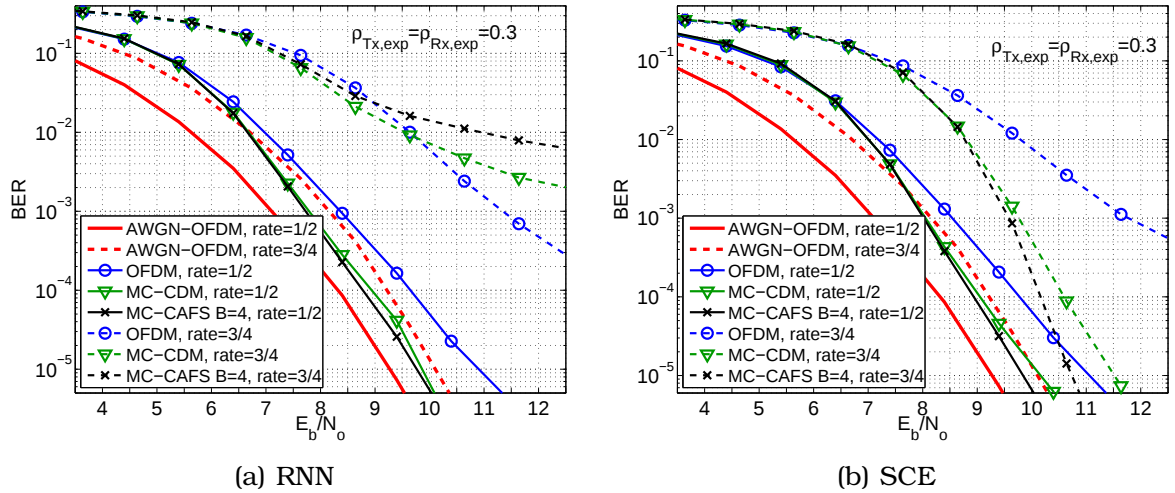


Figure 3.37: Coded transmission, convolutional code, memory 2, exponential correlation model, $\rho_{Tx} = \rho_{Rx} = 0.3$, $n_T = n_R = 4$, $N = 32$ and $L = 4$.

3.9 Summary

We have looked at spreading for MIMO-OFDM and proposed a family of spreading matrices in Sec. 3.5 that lead to maximum space-frequency diversity, $n_T n_R L$. The criteria for designing those matrices were given in Sec. 3.3. We have shown that MC-CAFS spreading leads to the lowest BER using all employed equalizers for low to moderate antenna correlations for moderate E_b/N_o values.

We have shown that both spreading and correlations increase the interference experienced on the channel which affects the performance of employed equalizers. The RNN, being the most sensitive to high interference, exhibits a fast degradation in the BER performance as the antenna correlations increase. We have also shown that, in presence of antenna correlations, increasing the number of receive antennas indefinitely does not improve the MFB or reduce the interference as with the uncorrelated case. Even with $n_R = 48$, the RNN does not converge and the other equalizers are at least 10 dB from the AWGN at 10^{-3} for $\rho_{Tx} = \rho_{Rx} = 0.8$.

In Sec. 3.7, we proposed two rotation possibilities for MC-CAFS in order to increase the signal space diversity, i. e. increase the number of distinct points after spreading. We have shown that only powerful equalizers (RNN and SCE) benefit from the rotations. The gains through rotations were shown to be most significant if the total diversity in the channel is low.

Finally, we looked at coded transmission, and showed that MC-CAFS spreading leads to the lowest BER using the SCE for all code rates and given antenna correlations. The performance of the RNN equalizer, which is sensitive to interference, however deteriorates fast as both the code rate and antenna correlations increase.

3.A Appendix to Chapter 3

3.A.1 Expected Values of Functions of Correlated Complex Random Variable for Kronecker Correlation Model

For the sake of clarity, we show the derivation for a 2×2 correlated channel matrix. However, all of the following results can be readily extended to any $m \times n$ matrix. First, the square root a 2×2 correlation matrix, with $|\rho| \leq 1$ is given as,

$$\underline{\underline{k}} = \begin{bmatrix} 1 & \rho \\ \rho^* & 1 \end{bmatrix} = \underline{\underline{k}}^{1/2} \underline{\underline{k}}^{1/2} = \begin{bmatrix} e & \varrho \\ \varrho^* & e \end{bmatrix} \begin{bmatrix} e & \varrho \\ \varrho^* & e \end{bmatrix} \quad (3.58)$$

where $e \in \mathbb{R}$, $\rho \in \mathbb{C}$, $\varrho \in \mathbb{C}$, $e^2 + |\varrho|^2 = 1$ and $2e\varrho = \rho$. The correlated channel matrix (transmit, ρ_2 , and receive correlations, ρ_1 , are considered)

$$\underline{\underline{h}}^c = \begin{bmatrix} e_1 & \varrho_1 \\ \varrho_1^* & e_1 \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} e_2 & \varrho_2 \\ \varrho_2^* & e_2 \end{bmatrix}, \quad (3.59)$$

To calculate the correlation between any two elements of the matrix $\underline{\underline{h}}^c$, we can make use of the above correlation model and the fact that $\mathcal{E}\{h_{ij}h_{lm}^*\} = 0$ for the uncorrelated elements of $\underline{\underline{h}} \forall i \neq l$ and $j \neq m$

$$\mathcal{E}\{h_{11}^c h_{12}^{c*}\} = e_1^2 e_2 \varrho_2^* |h_{11}|^2 + \varrho_1 \varrho_1^* e_2 \varrho_2 |h_{21}|^2 + e_1^2 e_2 \varrho_2^* |h_{12}|^2 + \varrho_1 \varrho_1^* e_2 \varrho_2^* |h_{22}|^2 \quad (3.60)$$

and $\mathcal{E}\{h_{ij}h_{ij}^*\} = \sigma^2 = 1$

$$\mathcal{E}\{h_{11}^c h_{12}^{c*}\} = e_1^2 \frac{\rho_2^*}{2} \sigma^2 + |\varrho_1|^2 \frac{\rho_2^*}{2} \sigma^2 + e_1^2 \frac{\rho_2^*}{2} \sigma^2 + |\varrho_1|^2 \frac{\rho_2^*}{2} \sigma^2 = \rho_2^* \quad (3.61)$$

Similarly

$$\mathcal{E}\{h_{11}^c h_{12}^{c*}\} = \mathcal{E}\{h_{21}^c h_{22}^{c*}\} = \rho_2^* \quad (3.62)$$

$$\mathcal{E}\{h_{11}^{c*} h_{12}^c\} = \mathcal{E}\{h_{21}^{c*} h_{22}^c\} = \rho_2 \quad (3.63)$$

$$\mathcal{E}\{h_{12}^c h_{22}^{c*}\} = \mathcal{E}\{h_{11}^c h_{21}^{c*}\} = \rho_1 \quad (3.64)$$

$$\mathcal{E}\{h_{12}^{c*} h_{22}^c\} = \mathcal{E}\{h_{11}^{c*} h_{21}^c\} = \rho_1^* \quad (3.65)$$

Note that the above correlations can also be obtained by making use of the Kronecker correlation model, $\underline{\underline{R}}_v = \underline{\underline{k}}_T^T \otimes \underline{\underline{k}}_R$, to find the correlation coefficient between any two elements of $\underline{\underline{h}}^c$. However, in the derivations that follow, the above form is more advantageous.

For the uncorrelated case,

$$\mathcal{E}\{h_{ij}h_{lm}\} = \mathcal{E}\{(h_{ijR} + j h_{ijI})(h_{lmR} + j h_{lmI})\} = 0, \quad (3.66)$$

3 Spreading

since the real and imaginary parts are assumed independent. From Eqn. 3.66, It can be shown that for $\sigma_{real}^2 = \sigma_{imag}^2 = \sigma^2/2$

$$\mathcal{E}\{h_{ij}h_{ij}\} = \sigma_{real}^2 - \sigma_{imag}^2 = 0, \quad (3.67)$$

Using the correlation model of Eqn. 3.59, the results of Eqns. 3.66, 3.67, and proceeding with the derivation as in Eqn. 3.60, it can be shown that

$$\mathcal{E}\{h_{ij}^c h_{lm}^c\} = 0, \quad (3.68)$$

$\forall i, j, l$ and m . Using Eqns. 3.66 to 3.68, we now can derive the expected values of functions of the correlated complex random variables.

If x_i are real jointly normal random variables with mean zero, variance one, and $\mathcal{E}\{x_i x_j\} = \rho_{ij}$, then [43]

$$\mathcal{E}\{x_1 x_2 x_3 x_4\} = \rho_{12}\rho_{34} + \rho_{13}\rho_{24} + \rho_{14}\rho_{23} \quad (3.69)$$

For complex random variables x_i ($CN(0, 1)$), Eqn. 3.70 can be derived by expanding x_i into their real and imaginary parts and using Eqns. 3.67, 3.68 and 3.69.

$$\mathcal{E}\{x_1^* x_2 x_3 x_4^*\} = \rho_{12}^* \rho_{34} + \rho_{13}^* \rho_{24} \quad (3.70)$$

For any two real random variables, x and y , that are jointly normal with zero mean [43]

$$\mathcal{E}\{x^2 y^2\} = \mathcal{E}\{x^2\}\mathcal{E}\{y^2\} + 2\mathcal{E}^2\{xy\} \quad (3.71)$$

Accordingly, for two $CN(0, 1)$ random variables, x and y , the following Eqn. can be derived

$$\mathcal{E}\{|x|^2 |y|^2\} = \mathcal{E}\{(x_{real}^2 + x_{imag}^2)(y_{real}^2 + y_{imag}^2)\} = 1 + |\rho_{xy}|^2 \quad (3.72)$$

From Eqn. 3.72,

$$\mathcal{E}\{|x|^4\} = \mathcal{E}\{|x|^2 |x|^2\} = 2, \quad (3.73)$$

since $|\rho_{xx}|^2 = 1$.

3.A.2 Expected values of α , β and the $\text{var}(\bar{r})$

The aim of the following derivations is to find a closed form expression for the $\mathcal{E}\{\alpha\}$ and $\mathcal{E}\{\beta\}$ as a function the antenna correlations and the number of transmit and receive antennas for the unspread MIMO-OFDM system. In all that follows, we assume that $H_{i_R i_T}(k)$ to be complex Gaussian with zero mean and variance $\sigma^2 = 1$, i. e. $(CN(0, 1))$. This is easily satisfied, even for non Gaussian channel taps, since, based on the central limit theory, the sum L of independent random variables tends to be Gaussian as $L \rightarrow \infty$. Thus, [43]

$$\mathcal{E}\{H_{i_R i_T}(k)H_{j_R j_T}^*(k)\} = \mathcal{E}\{H_{i_R i_T}(k)\}\mathcal{E}\{H_{j_R j_T}^*(k)\} + \rho_{ij}\sigma_i\sigma_j = \rho_{ij} = \rho_{ji}^* \quad (3.74)$$

Based on the Kronecker correlation model and the correlated channel matrix in the frequency domain (Eqn. 3.34),

$$\begin{aligned} \mathcal{E}\{H_{i_R i_T}(k)H_{i_R j_T}^*(k)\} &= \rho_{T \times m}, \quad m = j_T - i_T \\ \mathcal{E}\{H_{i_R i_T}(k)H_{j_R i_T}^*(k)\} &= \rho_{R \times m}, \quad m = j_R - i_R, \end{aligned} \quad (3.75)$$

where $\rho_{T \times m}$ and $\rho_{R \times m}$ are the transmit and receive correlations between transmit/receive antennas with spacing m (see Sec. 2.3).

Expected Values of the Diagonal Elements of $\underline{\underline{R}}$

The diagonal elements of $\underline{\underline{R}}_{\text{MO}}$ are real and are given by

$$r_{ii} = \frac{1}{n_R} \sum_{i_R=1}^{n_R} |H_{i_R i_T}(k)|^2 \quad (3.76)$$

where i in this case is $i = k + (i_T - 1)N$, $k = 1, \dots, N$. The square of the diagonal elements, r_{ii}^2 , is thus

$$\begin{aligned} \mathcal{E}\{r_{ii}^2\} &= \frac{1}{n_R^2} \mathcal{E}\{(|H_{1i_T}(k)|^2 + \dots + |H_{n_R i_T}(k)|^2)^2\} \\ &= \frac{1}{n_R^2} \left(\sum_{i_R=1}^{n_R} \mathcal{E}\{|H_{i_R i_T}(k)|^4\} + 2 \sum_{l=1}^{n_R} \sum_{m>l}^{n_R} \mathcal{E}\{|H_{li_T}(k)|^2 |H_{mi_T}(k)|^2\} \right) \\ &= \frac{2}{n_R} + \frac{2}{n_R^2} \left(\sum_{m=1}^{n_R-1} (1 + |\rho_{R \times m}|^2)(n_R - m) \right) \\ &= 1 + \frac{1}{n_R} + \frac{2}{n_R^2} \left(\sum_{m=1}^{n_R-1} |\rho_{R \times m}|^2 (n_R - m) \right) \end{aligned} \quad (3.77)$$

where we have made use of Appendix 3.A.1 to obtain $\mathcal{E}\{|H_{li_T}(k)|^2 |H_{mi_T}(k)|^2\}$.

3 Spreading

Expected Values of Off-Diagonal Elements of $\underline{\underline{R}}$

We shall denote the i th column of $\underline{\underline{H}}$ by $\underline{H}_i(k)$ where again $i = k + (i_T - 1)N$ and $k = 1 \dots N$. Thus, the off diagonal elements of $\underline{\underline{R}}_{\text{MO}}$, r_{ij} for $i \neq j$, can be written as

$$r_{ij} = \frac{1}{n_R} \underline{H}_i^H(k_1) \underline{H}_j(k_2) \quad (3.78)$$

Note that $r_{ij} = 0$ for $k_1 \neq k_2$. The expected value of $|r_{ij}|^2$ can thus be given by

$$\begin{aligned} \mathcal{E}\{|r_{ij}|^2\} &= \frac{1}{n_R^2} \mathcal{E}\{|\underline{H}_i^H(k) \underline{H}_j(k)|^2\} \\ &= \frac{1}{n_R^2} \sum_{m=1}^{n_R} \mathcal{E}\{|H_{mi}|^2 |H_{mj}|^2\} + \frac{1}{n_R^2} \sum_{m=1}^{n_R} \sum_{l \neq m}^{n_R} \mathcal{E}\{H_{mi}^* H_{mj} H_{li} H_{lj}^*\} \end{aligned} \quad (3.79)$$

From Eqn.3.74, 3.75 and Appendix 3.A.1,

$$\mathcal{E}\{|r_{ij}|^2\} = \frac{1}{n_R} + |\rho_{\text{Tx}(j-i)}|^2 + \sum_{m=1}^{n_R-1} \frac{(n_R - 1)}{n_R^2} |\rho_{\text{Rx}m}|^2 \quad (3.80)$$

Let $l = j - i$, the expected value of the Frobenius norm of $\underline{\underline{R}}_{\text{MO}}$ for any i can then be written as follows,

$$\mathcal{E}\{\|\underline{\underline{R}}_{\text{MO}}\|_F^2\} = N n_T \mathcal{E}\{r_{ii}^2\} + 2N \sum_{l=1}^{n_T-1} (n_T - l) \mathcal{E}\{|r_{i(l+i)}|^2\}. \quad (3.81)$$

Variance of $\bar{r}$

$\bar{r}$ is defined as the mean of the diagonal elements of $\underline{\underline{R}}$. Based on the channel normalization, the expected value of $\bar{r}$, $\mathcal{E}\{\bar{r}\} = 1$. Since the variance of $\bar{r}$ affects the value of α , in this section we derive the expression for variance of $\bar{r}$ as a function of the antenna correlations. We shall show that $\text{var}(\bar{r}) = 1/\Psi(\underline{\underline{R}}_v)$, where $\Psi(\underline{\underline{R}}_v)$ is the diversity measure in Eqn. 2.53.

To calculate the variance of $\bar{r}$, $\text{var}\{\bar{r}\}$, we shall make use of the following equation (see Chapter 2 for more details on normalization)

$$n_R \frac{\text{Tr}(\underline{\underline{R}})}{N} = n_T n_R \bar{r} = \sum_{l=1}^L \|\underline{h}(l)\|_F^2,$$

where the elements of $\underline{h}(l)$ are assumed to be complex Gaussian, $CN(0, 1/L)$, random variables. Accordingly, $\mathcal{E}\{|h_{ij}|^2\} = 1/L$, $\text{var}(|h_{ij}|^2) = 1/L^2$, and $\mathcal{E}\{\|\underline{h}(l)\|_F^2\} = n_T n_R / L$. Since the individual channel taps are assumed to be

3.A Appendix to Chapter 3

uncorrelated, with equal PDP, and to fade independently, we can make use of the central limit theory as follows,

$$\text{var}\left(\sum_{l=1}^L \|\underline{h}(l)\|_F^2\right) = \sum_{l=1}^L \text{var}(\|\underline{h}(l)\|_F^2), \quad (3.82)$$

and

$$\mathcal{E}\left\{\sum_{l=1}^L \|\underline{h}(l)\|_F^2\right\} = \sum_{l=1}^L \mathcal{E}\{\|\underline{h}(l)\|_F^2\} = n_T n_R. \quad (3.83)$$

Thus, the variance of $\bar{r}$ can be obtained as follows

$$\text{var}(\bar{r}) = \frac{\text{var}\left(\sum_{l=1}^L \|\underline{h}(l)\|_F^2\right)}{n_T^2 n_R^2} = \frac{\sum_{l=1}^L \text{var}(\|\underline{h}(l)\|_F^2)}{n_T^2 n_R^2}. \quad (3.84)$$

Now, we look at the individual channel taps to calculate $\text{var}(\|\underline{h}(l)\|_F^2)$

$$\begin{aligned} \text{var}(\|\underline{h}(l)\|_F^2) &= \mathcal{E}\{(\|\underline{h}(l)\|_F^2)^2\} - (\mathcal{E}\{\|\underline{h}(l)\|_F^2\})^2, \\ &= \mathcal{E}\{(\|\underline{h}(l)\|_F^2)^2\} - \frac{n_T^2 n_R^2}{L^2}, \end{aligned} \quad (3.85)$$

where

$$\mathcal{E}\{(\|\underline{h}(l)\|_F^2)^2\} = \left(\sum_{i=1}^{n_R} \sum_{j=1}^{n_T} |h_{ij}|^2\right)^2. \quad (3.86)$$

Since for complex Gaussian random variables ($CN(0, 1/L)$) with zero mean and variance $1/L$ (see Appendix 3.A.1)

$$\mathcal{E}\{|h_{ij}|^2 |h_{lk}|^2\} = \frac{1}{L^2}(1 + |\rho_{Tx_m}|^2 |\rho_{Rx_n}|^2), \quad (3.87)$$

where $m = |k - j|$, $n = |l - i|$, and

$$\mathcal{E}\{|h_{ij}|^4\} = \frac{2}{L^2}, \quad (3.88)$$

the $\text{var}(\|\underline{h}(l)\|_F^2)$ can be obtained by expanding Eqn. 3.86 and using Eqns. 3.87 and 3.88 and substituting into Eqn. 3.85. After some manipulations we obtain,

$$\begin{aligned} \text{var}(\|\underline{h}(l)\|_F^2) &= \frac{n_T n_R}{L^2} + \frac{2}{L^2} \left(n_R \sum_{m=1}^{n_T-1} (n_T - m) |\rho_{Tx_m}|^2 + n_T \sum_{n=1}^{n_R-1} (n_R - m) |\rho_{Rx_n}|^2 \right) \\ &+ \frac{4}{L^2} \left(\sum_{m=1}^{n_T-1} (n_T - m) |\rho_{Tx_m}|^2 \sum_{n=1}^{n_R-1} (n_R - m) |\rho_{Rx_n}|^2 \right). \end{aligned} \quad (3.89)$$

3 Spreading

Thus, from Eqn, 3.84,

$$\begin{aligned} \text{var}(\bar{r}) = & \frac{1}{n_T n_R L} + \frac{2}{n_T^2 n_R^2 L} \left(n_R \sum_{m=1}^{n_T-1} (n_T - m) |\rho_{Tx_m}|^2 + n_T \sum_{n=1}^{n_R-1} (n_R - m) |\rho_{Rx_n}|^2 \right) \\ & + \frac{4}{n_T^2 n_R^2 L} \left(\sum_{m=1}^{n_T-1} (n_T - m) |\rho_{Tx_m}|^2 \sum_{n=1}^{n_R-1} (n_R - m) |\rho_{Rx_n}|^2 \right). \end{aligned} \quad (3.90)$$

In the uncorrelated case

$$\text{var}(\bar{r}) = \frac{1}{n_T n_R L}. \quad (3.91)$$

In case of full transmit correlations and zero receive correlations

$$\text{var}(\bar{r}) = \frac{1}{n_R L}, \quad (3.92)$$

and for full receive correlations and zero transmit correlations

$$\text{var}(\bar{r}) = \frac{1}{n_T L}. \quad (3.93)$$

In the fully correlated case, $\rho_{Tx_m} = \rho_{Rx_n} = 1$ for all n and m ,

$$\text{var}(\bar{r}) = \frac{1}{L}, \quad (3.94)$$

i. e. the variance of $\text{var}(\bar{r})$ is maximum. Thus $\frac{1}{L} \leq \text{var}(\bar{r}) \leq \frac{1}{n_T n_R L}$ and for high diversity, the $\text{var}(\bar{r}) \approx 0$.

Now, we take a closer look at the diversity measure in Eqns. 2.48, 2.51 and 2.53, where, for the frequency selective MIMO channel and $\underline{k}_T(l) = \underline{k}_T$ and $\underline{k}_R(l) = \underline{k}_R$ for all $l = 1 \cdots L$, we can write

$$\frac{1}{\Psi(\underline{R}_v)} = \frac{1}{L} \frac{1}{\Psi(\underline{k}_T)} \frac{1}{\Psi(\underline{k}_R)} = \frac{1}{L} \frac{\|\underline{k}_T\|_F^2}{n_T^2} \frac{\|\underline{k}_R\|_F^2}{n_R^2}, \quad (3.95)$$

and

$$\begin{aligned} \|\underline{k}_T\|_F^2 &= n_T + 2 \sum_{i=1}^{n_T-1} (n_T - i) |\rho_{Tx_i}|^2, \\ \|\underline{k}_R\|_F^2 &= n_R + 2 \sum_{i=1}^{n_R-1} (n_R - i) |\rho_{Rx_i}|^2 \end{aligned} \quad (3.96)$$

By substituting Eqns. 3.96 in Eqn 3.95 we get,

$$\text{var}(\bar{r}) = \frac{1}{\Psi(\underline{R}_v)}. \quad (3.97)$$

Chapter 4

Precoding

So far, we have assumed that only the receiver has full channel knowledge. In this Chapter, we consider the case where either full or partial channel knowledge is available at the transmitter. While full channel knowledge at the receiver is usually available, at the transmitter such knowledge is typically limited. In wireless communications, channel feedback from the receiver to the transmitter is common. However, the channel may be varying too fast that the feedback does not give an accurate estimate of the current channel. However, partial channel knowledge at the transmitter, e.g only transmit correlations, can be available. Correlations can be statistically calculated for a short time duration and do not change as fast as the actual channel [74, 75]. In this chapter, we show the precoder for the above two cases. However, we focus on the case, when only partial CSIT is available at the transmitter.

4 Precoding

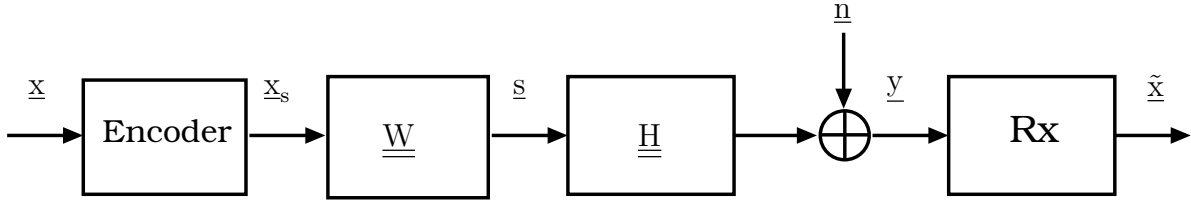


Figure 4.1: Matrix Vector transmission model with precoding.

4.1 Linear Precoding

Figure 4.1 shows the matrix vector transmission model with precoding. The extra block $\underline{W}$, represents the linear precoding matrix which connects the encoder with the channel. The encoder shown stands for any type of encoding done (e.g. space-frequency coding or spreading matrix) and Rx for the receiver which can include an equalizer and/or a decoder. In this Chapter, we shall only consider orthogonal spreading matrices. In addition, we assume that the covariance of the transmitted symbols to be $\mathcal{E}\{\underline{x}\underline{x}^H\} = \mathcal{E}\{\underline{x}_s\underline{x}_s^H\} = \underline{I}$. The aim of the precoder is thus to generate an optimal signal covariance, $\underline{Q} = \mathcal{E}\{\underline{s}\underline{s}^H\}$, based on the available channel state information at the transmitter (CSIT) and the given performance criterion. Several precoding design criteria exist, for example: mutual information or receive signal to noise ratio (SNR) maximization. Minimization of the average pair-wise error probability (PEP), the BER, the symbol error rate (SER) or the detection mean square error (MSE) are other examples. Accordingly, the symbols are first spread, precoded and then transmitted over the channel. The received symbol vector is thus,

$$\underline{y} = \underline{H}_{eff} \underline{x} + \underline{n}, \quad (4.1)$$

where the channel matrix in the frequency domain is represented by an effective channel matrix that includes both the encoder and precoder as follows,

$$\underline{H}_{eff} = \underline{H}^c \underline{W} \underline{U} \quad (4.2)$$

At the receiver side (Rx), channel matched filtering and equalization are applied. The channel correlation matrix, $\underline{R}$, for this MIMO-OFDM system can thus be described by the following equation

$$\underline{R} = \frac{1}{n_R} \underline{U}^H \underline{W}^H \underline{R}_{MO} \underline{W} \underline{U} = \underline{H}_{eff}^H \underline{H}_{eff}, \quad (4.3)$$

where $\underline{R}_{MO} = \underline{H}^{cH} \underline{H}^c$.

4.2 Full Channel Knowledge at the Transmitter

In this Section, the channel is assumed to be fully known at both the transmitter and the receiver. That is, full CSI is available at both the transmitter and the receiver. If the goal is to maximize the mutual information [53], or to minimize the average pairwise error probability (PEP) [76], the sum or weighted sum of the mean square errors (MSE) of all subchannels [77, 78], the precoder is given by,

$$\underline{\underline{W}} = \underline{\underline{V}} \underline{\underline{\Lambda}}^{1/2}, \quad (4.4)$$

where $\underline{\underline{V}}$ is obtained by the singular value decomposition (SVD) of the channel matrix, $\underline{\underline{H}} = \underline{\underline{U}} \underline{\underline{\Sigma}} \underline{\underline{V}}^H$ and $\underline{\underline{\Lambda}}$ is the power allocation matrix. At the receiver, also assuming full channel knowledge, the signal is left-multiplied by $\underline{\underline{U}}$ to obtain

$$\tilde{\underline{\underline{x}}} = \underline{\underline{\Sigma}} \tilde{\underline{\underline{s}}} + \underline{\underline{n}}, \quad (4.5)$$

where $\underline{\underline{\Sigma}}$ is a diagonal matrix containing the singular values of the channel matrix $\underline{\underline{H}}$ and $\tilde{\underline{\underline{s}}} = \underline{\underline{\Lambda}}^{1/2} \underline{\underline{x}}_s$. The covariance matrix of $\tilde{\underline{\underline{s}}}$ is $\underline{\underline{\Lambda}}$ and is subjected to the power constraint: $\text{Tr}(\underline{\underline{\Lambda}}) = p_o$, where p_o is available transmit power. In other words, the channel diagonalizing structure is optimal and $\underline{\underline{V}}$ represents the optimal beam direction for perfect CSIT at the transmitter. The choice of $\underline{\underline{\Lambda}}$ on the other hand, as we shall show next, depends on the optimization criterion. In case of frequency selective channels, the optimum precoder and decoder as shown above can be applied to the individual OFDM subchannels [78, 79] since this multicarrier approach is known to be capacity-lossless [79].

For other criteria, such as the minimization of the average BER or the maximum MSE, the precoding matrix is given by [77]

$$\underline{\underline{W}} = \underline{\underline{V}} \underline{\underline{\Lambda}}^{1/2} \underline{\underline{U}}_{\text{Rot}}^H, \quad (4.6)$$

where $\underline{\underline{U}}_{\text{Rot}}^H$ is a unitary matrix which can be chosen to be any rotation matrix, e.g. Hadamard or DFT matrix [77]. Note that the covariance matrix of the transmit signal does not change by $\underline{\underline{U}}_{\text{Rot}}^H$. The authors showed that the MMSE-BLE is the optimal receiver in this case. Again, for frequency selective channels, the results can be directly extended to the individual OFDM subcarriers [77].

The difference between the above design criteria is whether the objective functions used for optimization are Schur-concave or Schur-convex. For Schur-convex objective functions, the channel diagonalizing structure (Eqn.4.5) is optimum while for Schur-concave objective functions the diagonal structure is optimal only in combination with a specific rotation of the transmitted symbols (Eqn. 4.6) [77].

4 Precoding

Power Allocation Strategies

If the goal is to maximize the channel ergodic capacity, then the optimum $\underline{\Lambda}_{opt}$ is obtained through the well known water-filling solution on the square of the channel singular values $\lambda_i = \sigma_i^2$ [4, 3, 53],

$$p_i = \left(\mu - \frac{N_o}{\lambda_i} \right)_+, \quad (4.7)$$

where N_o is the noise power, and μ is chosen such that $\sum_i p_i = p_o$ and $()_+$ denotes $\max(0, p_i)$. Thus, depending on the amount of available transmit power, some of the weak modes might receive zero power and the signals transmitted in direction of eigenvectors with good eigenvalues are assigned more power than those transmitted over eigenvectors corresponding to small eigenvalues. MIMO channel capacity will be discussed in more details in Chapter 5.

Sampath et al. in [78] obtained the optimum linear precoder and decoder using the weight minimum mean-squared error (weight MMSE) criterion subjected to transmit power constraint. As mentioned previously, the optimum linear precoder and decoder were found to diagonalize the channel into eigen subchannels for any chosen set of error weights. Only the power allocation strategy is dependent on the error weights. The weight matrix is chosen based on several design criteria, eg. quality of service (QoS), or equal error (i. e. equal SNR on all subchannels), or information rate maximization (in this case, the power allocation is as given in Eqn. 4.7). For example, assuming the design criterion is the equal error, then the power allocation matrix is given by [78],

$$\underline{\Lambda}^{1/2} = \sqrt{\frac{p_o}{\text{Tr}(\underline{\Sigma}^{-1})}} \underline{\Sigma}^{-1/2}, \quad (4.8)$$

If the aim is to minimize the PEP per distance (i. e. minimizing the PEP based on a given codeword distance, for example the minimum distance codeword), the optimal solution would be to allocate all of the power to the strongest eigenmode [76]. In this case, precoding is reduced to single beam-forming and represents an extreme case of selective power allocation. Yet, it maximizes the receive SNR and extracts the full spatial diversity [22].

4.3 Partial Channel Knowledge at the Transmitter

In this subsection, only partial channel knowledge is assumed at the transmitter. In this case, the transmission model in Fig. 4.1 still remains valid. However, the precoding matrix $\underline{W}$ is not based on the full channel knowledge, but only on partial knowledge. We assume that only the transmit

4.3 Partial Channel Knowledge at the Transmitter

correlation matrix is known at the transmitter. As aforementioned, this is a well-founded assumption, since, while the channel might be fast changing, the channel correlations remain constant for longer periods of time [74, 75] .

4.3.1 Flat Fading Channels

Several authors have looked at precoding in presence of partial channel knowledge, e. g. [80, 81, 82, 76, 83]. All have found that transmitting in the direction of the eigenvectors of the transmit correlations is the optimum direction for both capacity maximization ([80, 82]) and pairwise error probability minimization ([81, 76]). It was shown in [80], that the optimal transmit strategy for capacity maximization is to transmit along the eigenvectors of the transmit correlation matrix. The power allocation is done through numerical optimization and resembles that of waterfilling, where stronger modes are allocated more power than weaker ones. The authors in [82] considered both transmit and receive correlations and have shown that, although one still needs to transmit in direction of the transmit correlations eigenvectors, the energy distribution at the transmitter is dependent on both the transmit and receive correlations. In [84], it was shown that taking the receive correlations into account has little to no effect on the BER.

In [81], it was shown that the optimum precoder, $\underline{w}$, that minimizes the average pairwise error probability for a flat fading MIMO channel employing STBC encoding is

$$\underline{w} = \underline{v}_T \phi \underline{v}_C^H, \quad (4.9)$$

where $\underline{k}_T = \underline{v}_T \underline{e}_T \underline{v}_T^H$ and $\underline{c} \underline{c}^H = \underline{v}_C \underline{e}_C \underline{v}_C^H$ and $\underline{c} = \underline{x}_1 - \underline{x}_2$ is the error matrix between any two codewords. ϕ is a diagonal matrix describing the power allocation strategy for the transmit symbols under the power constraint $\text{Tr} \{ \underline{w} \underline{w}^H \} = p_o$, where p_o is the total available power. In this precoding strategy, both the error matrix and the transmit correlations are taken into account. For orthogonal STBC, $\underline{c} \underline{c}^H = \alpha \underline{I}$. That is, the code word error matrix is a scaled identity matrix, where α can be chosen to be either the minimum or average distance. In this case, $\underline{v}_C$ can be omitted and the precoder reduces to

$$\underline{w} = \underline{v}_T \phi \quad (4.10)$$

The matrix, ϕ^2 , is obtained by waterfilling over the eigenvalues of the channel correlation matrix [81]. In case of quasi-orthogonal space time block codes (QSTBC), the $\underline{c} \underline{c}^H$ is not a scaled identity matrix. However, $\mathcal{E} \{ \underline{c} \underline{c}^H \}$ is a scaled identity matrix and $\underline{v}_C$ can also be omitted if the average distance is considered [76, 85].

4 Precoding

To summarize, we can think of the precoder as a link between the encoder and the channel. The precoder has the task of decorrelating the coming encoded symbols (if the covariance matrix is different from an identity matrix) and adjusting the symbol covariance to that suitable to the channel, based on the available channel knowledge.

4.3.2 Frequency Selective Channels

Now, we shall extend the above precoding to MIMO-OFDM. As aforementioned, flat fading techniques can be applied to frequency selective channels by implementing the precoding at the individual subcarriers of the OFDM system [21, 76, 77, 86, 87]. We again use the channel matrix in the frequency domain from Chapter 3. However, in this Chapter, we shall permute the channel matrix $\underline{\underline{H}}$ such that the entries are grouped by frequency. In this case, $\underline{\underline{H}}$ is a block diagonal matrix and we denote it by $\underline{\underline{H}}_B$,

$$\underline{\underline{H}}_B^c = \underline{\underline{K}}_R^{1/2} \underline{\underline{H}}_B \underline{\underline{K}}_T^{1/2}, \quad (4.11)$$

The block matrix $\underline{\underline{H}}_B$ is as follows

$$\underline{\underline{H}}_B = \begin{bmatrix} \underline{\underline{H}}_1 & \dots & \dots & \dots & 0 \\ \vdots & \ddots & \underline{\underline{H}}_k & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \underline{\underline{H}}_N \end{bmatrix} \quad (4.12)$$

where $\underline{\underline{H}}_k$ are $n_R \times n_T$ matrices containing the channel transfer function at the k th frequency. The matrices $\underline{\underline{K}}_T$ and $\underline{\underline{K}}_R$ are given by

$$\underline{\underline{K}}_T = \underline{\underline{I}}_N \otimes \underline{\underline{k}}_T \quad (4.13)$$

$$\underline{\underline{K}}_R = \underline{\underline{I}}_N \otimes \underline{\underline{k}}_R, \quad (4.14)$$

where $\otimes$ is the Kronecker product and $\underline{\underline{I}}_N$ is the identity matrix of size $N \times N$. Note that both $\underline{\underline{K}}_T$ and $\underline{\underline{K}}_R$ always have the same structure as $\underline{\underline{H}}$. That is, if the elements of $\underline{\underline{H}}$ are grouped by frequency, so are $\underline{\underline{K}}_T$ and $\underline{\underline{K}}_R$ (i. e. $\underline{\underline{K}} = \underline{\underline{k}} \otimes \underline{\underline{I}}_N$) as was the case in Chapter 3. Both forms are completely equivalent. However, when spreading is considered, the antenna grouping is a more convenient representation while when precoding is treated grouping by frequency is the more suitable one.

We make use of the PEP from [88], where the transmit antenna correlations are included in the Eqn. 3.8.

$$\underline{\underline{C}}_Y = \sum_{l=0}^L \left[\underline{\underline{D}}^l (\underline{\underline{C}}_p - \underline{\underline{E}}_p)^T \underline{\underline{k}}_T^*(l) (\underline{\underline{C}}_p - \underline{\underline{E}}_p)^* \underline{\underline{D}}^{-l} \right] \otimes \underline{\underline{I}}_{n_R} = \underline{\underline{Y}} \otimes \underline{\underline{I}}_{n_R}, \quad (4.15)$$

4.3 Partial Channel Knowledge at the Transmitter

where $\underline{\underline{C}}_p$ and $\underline{\underline{E}}_p$ are the precoded transmit matrices and $\underline{\underline{k}}_T$ is assumed to be full rank. As mentioned in Chapter 3, $\underline{\underline{C}}_p$ and $\underline{\underline{E}}_p$ are simply obtained by reshaping the transmit vectors $\underline{C}_p$ and $\underline{E}_p$,

$$\underline{\underline{C}}_p = \underline{\underline{W}} \underline{\underline{C}} = (\underline{\underline{I}}_N \otimes \underline{w}) \underline{\underline{C}} = \begin{bmatrix} \underline{w} \underline{C}_1 \\ \underline{w} \underline{C}_2 \\ \vdots \\ \underline{w} \underline{C}_N \end{bmatrix} = \begin{bmatrix} \underline{C}_{p1} \\ \underline{C}_{p2} \\ \vdots \\ \underline{C}_{pN} \end{bmatrix}, \quad (4.16)$$

where $\underline{C}_j$ are $n_T \times 1$ subvectors of the $n_T N \times 1$ transmit vector $\underline{C}$. The $n_T \times N$ reshaped transmit matrix is thus $\underline{\underline{C}}_p = [\underline{C}_{p1} \ \underline{C}_{p2} \ \underline{C}_{pj} \ \cdots \ \underline{C}_{pN}]$. We define the error vector between any two transmit vectors as $\underline{\underline{\Delta E}} = \underline{\underline{C}} - \underline{\underline{E}}$. Thus, after precoding and reshaping, the error matrix $\underline{\underline{\Delta E}}_p$ can be written as,

$$\underline{\underline{\Delta E}}_p = \underline{\underline{C}}_p - \underline{\underline{E}}_p = \underline{w} \underline{\underline{\Delta E}}. \quad (4.17)$$

Since the eigenvalues $\underline{\underline{C}}_Y$ are equal to those of $\underline{Y}$ occurring with n_R multiplicity (as was also the case in Chapter 3), in all what follows, we concentrate on $\underline{Y}$. By precoding at each OFDM subcarrier, assuming that $\underline{\underline{k}}_T(l) = \underline{\underline{k}}_T$, the precoder can be described as follows,

$$\underline{\underline{W}} = \underline{\underline{I}}_N \otimes \underline{w}. \quad (4.18)$$

Our goal is thus to find the precoding matrix $\underline{\underline{W}}$ (or equivalently $\underline{w}$), to minimize the average PEP which is equivalent to maximizing

$$\max_{\underline{\underline{W}}} J = \prod_{i=1}^{\text{rank}(\underline{Y})} \left(1 + \frac{E_s}{4N_o} \lambda_i(\underline{Y}) \right), \quad (4.19)$$

subjected to the power constraint $\text{Tr}\{\underline{\underline{W}} \underline{\underline{W}}^H\} = p_o$, and where $\lambda_i(\underline{Y})$ are the eigenvalues of $\underline{Y}$. We now proceed as in [81], assuming $\underline{w} = \underline{v}_T \underline{\phi} \underline{v}_{\Delta E}^H$, where $\underline{\underline{\Delta E}} \underline{\underline{\Delta E}}^H = \underline{v}_{\Delta E} \underline{e}_{\Delta E} \underline{v}_{\Delta E}^H$ and substituting Eqn. 4.17 into $\underline{Y}$ as given in Eqn. 4.15. Accordingly,

$$\begin{aligned} \underline{Y} &= \sum_{l=0}^{L-1} \left[\underline{D}^l \underline{\underline{\Delta E}}^T \underline{w}^T \underline{\underline{k}}_T^* \underline{w}^* \underline{\underline{\Delta E}}^* \underline{D}^{-l} \right] \\ &= \sum_{l=0}^{L-1} \left[\underline{D}^l \underline{\underline{\Delta E}}^T \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{v}_T^T \underline{k}_T^* \underline{v}_T^* \underline{\phi}^* \underline{v}_{\Delta E}^T \underline{\underline{\Delta E}}^* \underline{D}^{-l} \right] \\ &= \sum_{l=0}^{L-1} \left[\underline{D}^l \underline{\underline{\Delta E}}^T \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{e}_T \underline{\phi}^* \underline{v}_{\Delta E}^T \underline{\underline{\Delta E}}^* \underline{D}^{-l} \right] \\ &= \underline{F} \underline{F}^H, \end{aligned} \quad (4.20)$$

4 Precoding

where $\underline{v}_T^T k^* \underline{v}_T^* = \underline{e}_T$ and $\underline{F}$ is given by,

$$\underline{F} = \left[\underline{D}^0 \underline{\Delta E}^T \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{e}_T^{1/2} \dots \underline{D}^{L-1} \underline{\Delta E}^T \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{e}_T^{1/2} \right] \quad (4.21)$$

since the eigenvalues of $\underline{F} \underline{F}^H$ are equal to those of $\underline{F}^H \underline{F}$, we shall now consider the latter expression,

$$\underline{F}^H \underline{F} = \begin{bmatrix} \underline{G} \underline{G}^H & \underline{G} \underline{D} \underline{G}^H & \dots & \underline{G} \underline{D}^{L-1} \underline{G}^H \\ \underline{G} \underline{D}^{-1} \underline{G}^H & \underline{G} \underline{G}^H & \dots & \underline{G} \underline{D}^{L-2} \underline{G}^H \\ \vdots & \vdots & \underline{G} \underline{G}^H & \vdots \\ \underline{G} \underline{D}^{-(L-1)} \underline{G}^H & \dots & \dots & \underline{G} \underline{G}^H \end{bmatrix} \quad (4.22)$$

where

$$\underline{G} \underline{G}^H = \underline{e}_T^{1/2} \underline{\phi}^* \underline{v}_{\Delta E}^T \underline{\Delta E}^* \underline{\Delta E}^T \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{e}_T^{1/2} \quad (4.23)$$

and

$$\underline{G} \underline{D}^l \underline{G}^H = \underline{e}_T^{1/2} \underline{\phi}^* \underline{v}_{\Delta E}^T \underline{\Delta E}^* \underline{D}^l \underline{\Delta E}^T \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{e}_T^{1/2}. \quad (4.24)$$

As in [76, 85], from now on we shall only look at the average PEP and thus only consider the average distance (covariance of the error matrix) for the optimization in Eqn. 4.19. That is, we consider $\mathcal{E}\{\underline{\Delta E} \underline{\Delta E}^H\} = \underline{v}_{\Delta E} \underline{e}_{\Delta E} \underline{v}_{\Delta E}^H$, instead of $\underline{\Delta E} \underline{\Delta E}^H$. Since we assume that all transmit symbols powers are equal and $\mathcal{E}\{\underline{C} \underline{C}^H\} = \underline{I}$, the $\mathcal{E}\{\underline{\Delta E}^* \underline{\Delta E}^T\} = \alpha \underline{I}$, is a scaled identity matrix and thus $\underline{v}_{\Delta E}$ can be any arbitrary orthonormal matrix. Accordingly, by taking the average over all $\underline{\Delta E}$, we get

$$\underline{G} \underline{G}^H = \underline{e}_T^{1/2} \underline{\phi}^* \underline{v}_{\Delta E}^T \mathcal{E}\{\underline{\Delta E}^* \underline{\Delta E}^T\} \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{e}_T^{1/2} = \alpha \underline{e}_T^{1/2} \underline{\phi}^* \underline{\phi}^T \underline{e}_T^{1/2}, \quad (4.25)$$

and

$$\underline{G} \underline{D}^l \underline{G}^H = \underline{e}_T^{1/2} \underline{\phi}^* \underline{v}_{\Delta E}^T \mathcal{E}\{\underline{\Delta E}^* \underline{D}^l \underline{\Delta E}^T\} \underline{v}_{\Delta E}^* \underline{\phi}^T \underline{e}_T^{1/2} = \underline{0}, \quad (4.26)$$

since $\sum (\underline{D}^l)_{ii} = 0$, and accordingly $\mathcal{E}\{\underline{\Delta E}^* \underline{D}^l \underline{\Delta E}^T\} = \underline{0}$. The matrix defined by Eqn. 4.22 thus reduces to a block diagonal matrix. Now, we look again at Eqn. 4.19, and using the Hadamard inequality, we can upperbound Eqn. 4.19 as follows

$$J \leq \prod_{i=1}^{n_T L} \left(1 + \frac{E_s}{4N_o} (\underline{F}^H \underline{F})_{ii} \right), \quad (4.27)$$

where $(\underline{F}^H \underline{F})_{ii}$ are the diagonal elements of $\underline{F}^H \underline{F}$. The upperbound is achieved if and only if $\underline{F}^H \underline{F}$ is diagonal, that is if $\underline{\phi}$ is diagonal. In this case, Eqn. 4.27 becomes

$$J = \prod_{i=1}^{n_T L} \left(1 + \frac{\alpha E_s}{4N_o} \lambda_{Tii} \phi_{ii}^2 \right), \quad (4.28)$$

where λ_{Tii} and ϕ_{ii} are the diagonal elements of $\underline{\underline{e}}_T$ and $\underline{\underline{\phi}}$ respectively. Now we look at the power distribution to find the optimum values of ϕ_{ii}^2 that maximizes J subjected to the power constraint $\text{Tr}\{\underline{\underline{W}}\underline{\underline{W}}^H\} = p_o$. Maximizing J is equivalent to maximizing its log, i.e. maximizing $\log_2(J) = \sum_{i=1}^{n_T L} \log_2 \left(1 + \frac{\alpha E_s}{4N_o} \lambda_{Tii} \phi_{ii}^2 \right)$ and the solution to which is the well know water-filling method on the eigenvalues of $\underline{\underline{k}}_T$ [53, 89]. Thus the optimum precoder that minimizes the average PEP transmits in the direction of the transmit antenna correlations at each subcarrier and pours power on the eigenmodes of $\underline{\underline{k}}_T$. As was shown [21], transmitting in the direction of the transmit antenna correlations at each subcarrier is also the optimum direction for capacity maximization. In this case however, the optimum power allocation is obtained through numerical optimization. Nonetheless, it was shown in [21] that waterfilling on the eigenmodes of the transmit correlations, although is suboptimal, leads to almost optimal performance.

As aforementioned, $\underline{\underline{v}}_{\Delta E}$ can be any arbitrary chosen unitary matrix, for example, as in [81, 76], we can choose $\underline{\underline{v}}_{\Delta E} = \underline{\underline{I}}$. However, we can consider our spreading matrices, which are orthonormal, as part of our precoder. That is we can write

$$\underline{\underline{W}} = (\underline{\underline{I}}_N \otimes \underline{\underline{w}}) \underline{\underline{U}}, \quad (4.29)$$

as in [77]. As was shown in Sec. 3.6, the spreading matrix, $\underline{\underline{U}}$, as well as the correlations affect the amount of interference in the channel. In addition, the BER was shown to also depend on the equalizer employed at the receiver. Similarly, in the case of precoded transmission, the chosen spreading matrix, which although does not affect the covariance of the transmitted symbols, is expected to affect the BER.

4.4 Uncoded Transmission

In this Section, we look at the BER performance with precoding. As with previous Chapters, we consider the MMSE-BLE, MMSE-BDFE, RNN, and SCE equalizers. In addition, we also assume a 4×4 MIMO channel, with $L = 4$, $N = 32$ and uncoded transmission using 4 PSK.

Figures 4.2 and 4.3 show the BER at $E_b/N_o = 14$ dB for different spreading matrices and the unspread MIMO-OFDM versus the correlation coefficient ρ , where the exponential correlation model is assumed, i.e. $c_{ij} = \rho^{j-i}$. The transmit and receive correlations are assumed equal. For transmission without precoding and at low to moderate correlation values, the lowest BER corresponds to the spreading matrix with highest diversity: MC-CAFS. As the

4 Precoding

correlations increase (higher interference), spreading with a lower diversity matrix MC-CDM (lower interference) leads to lower BER. As the correlations further increase, OFDM becomes the better choice. The crossing points of the BER curves for the different spreading matrices depend on the equalizer, the E_b/N_o and the correlation value, and shift to the right as E_b/N_o increases. In case of precoded transmission, all equalizers but the MMSE-BLE seem to benefit from precoding. In fact, the BER worsens for OFDM, CDM and since the matched filter bound (MFB) worsens through precoding. Only for MC-CAFS (MFB does not worsen) does the BER remain unchanged. In addition, for low E_b/N_o , the precoding matrix is not an orthonormal matrix. As a matter of fact, $\underline{W}^H \underline{W}$ is a diagonal matrix which tends to $\underline{I}$ as the E_b/N_o increases. Thus, the channel condition number worsens through precoding, which in turn affects the MMSE-BLE performance in a negative fashion. The MMSE-BDFE on the other hand benefits tremendously from precoding due to error propagation reduction, which is achieved by detecting symbols transmitted in the direction of the strong eigenmodes (λ_{max}) first. To show this precoding effect on reducing the propagation error, the BER for MMSE-BDFE, where the symbols transmitted in the direction of weak eigenmodes are detected first (λ_{min}), are shown in Fig. 4.4(a). In this case, the BER deteriorates and is even higher than in the case of transmission without precoding. Note that MC-CAFS is not affected by precoding, since spreading is done over all diversity branches and thus the order of detection is irrelevant in this case.

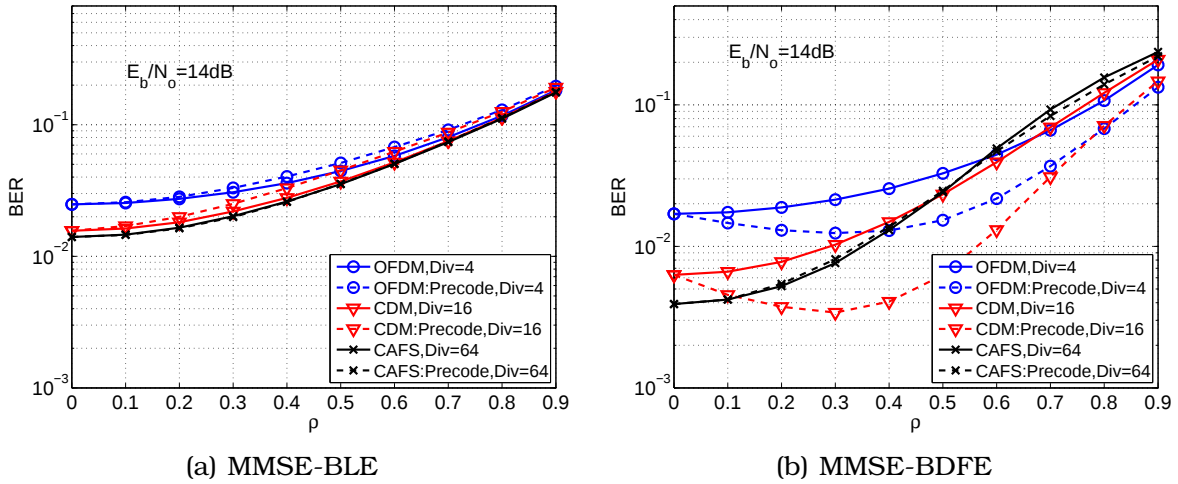


Figure 4.2: BER versus ρ for MIMO-OFDM, CDM, and MC-CAFS, exponential correlation model, $\rho_{Tx} = \rho_{Rx} = \rho$, $n_T = n_R = 4$, $L = 4$, $N = 32$ and $E_b/N_o = 14$ dB.

Figure 4.3 shows the BER versus ρ for the RNN and SCE. For MC-CDM and

4.4 Uncoded Transmission

OFDM, both equalizers show a improved BER performance through precoding. MC-CAFS also benefits from precoding, but only for the SCE. In case of the RNN, no improvement is observed. As mentioned in earlier Chapters, RNN is sensitive to high interference and does not always converge. This can be clearly seen in Fig. 4.3(a), where the BER using MC-CAFS can be seen to deteriorate fast as the antenna correlations increase. It is important to note here, that the RNN and SCE are less sensitive to propagation errors due to their iterative nature and BER reduction is always observed irrespective of the order of detection. However, lower BERs are still observed when symbols transmitted in the direction of the strong eigenmodes are detected first (see Figs. 4.4(b)). The BER reduction is probably due to the reduced total interference that is achieved through precoding and can be utilized by those two equalizers.

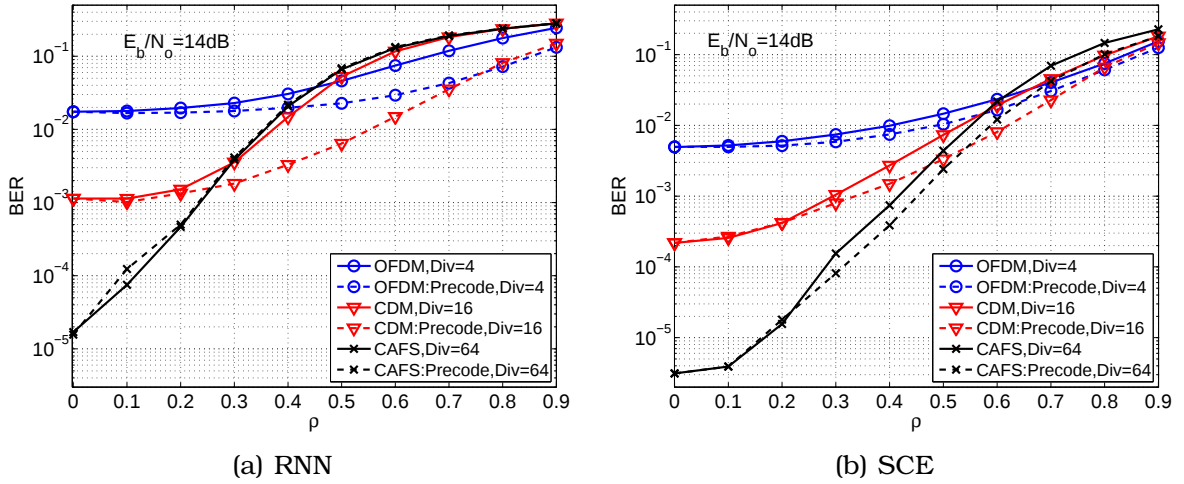


Figure 4.3: BER versus ρ for MIMO-OFDM, CDM, and MC-CAFS, exponential correlation model, $\rho_{Tx} = \rho_{Rx} = \rho$, $n_T = n_R = 4$, $L = 4$, $N = 32$ and $E_b/N_o = 14$ dB.

Now, we compare the BER performance assuming the constant correlation model to that of the complex exponential and exponential models. Figure 4.5 shows the BER for different spreading matrices versus ρ using the MMSE-BDFE for the constant and complex exponential correlation models. As expected, by comparing Figs. 4.5(a), 4.5(b) and 4.2(b), it is clear that the complex correlation model leads to worst BER for a given ρ . However, on considering the BER at different ρ values corresponding to the same correlation measure (see Fig. 2.14), then the BER for all correlation models are quite comparable. The correlation measure therefore seems to be a good indicator of the expected BER.

4 Precoding

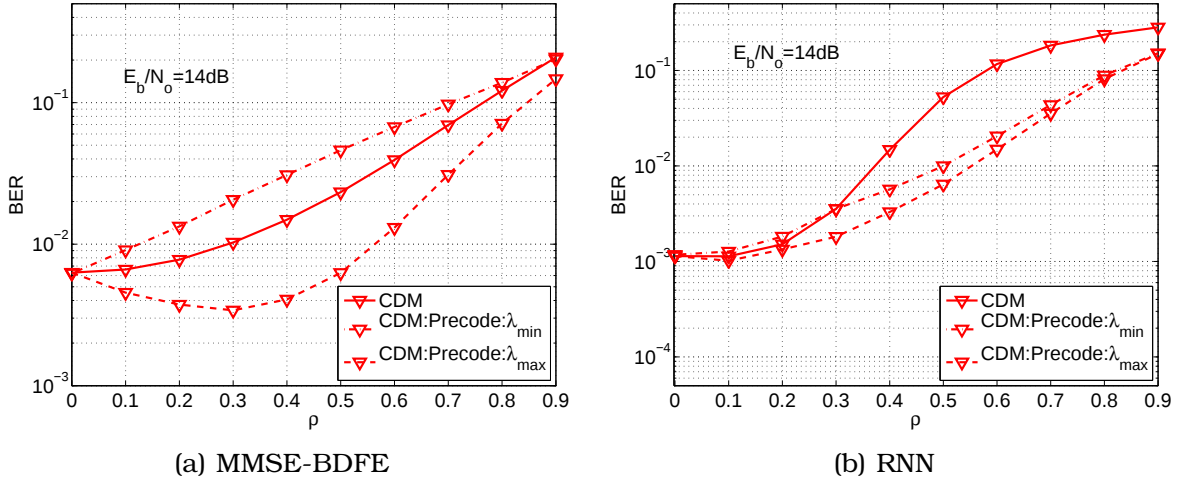


Figure 4.4: Effect of detection order: exponential correlation model, BER versus ρ for CDM, $n_T = n_R = 4$, $L = 4$, $N = 32$ and $E_b/N_o = 14$ dB.

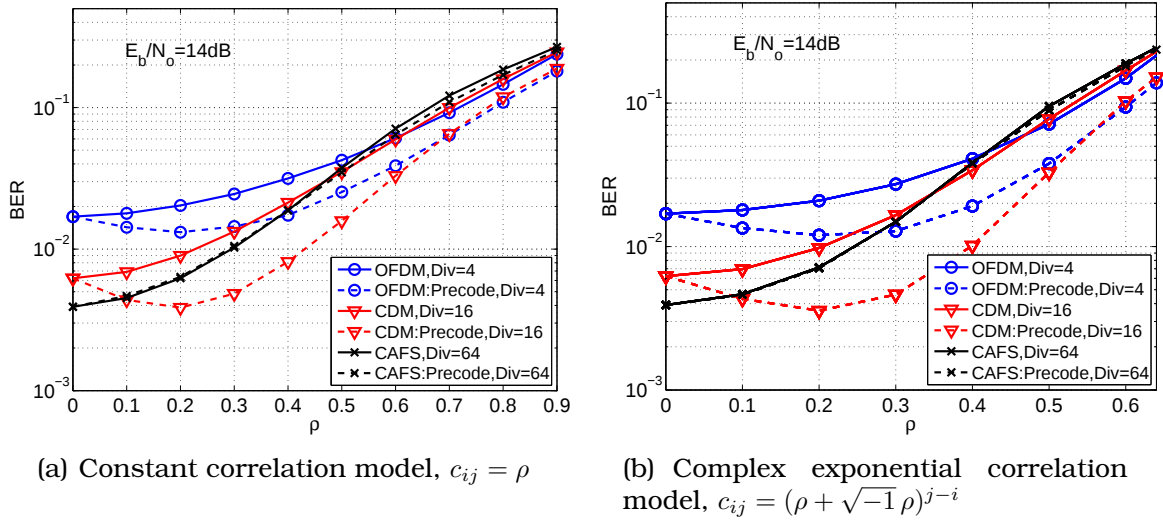


Figure 4.5: MMSE-BDFE: BER for constant and complex exponential correlation models, for $n_T = n_R = 4$, $L = 4$, $N = 32$ at $E_b/N_o = 14$ dB.

4.4.1 Imperfect Transmit Correlation Knowledge

So far we have assumed perfect knowledge of the transmit correlations at the transmitter. Now, we look at the case of imperfect knowledge. To simulate the imperfect knowledge of transmit antenna correlations, we perturb the channel matrix $\underline{\underline{H}}$ and use the resulting perturbed $\underline{\underline{H}}_p$ to calculate the channel

4.4 Uncoded Transmission

correlations [90]. We have opted for this method, since directly perturbing $\underline{k}_T$ does not guarantee that they remain positive semi-definite. Accordingly, we model the perturbed channels as follows,

$$\underline{H}_{kp} = \underline{H}_k^c + \underline{E}_k \quad (4.30)$$

where $\underline{H}_{kp}$ and $\underline{E}_k$ are the perturbed channel and the perturbation matrix at the k -th frequency respectively. The elements of $\underline{E}_k$ are modeled as uncorrelated $CN(0, \sigma_e^2)$. Note that the elements of $\underline{E}_k$ need not be uncorrelated. $\underline{E}_k$ can be assumed to have covariance matrix different from a scaled identity matrix. By assuming uncorrelated elements, we are investigating the case where the transmitter assumes lower correlations than the true ones, especially as $\sigma_e^2 \rightarrow 1.0$. If we would assume $\mathcal{E}\{\underline{E}_k^H \underline{E}_k\} = \sigma_e^2 \underline{1}_{n_T}$ (an $n_T \times n_T$ matrix of ones) then we would be looking at the other extreme, where the transmitter assumes higher correlations than the true ones. We assume the perturbations to be independent of the channel coefficients. The channel coefficients are normalized such that $\sigma_e^2 + \sigma_h^2 = 1.0$, where σ_h^2 is the variance of the elements of $\underline{H}_k^c$. Accordingly, the transmit antenna correlations at each frequency are obtained as follows,

$$\underline{k}_{Te} = \frac{1}{n_R} \mathcal{E}\{\underline{H}_{kp}^H \underline{H}_{kp}\} \quad (4.31)$$

Based on the above assumptions,

$$\underline{k}_{Te} = \sigma_h^2 \underline{k}_T + \sigma_e^2 \underline{I}_{n_T} \quad (4.32)$$

Thus, $\underline{k}_{Te} = \underline{k}_T$ only if $\sigma_e^2 = 0$. However, it is not important for $\underline{k}_{Te}$ be exactly or almost equal to $\underline{k}_T$, but that the eigenvectors of $\underline{k}_{Te}$ diagonalize $\underline{k}_T$ (i.e. $\underline{v}_{Te} \approx \underline{v}_T$, where $\underline{v}_{Te}$ are the eigenvectors of $\underline{k}_{Te}$). As a measure for that, we introduce the following norm, F_{vv} ,

$$F_{vv} = \sqrt{\|\underline{v}_{Te}^H \underline{k}_T \underline{v}_{Te} - \underline{e}_T\|_F^2}. \quad (4.33)$$

If $\underline{v}_{Te} = \underline{v}_T$, then $F_{vv} = 0$. The worst case for this model would be to estimate uncorrelated transmit antennas while the antennas at the transmitter are fully correlated and thus transmitting in direction of eigenmodes with zero eigenvalues. $\sigma_e^2 = 1$ corresponds to this worst case. For $\underline{k}_T = \underline{1}_{n_T}$, and $\underline{k}_{Te} = \underline{I}_{n_T}$ ($\underline{v}_{Te} = \underline{I}_{n_T}$),

$$F_{vv} = \sqrt{2n_T(n_T - 1)} = F_{max}.$$

Figure 4.6 show F_{vv}/F_{max} for 100 and 1000 channel realizations used for the estimation of $\underline{k}_T$. The exponential correlation model is assumed. The figures clearly show that for low σ_e^2 values, F_{vv} remains low. As expected, the higher

4 Precoding

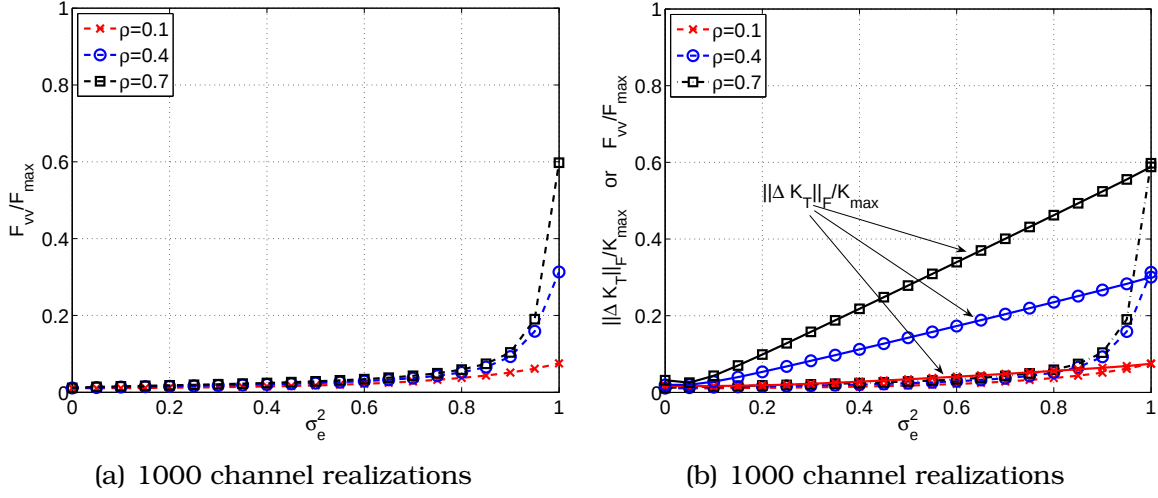


Figure 4.6: Normalized F_{vv} and $\|\Delta K_T\|_F$ for 1000 channel realizations.

σ_e^2 , the worse F_{vv} gets, since $\underline{k}_{T_e} \rightarrow \underline{I}_{n_T}$ as $\sigma_e^2 \rightarrow 1.0$ even though the actual antenna correlations may be high.

In order to see how much faster the difference between $\underline{k}_T$ and $\underline{k}_{T_e}$ increases with the increase of σ_e^2 compared to F_{vv} , we define the following norm,

$$\|\Delta K_T\|_F = \sqrt{\|\underline{k}_{T_e} - \underline{k}_T\|_F^2} = \sqrt{\|\sigma_e^2(\underline{k}_T - \underline{I}_{n_T})\|_F^2}. \quad (4.34)$$

The norm $\|\Delta K_T\|_F$, also normalized by the worse case norm value $K_{\max} = \sqrt{n_T(n_T - 1)}$, is shown in Fig. 4.6(b) (solid lines). As is clear from the figure, $\|\Delta K_T\|_F$ grows faster than F_{vv} , which indicates that the estimates $\underline{k}_{T_e}$ worsen with σ_e^2 faster than its eigenvectors. That is, the change in the eigenvectors is slower than the change in the eigenvalues. It is important to mention at this point, that the power allocation is still done based on the eigenvalues of $\underline{k}_{T_e}$ and the more even the eigenvalues are ($\sigma_e^2 \rightarrow 1$), the more uniform is the power allocation. However, it was shown through simulation results for uncoded transmission that $\underline{\Lambda}$ (power allocation matrix $\underline{\Lambda} = \underline{I}_{n_T} \otimes \underline{\Lambda}$) does not seem to affect the BER, especially at high E_b/N_o ($\underline{\Lambda} \rightarrow \underline{I}$), where the effect of precoding was shown to be most significant. Note that this is not true if $\underline{k}_{T_e}$ is rank deficient. In this case, transmitting in direction of eigenmodes with zero eigenvalues only worsens the BER performance. To isolate the effect of imperfect knowledge of $\underline{k}_T$ at the transmitter, perfect channel knowledge is still assumed at the receiver. Power allocation in case of coded transmission however, as we shall see Sec. 4.5, sometimes plays an important role in the BER reduction.

4.4 Uncoded Transmission

Figure 4.7 compares the BER versus E_b/N_o for MIMO-OFDM and CDM assuming both perfect and imperfect knowledge of $\underline{k}_T$ and for different power allocation strategies. Power allocation is done either through waterfilling on the eigenvalues of $\underline{k}_T$ or $\underline{k}_{Te}$, or is assumed uniform for all E_b/N_o (i. e. $\underline{\Lambda} = \underline{I} \forall E_b/N_o$). Zero receive correlations and $\rho_{Tx} = 0.7$ are assumed. Precoding using either $\underline{k}_T$ or $\underline{k}_{Te}$ leads to almost identical BER performance. The power allocation strategy also does not seem to affect the BER performance. All BER curves for the precoded OFDM or CDM, irrespective of the power allocation strategy, almost coincide.

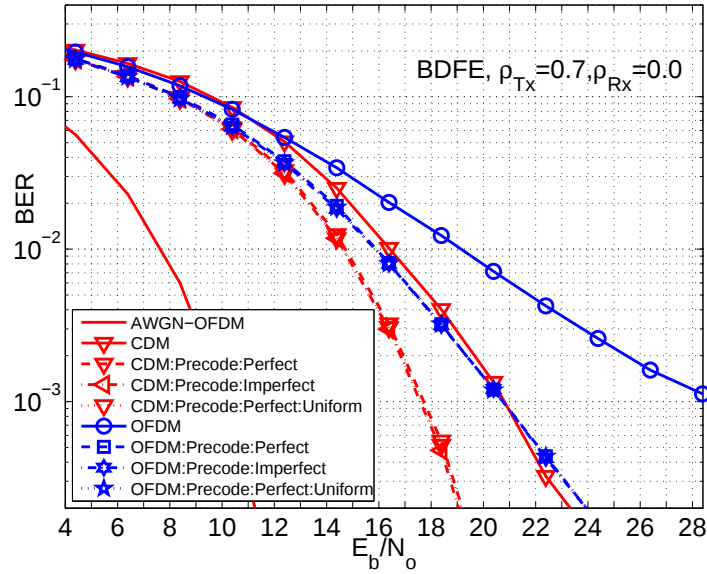


Figure 4.7: MMSE-BDFE: BER versus E_b/N_o , imperfect $\underline{k}_T$, $\sigma_e^2 = 0.5$ for $n_T = n_R = 4$, $L = 4$ and $N = 32$.

4.4.2 Different Antenna Correlation at each Channel Tap

So far, we have assumed that all the transmit correlations at all taps are the same. Now, we investigate the case when they are not. That is, if $\underline{k}_T$ and $\underline{k}_R$ do not satisfy Eqn. 2.42, the antenna correlation matrices in the frequency domain are calculated at each frequency as follows,

$$\underline{k}_k = \frac{1}{n_R} \mathcal{E} \{ \underline{H}_k^{cH} \underline{H}_k^c \}, \quad (4.35)$$

and are used for precoding at the corresponding frequency. In Appendix 4.A, we show that, if the antenna correlations are not the same for all taps, the

4 Precoding

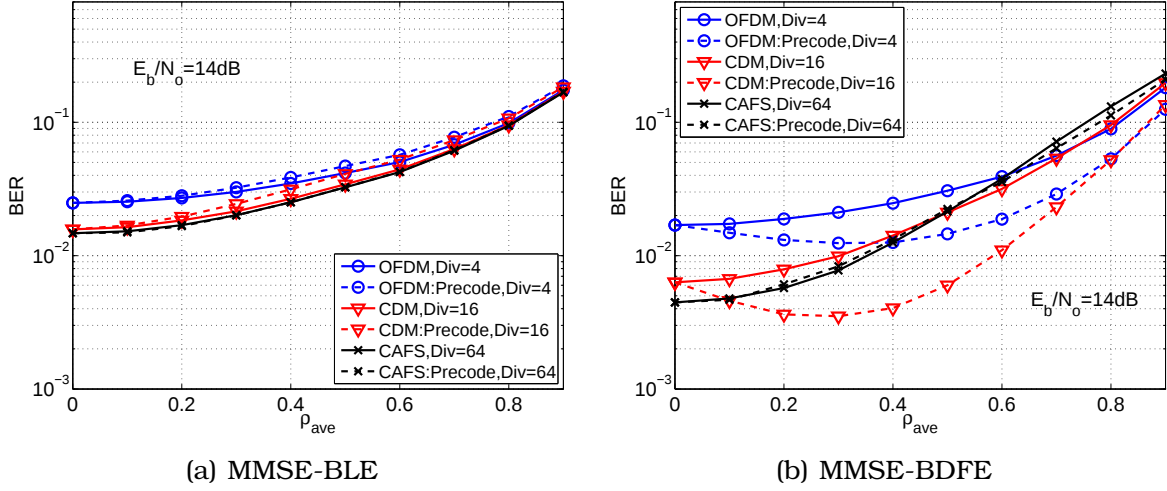


Figure 4.8: BER for Precoding using $\underline{K}_{Tave}$ at $E_b/N_o = 14$ dB, $\underline{k}_T(l)$ follow the exponential correlation model, $n_T = n_R = 4$, $L = 4$, $N = 32$.

resulting correlations in the frequency domain, assuming uncorrelated taps, is the weight average of those correlations at each tap. Thus, assuming equal PDP (i. e. $\sigma_l^2 = 1/L$),

$$\underline{k}_k = \underline{k}_{Tave} = \frac{1}{L} \sum_{l=1}^L \underline{k}_T(l), \quad (4.36)$$

for all frequencies, $k = 1, \dots, N$. Thus,

$$\underline{K}_{Tave} = \underline{I}_N \otimes \underline{k}_{Tave}. \quad (4.37)$$

Since the sum of positive definite matrices is positive definite, $\underline{k}_{Rave}$ is thus positive definite. The above also applies to the receive correlations. That is,

$$\underline{K}_{Rave} = \underline{I}_N \otimes \underline{k}_{Rave}, \quad (4.38)$$

where again $\underline{k}_{Rave} = \frac{1}{L} \sum_{l=1}^L \underline{k}_R(l)$. For nonequal PDP, the $\underline{k}_T(l)$ should be scaled by its corresponding σ_l^2 (see Appendix 4.A). It is important to not here, that unlike the case where $\underline{k}(l) = \underline{k} \quad \forall l$, $\underline{H}_B^c$ is not equal to $\underline{K}_{Rave}^{1/2} \underline{H}_B \underline{K}_{Tave}^{1/2}$. $\underline{H}_B^c$ has to be calculated by replacing $\underline{h}_{i_R i_T}$ by $\underline{h}_{i_R i_T}^c$ in Eqn. 2.24. The transmit correlation matrix of Eqn. 4.37 is used only for precoding. In addition, even if the individual $\underline{k}_T(l)$ may for example follow the exponential model, the resulting $\underline{k}_{Tave}$ in general may have a different structure.

Figures 4.8 show the BER for MMSE-BLE and MMSE-BDFE versus ρ_{ave} , where $\rho_{ave} = 1/L \sum_{l=1}^L \rho_l$, $\rho_l \leq 1.0 \quad \forall l$ and the individual tap correlations were assumed to follow the exponential correlation model. Comparing those figures, with Figs. 4.2, it is clear that the performance is quite comparable.

4.5 Coded Transmission

In this Section, we first consider coded transmission over time varying channels. We assume a block fading model where the channel remains constant during the transmission of one codeword. Again, we look at a 4×4 MIMO channel, with $L = 4$ and transmission using 4PSK. At the receiver, iterative equalization and decoding is performed using either the RNN or SCE as explained in Chapter 2. We consider the convolutional code with generator polynomial $[7, 5]_8$ with memory 2 and rate $1/2$. Note that in all what follows, the BER is always plotted versus the transmit E_b/N_o . That way, the effect of transmit power distribution can be captured.

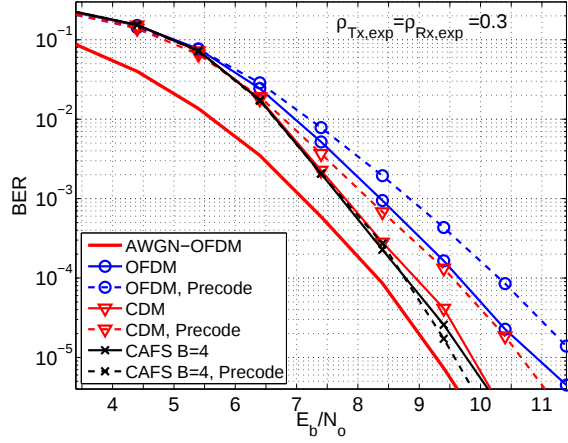
Figure 4.9 shows the BER versus the transmit E_b/N_o using the RNN and SCE equalizers the assuming exponential correlation model. We consider three cases: $\rho_{Tx} = \rho_{Rx} = 0.3$, $\rho_{Tx} = 0.7$, $\rho_{Rx} = 0.0$ and $\rho_{Tx} = 0.7$, $\rho_{Rx} = 0.7$. As expected, the BER deteriorates with increasing correlations. The RNN and SCE have a very comparable performance for low correlation values (Figs. 4.9(a) and 4.9(b)). Both benefit from precoding only when MC-CAFS spreading is applied. Precoding with OFDM and MC-CDM leads to worse BERs than without.

With increased correlations ($\rho_{Tx} = 0.7$), the SCE continues to benefit from precoding when MC-CAFS spreading is employed. In case of OFDM and MC-CDM, precoding only improves the performance at low to moderate E_b/N_o values. The range of E_b/N_o values where this improvement is observed increases as the amount of correlation present in the channel increases (compare Figs. 4.9(b) 4.9(d) and 4.9(f)). However, at high E_b/N_o , precoding with OFDM and MC-CDM only worsens the BER. The RNN on the other hand, exhibits a totally different behavior. Precoding with MC-CDM spreading seems now to improve the BER. When correlations are high at both the transmitter and receiver ($\rho_{Tx} = \rho_{Rx} = 0.7$), precoding with OFDM also leads to improved BER performance.

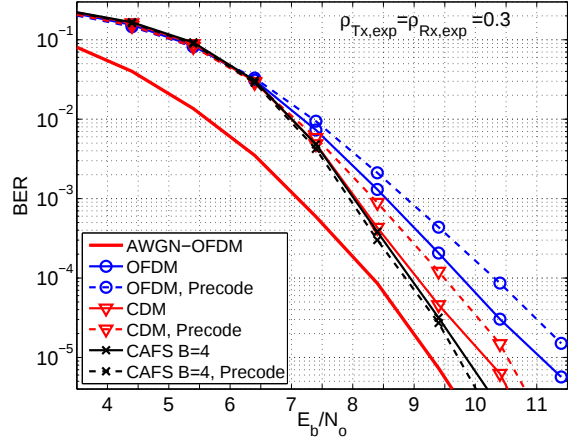
To better understand the behavior of those equalizers as part of the iterative equalization and decoding loop, we shall now consider the EXIT charts as an analysis tool. However, in order for the EXIT charts to make sense, we have to consider one channel at a time, and look at the EXIT chart for each channel realization. Due to the computational complexity and volume involved, we instead look at three MIMO 4×4 channel groups:

- Group 1 with four channels having low correlations at both the transmitter and receiver $\rho_{Tx} = \rho_{Rx} = 0.3$.
- Group 2 with four channels having high correlations at the transmitter and zero receive correlations $\rho_{Tx} = 0.7$, $\rho_{Rx} = 0.0$.
- Group 3 also with four channels having high correlations at both the trans-

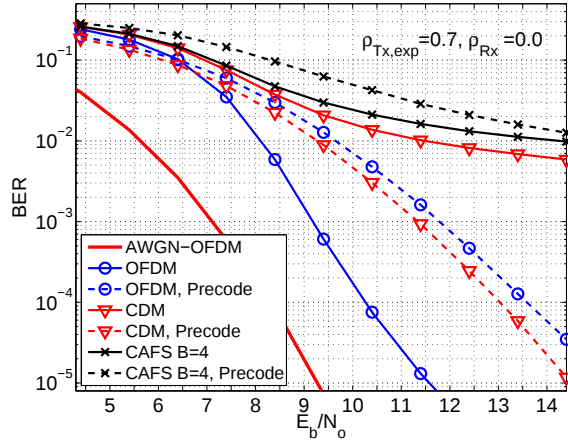
4 Precoding



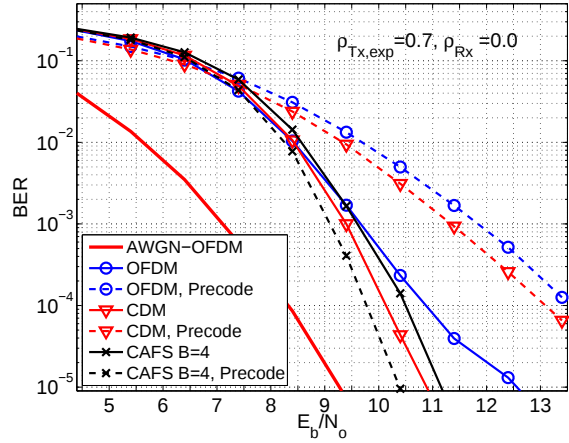
(a) RNN: $\rho_{T_x} = \rho_{R_x} = 0.3$



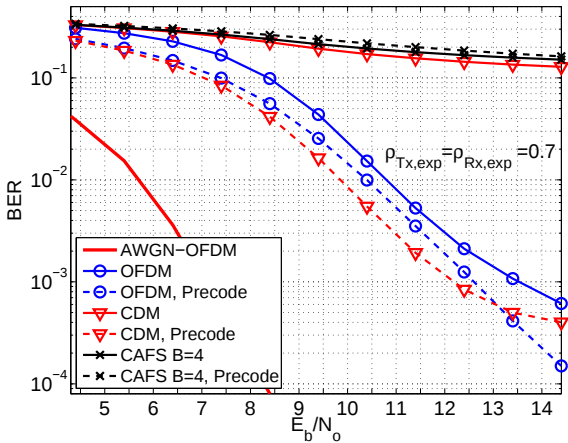
(b) SCE: $\rho_{T_x} = \rho_{R_x} = 0.3$



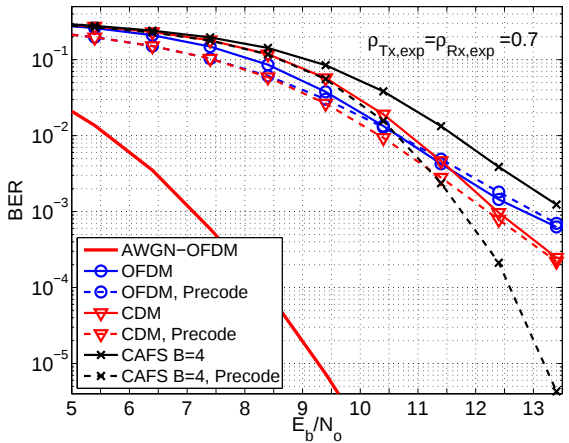
(c) RNN: $\rho_{T_x} = 0.7, \rho_{R_x} = 0.0$



(d) SCE: $\rho_{T_x} = 0.7, \rho_{R_x} = 0.0$



(e) RNN: $\rho_{T_x} = \rho_{R_x} = 0.7$



(f) SCE: $\rho_{T_x} = \rho_{R_x} = 0.7$

Figure 4.9: Time varying channels: BER curves for RNN and SCE for different correlation values, $n_T = n_R = 4$, $L = 4$, $N = 32$ and with a convolutional code with rate $= 1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

4.5 Coded Transmission

mitter and receiver $\rho_{Tx} = 0.7$, $\rho_{Rx} = 0.7$.

Again, the exponential correlation model is assumed for all groups. The four channels in each group were chosen from a sample containing 10000 randomly generated channels, with the statistics given in Table 4.1, where μ stands for the mean and σ for the standard deviation. Each of the chosen

LOW CORRELATION SAMPLE: $\rho_{Tx} = 0.3$, $\rho_{Rx} = 0.3$					
Channel Characteristics	min	max	μ	σ	Median
Condition number	63	148.5×10^6	27829	787893	3513
CN without outer 10%	836	25317	5536	5520	3513
$\beta_{1/2}$	0.91	2.05	1.45	0.13	1.45
HIGH CORRELATION SAMPLE: $\rho_{Tx} = 0.7$, $\rho_{Rx} = 0.0$					
Channel Characteristics	min	max	μ	σ	Median
Condition number	187	1194×10^6	83141	5453708	8130
CN without outer 10%	1857	57900	12740	12013	8130
$\beta_{1/2}$	1.2	2.55	1.96	0.18	1.96
HIGH CORRELATION SAMPLE: $\rho_{Tx} = 0.7$, $\rho_{Rx} = 0.7$					
Channel Characteristics	min	max	μ	σ	Median
Condition number	352	850×10^6	229815	6106479	24068
CN without outer 10%	5084	182155	38240	37149	24068
$\beta_{1/2}$	1.23	2.74	2.14	0.21	2.15

Table 4.1: Channel characteristics for different correlation values.

channels is characterized by its condition number, $\max\{|r_{ij}|\}$ and $\beta_{1/2}$ (see Chapter 3). The chosen channels and their characteristic values are given in Table 4.2.

The digits in the channel names correspond to the correlations values, with the first two digits representing the transmit correlations and the last two the receive correlations. The AXXXX channels are channels with condition numbers close to the minimum of the corresponding sample (excluding the outer 10% channels), while the BXXXX channels are those with condition numbers close to the maximum, again excluding the outer 10%. The CXXXX and DXXXX are channels whose condition number are close to that of the median of the sample. The difference being only in the amount of interference, with CXXXX having low interference ($\beta_{1/2} = \mu - 2\sigma$) and the DXXXX channels having high interference ($\beta_{1/2} = \mu + 2\sigma$). In this Section, we only look at the C and D 0700 and 0707 channels from the above table. The BER curves and

4 Precoding

Name	ρ_{Tx}	ρ_{Tx}	Condition Number	$\beta_{1/2}$	$\max\{ r_{ij} \}$
A0303	0.3	0.3	800	1.47	1.04
B0303	0.3	0.3	25016	1.43	1.12
C0303	0.3	0.3	4302	1.19	1.17
D0303	0.3	0.3	4302	1.71	1.72
A0700	0.7	0.0	2000	1.97	1.96
B0700	0.7	0.0	60495	1.95	1.22
C0700	0.7	0.0	10006	1.59	1.63
D0700	0.7	0.0	10007	2.32	1.98
A0707	0.7	0.7	5000	2.12	2.26
B0707	0.7	0.7	180191	2.18	2.64
C0707	0.7	0.7	29738	1.72	2.15
D0707	0.7	0.7	29740	2.57	3.50

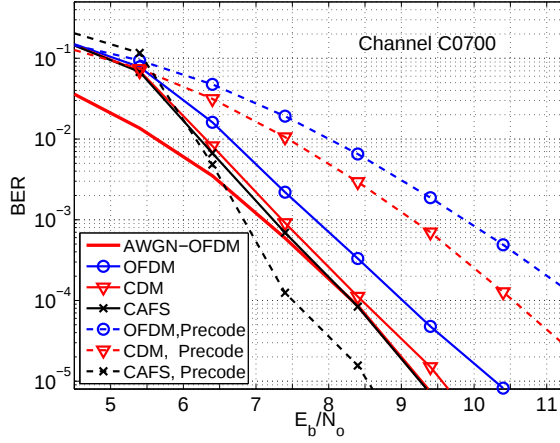
Table 4.2: Chosen channels and their characteristic values.

EXIT-Charts for all other channels can be found in Appendix 4.A.2.

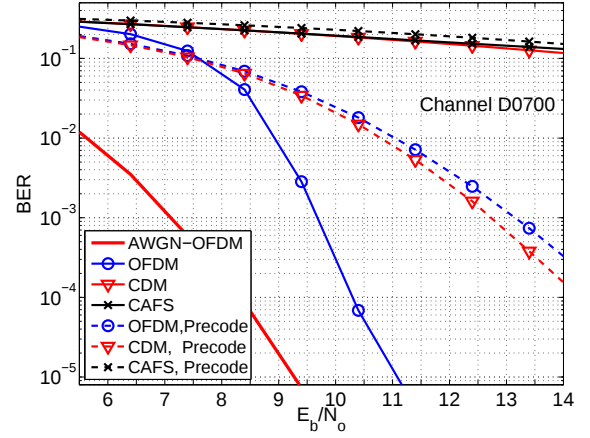
Figures 4.10 and 4.11 show the BER as well as the EXIT charts (after 2 iterations) for the C0700 and D0700 channels using the RNN and the SCE respectively. Although both channels have the same condition number, the D0700 has a higher amount of interference (a higher β and $\max\{|r_{ij}|\}$ compared to the C0700 channel). As can be seen from the BER curves, the performance of both channels are quite different, with D0700 BER curves being noticeably worse. In case of the C0700 channel, irrespective of the equalizer used, precoding always leads to worse BER than without precoding, except for MC-CAFS. In this case, the BER performance even outperforms the AWGN BER curves due to the better power distribution done by the precoder. This effect can be clearly captured here since, as aforementioned, the BER is plotted versus the the transmit E_b/N_o . For the D0700 and RNN on the other hand, only the OFDM BER curve falls with increased E_b/N_o . Precoding worsens the OFDM BER at high E_b/N_o . The BER performance of MC-CAFS with and without precoding as well as MC-CDM remains very poor even at high E_b/N_o . For SCE however, MC-CAFS with and without precoding and MC-CDM lead to the lowest BER rates as can be seen in Fig.4.11(b). The BER results therefore, although are worse than those of the C0700 channel, still exhibit the same behavior.

By looking at the EXIT-charts, it is clear that the tunnels between all spreading and precoding curves and that of the decoder are more widely open in case of C0700 channel compared to those of the D0700, which means that the waterfall region occurs at lower E_b/N_o values and convergence is faster. These observations agree with the BER curves of the corresponding figures.

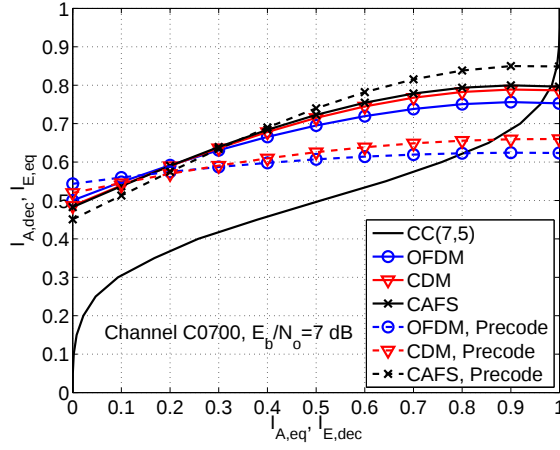
4.5 Coded Transmission



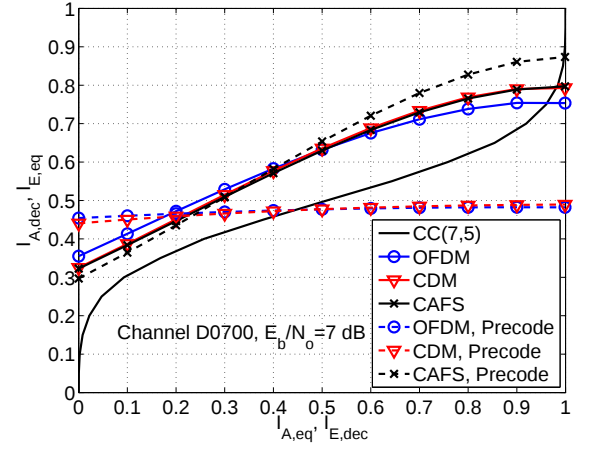
(a) BER curves for Channel C0700



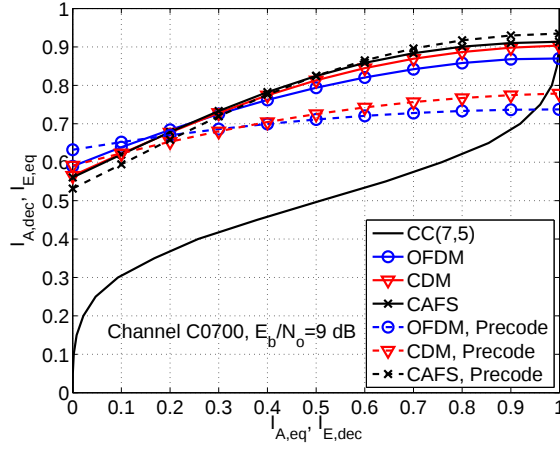
(b) BER curves for Channel D0700



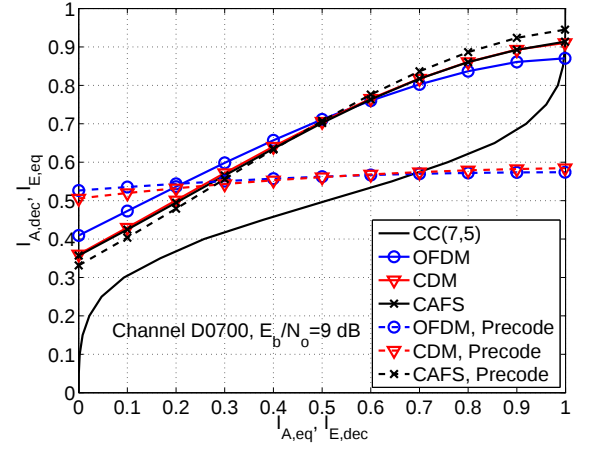
(c) EXIT-Chart: Channel C0700 at 7 dB



(d) EXIT-Chart: Channel D0700 at 7 dB



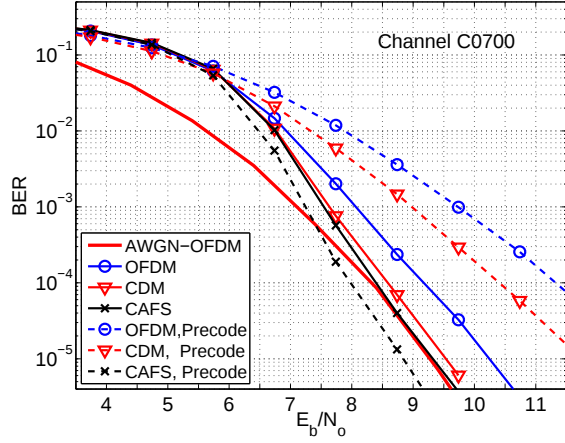
(e) EXIT-Chart: Channel C0700 at 9 dB



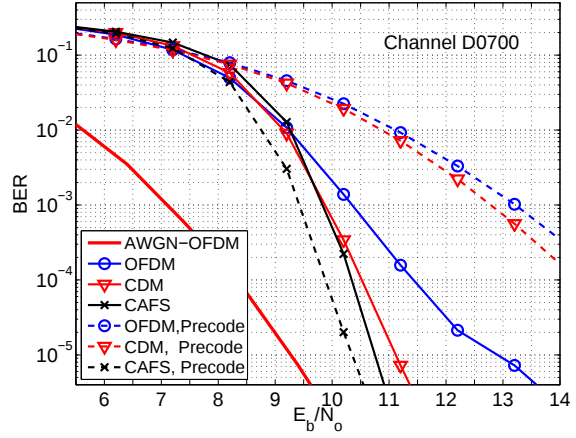
(f) EXIT-Chart: Channel D0700 at 9 dB

Figure 4.10: RNN: BER curves and corresponding EXIT charts for channels C0700 and D0700, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

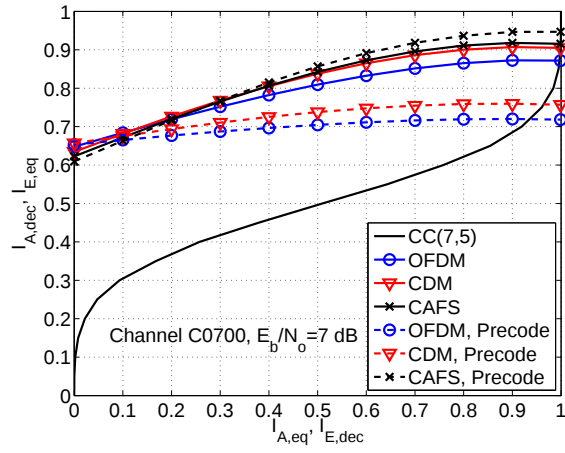
4 Precoding



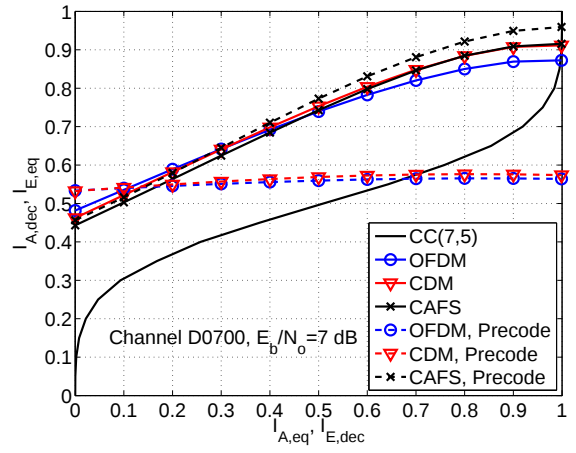
(a) BER curves for Channel C0700



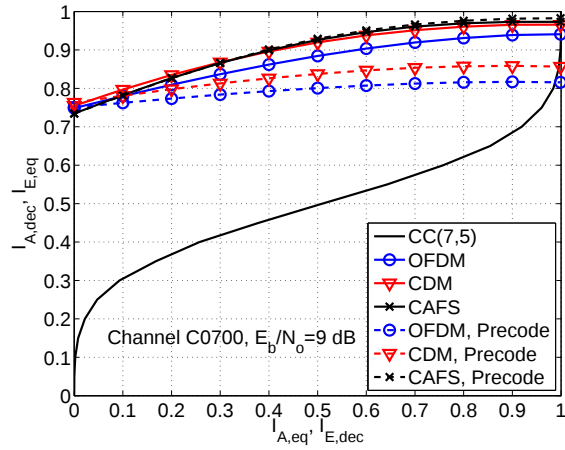
(b) BER curves for Channel D0700



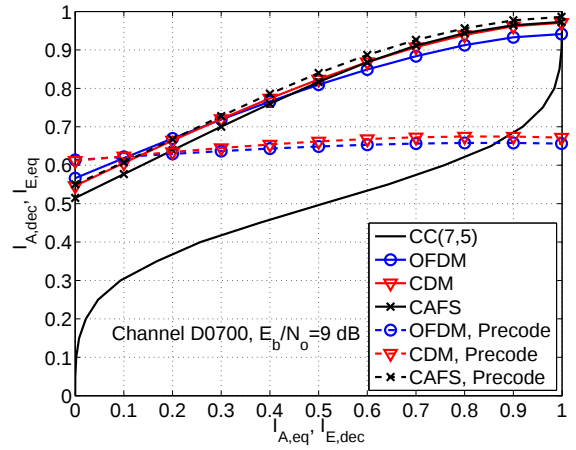
(c) EXIT-Chart: Channel C0700 at 7 dB



(d) EXIT-Chart: Channel D0700 at 7 dB



(e) EXIT-Chart: Channel C0700 at 9 dB



(f) EXIT-Chart: Channel D0700 at 9 dB

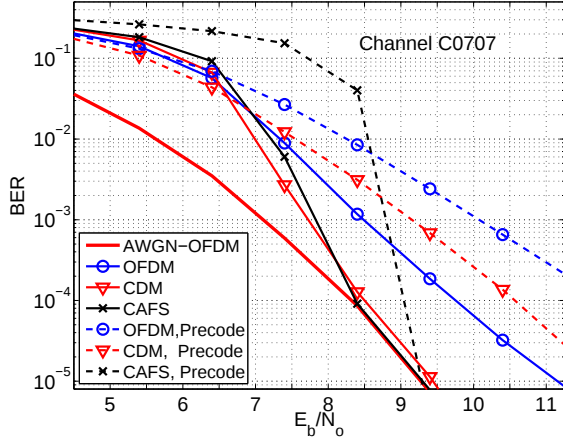
Figure 4.11: SCE: BER curves and corresponding EXIT charts for channels C0700 and D0700, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

We can also see that the starting point ($I_{A,dec}$ at $I_{E,dec} = 0$) for OFDM and CDM for the precoded case are higher than those without precoding. As mentioned in Chapter 2, this initial point is dependent on the amount of interference in the channel, being higher for less interference. This is especially important when the tunnel opening is small i. e. at low E_b/N_o . Since precoding reduces the amount of total interference, the starting point shifts upwards, which explains why precoding with OFDM and MC-CDM improves the BER at low E_b/N_o . On the other hand, the intersection points between those curves and that of the decoder become more important as E_b/N_o increases (wider tunnel). This intersection point is dependent on the MFB, and shifts to the right as the MFB improves. Since the hard decisions are made after the decoder, a higher $I_{E,dec}$ at the intersection point means in general better BER. This explains why the OFDM and MC-CDM do not converge to the AWGN performance as the E_b/N_o increases. The starting points for all EXIT-chart curves in case of SCE are higher than for those of the RNN at the same E_b/N_o . In addition, the SCE EXIT-charts tunnel openings are wider than those of the RNN. Both factors thus explain the better BER performance and the faster convergence using the SCE.

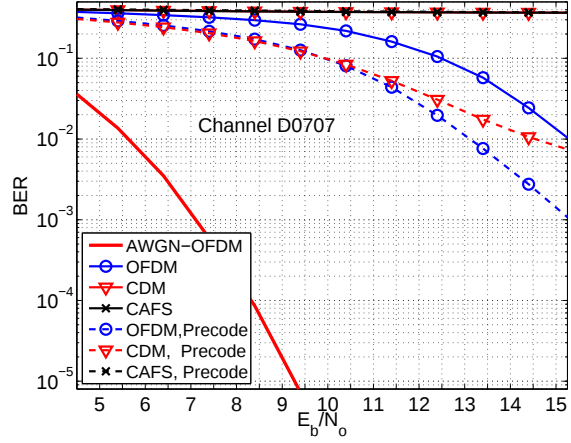
Figures 4.12 and 4.13 show the BER curves and corresponding EXIT-charts for the C0707 and D0707 channels again using the RNN and SCE respectively. Due to the presence of the receive correlations, the amount of interference increases compared to the C0700 and D0700 channels (see Chapter 3). Due to increased interference values, both the RNN and SCE BER curves converge slower and the waterfall region starts at later E_b/N_o values compared to the 0700 channels. For the D0707, the BER remain very high even at $E_b/N_o = 11$ dB for both equalizers. In case of the RNN, and as can be seen in the EXIT-charts in Figs. 4.12(d) and 4.12(f), that even at 13 dB, the curves of the MC-CDM, MC-CAFS with and without precoding lie below that of the decoder. That is, the tunnel is closed and no exchange of information is possible between the RNN and the decoder. As a result, the BER remains very high (almost 0.4). A tunnel opening exists for OFDM with and without precoding and MC-CDM with precoding. As can be seen in Fig. 4.12(f), although the intersection point of OFDM without precoding with the decoder curve occurs at higher $I_{E,dec}$ value than for OFDM with precoding, the BER is worse. This can be explain by looking at the $I_{A,dec}$ value at $I_{E,dec} = 0$, which is lower in case of OFDM without precoding than for OFDM with precoding. The starting point seems to be a more important factor even at this E_b/N_o due to the high amount of interference. All of the above can be applied to the SCE, except that the tunnels are already open at much lower E_b/N_o values compared to the RNN equalizer.

In case of the C0707 channel, the tunnels for all spreading and precoding curves are open and convergence can be seen to occur at much lower

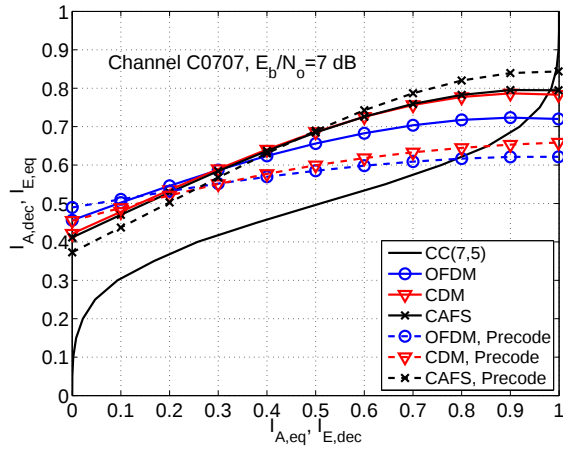
4 Precoding



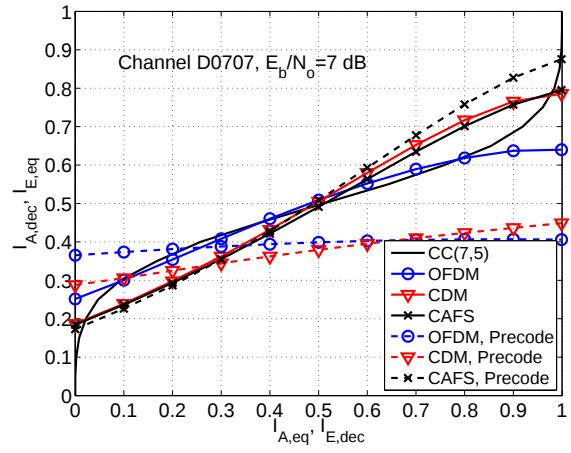
(a) BER curves for Channel C0707



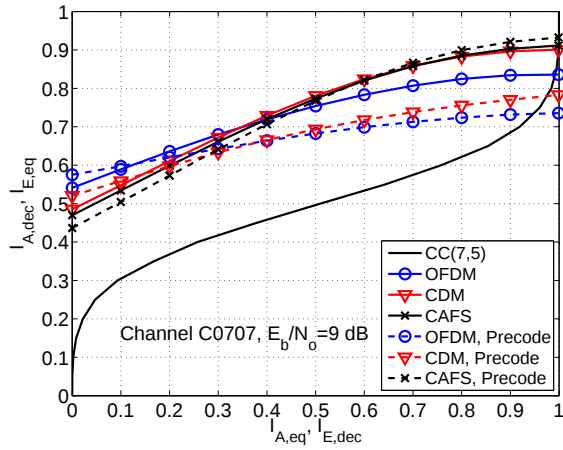
(b) BER curves for Channel D0707



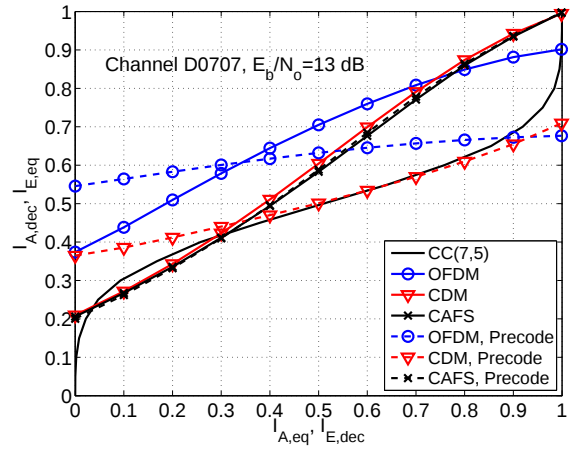
(c) EXIT-Chart: Channel C0707 at 7 dB



(d) EXIT-Chart: Channel D0707 at 7 dB



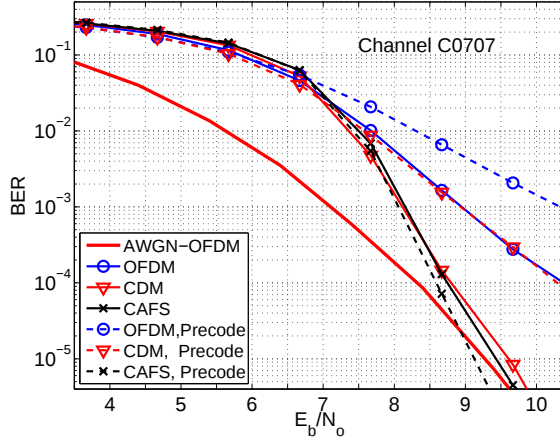
(e) EXIT-Chart: Channel C0700 at 9 dB



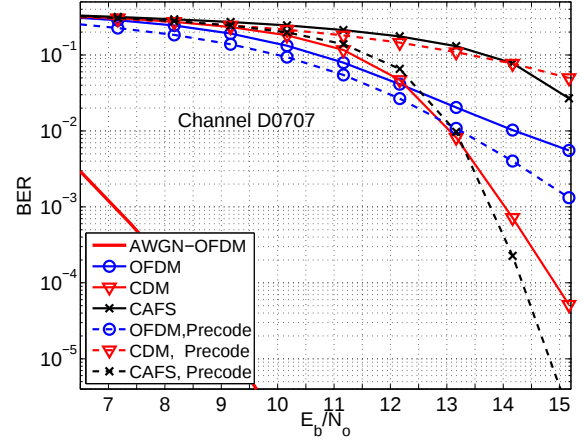
(f) EXIT-Chart: Channel D0707 at 13 dB

Figure 4.12: RNN: BER curves and corresponding EXIT charts for channels C0707 and D0707, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

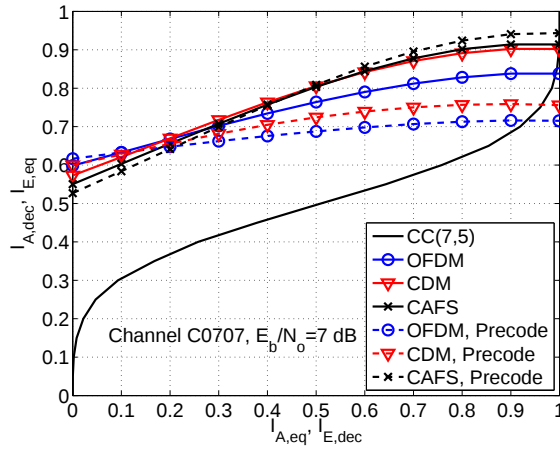
4.5 Coded Transmission



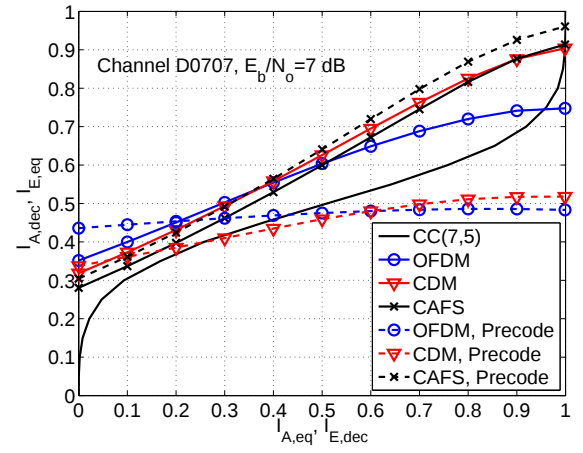
(a) BER curves for Channel C0707



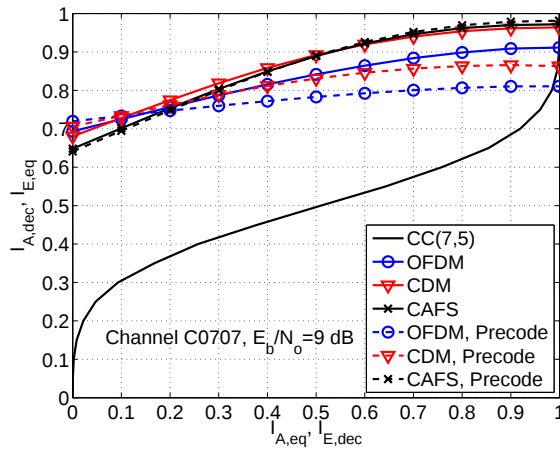
(b) BER curves for Channel D0707



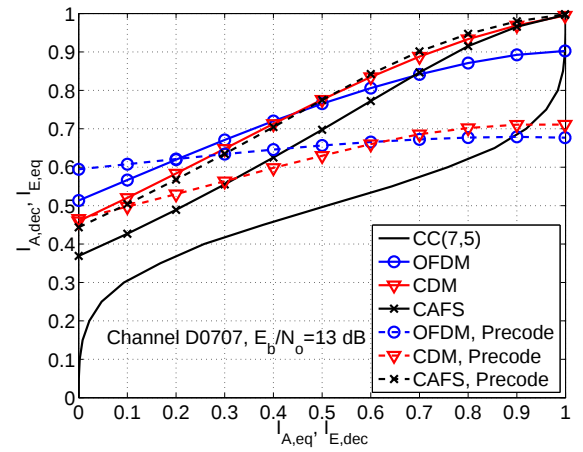
(c) EXIT-Chart: Channel C0707 at 7 dB



(d) EXIT-Chart: Channel D0707 at 7 dB



(e) EXIT-Chart: Channel C0700 at 9 dB



(f) EXIT-Chart: Channel D0707 at 13 dB

Figure 4.13: SCE: BER curves and corresponding EXIT charts for channels C0707 and D0707, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

4 Precoding

E_b/N_o values. The BER for MC-CDM, MC-CAFS with and without precoding all converge to the AWGN performance at high E_b/N_o for both equalizers. In case of the SCE the BER for MC-CAFS with precoding outperforms the AWGN channel. Again, this is due the better power distribution obtained through waterfilling. For all other cases (OFDM with and with precoding and MC-CDM with precoding), the BER is lower the higher the $I_{E,dec}$ value at the intersection point between the decoder curve and the corresponding RNN/SCE curves in the EXIT-charts. From the above and by looking at the BER curves and EXIT-charts in Appendix 4.A.2, we can deduce that the amount of interference seems to be the most important factor affecting the BER. The larger the interference, irrespective of the condition number, the worse the BER and the slower the convergence (waterfall region occurs at higher E_b/N_o values). Using the SCE, MC-CAFS spreading with precoding was shown to outperform all others, even the AWGN in some cases. Same results are observed with the RNN when the amount of interference was not high (e. g. CXXXX and X0303 channels).

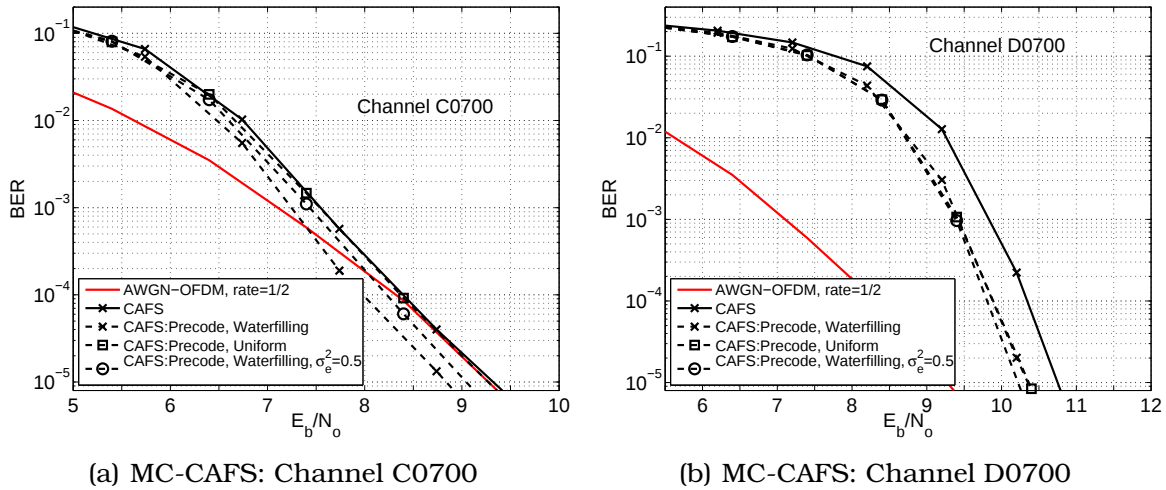


Figure 4.14: SCE: Effect of power allocation strategy on the BER, convolutional code, rate = 1/2, $m = 2$, generator polynomial $[7, 5]_8$.

Now we take a closer look at the effect of the power allocation strategy on the BER. We again consider channels C0700 and D0700. Figure 4.14 shows the BER with precoding using different power distribution techniques: waterfilling on eigenvalues of $\underline{k}_{\underline{T}}$, uniform power distribution and waterfilling over the eigenvalues of $\underline{k}_{\underline{T}_e}$ with estimation errors $\sigma_e^2 = 0.5$. Figure 4.14(a) shows that the BER for the C0700 channel is dependent on the power allocation. The lowest BER is achieved for waterfilling on the eigenvalues of $\underline{k}_{\underline{T}}$. Uniform power distribution leads to the same BER as without precoding. The BER in

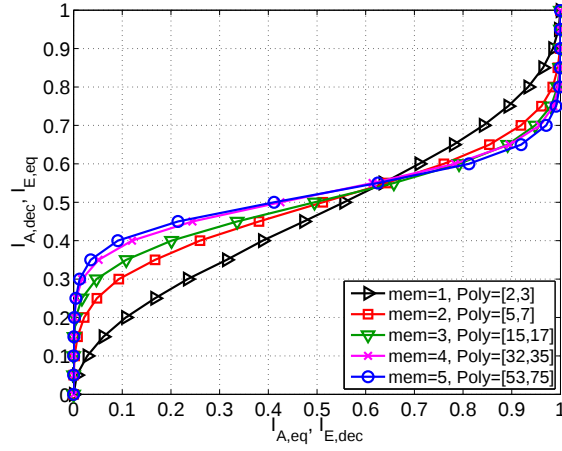


Figure 4.15: Rate=1/2 convolutional codes with different memory.

case of $\underline{k}_T$ with estimation errors lies in between the above two. In this latter case, the power allocation is less selective since the difference in the values between the eigenvalues of $\underline{k}_{T_e}$ is less than for the actual $\underline{k}_T$ (see Sec. 4.4.1). In case of the D0700, the power allocation strategy does not seem to affect the BER as can be seen in Fig. 4.14(b). The D0700 channel has a higher amount of interference that is partially reduced through precoding. In this case, the improvement in the BER seems to come mainly from transmitting in the direction of the eigenvectors of $\underline{k}_T$ and less on the power allocation strategy. Thus, in contrast to the uncoded transmission, the power allocation strategy does seem to play an important factor in the BER performance.

In all of the above, we have only considered a memory 2 convolutional code ($[7, 5]_8$). However, other convolutional codes with lower or higher memory can be used each with a different transfer function. For example, Fig. 4.15 shows the curves for some chosen convolutional codes with rate=1/2, but with memory values ranging from 1 to 5. This figure clearly shows that the code memory has a strong effect on the code transfer function. The higher the memory, the larger the $I_{A,dec}$ value at $I_{E,dec} = 0$. On the other hand, the curves of the high memory codes, (example mem 4 and 5 codes), show a slower increase of $I_{A,dec}$ with increasing $I_{E,dec}$ compared to lower memory codes. However, at high $I_{E,dec}$ values ($I_{E,dec} \approx 0.9$) the $I_{A,dec}$ values suddenly jump to values close to 1.0. It seems logical to expect the low memory codes to lead to better BER either at low E_b/N_o or/and in presence of high interference since the tunnel opening between the equalizer and the code curves in this case would be larger. However, at high E_b/N_o or for low interference, worse BER is expected since the $I_{E,dec}$ value at the intersection point between

4 Precoding

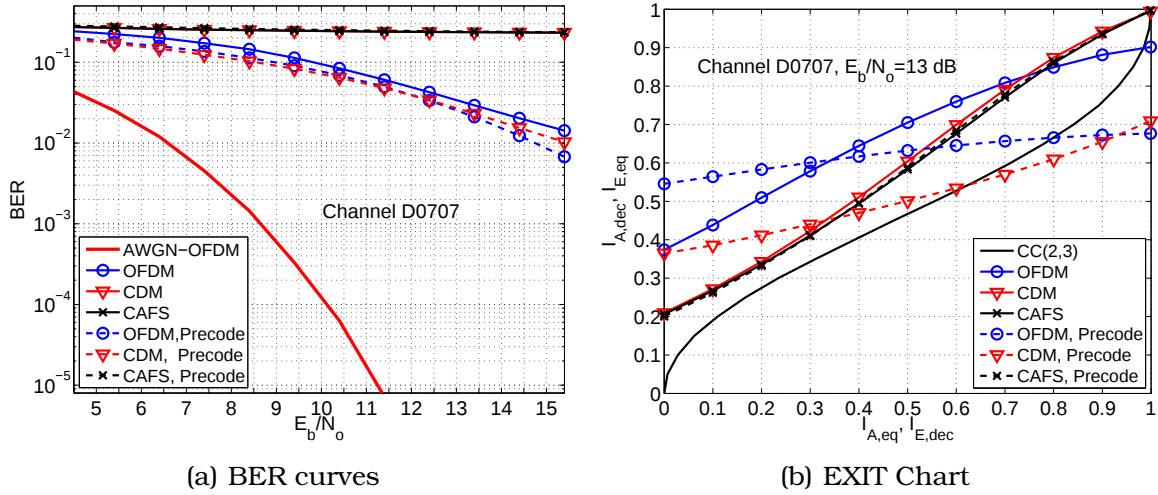


Figure 4.16: RNN: Effect of code memory on the BER of D0707 channel, convolutional code, rate = $1/2$, $m = 1$, generator polynomial $[2, 3]_8$.

the equalizer and code curves is smaller, the lower the code memory. Figure 4.16 shows the BER for the D0707 channel using memory 1 code with rate = $1/2$ and polynomial $[2, 3]_8$. The BER and the EXIT chart results confirm to our expectations. The BER, especially at low E_b/N_0 , is lower for this memory 1 code than for the memory 2 $[5, 7]_8$ code. The EXIT charts tunnels are also open for all spreading matrices at $E_b/N_0 = 13$ dB in contrast to Fig. 4.12(f). However, at high E_b/N_0 the performance is worse. Again, the $I_{E,dec}$ at the intersection point between the equalizer and the decoder curves are lower than those in Fig. 4.12(f), explaining the worse performance at higher E_b/N_0 values. Similar remarks can also be made about the C0707 channel shown in Fig. 4.17. Nonetheless, the BER gains obtained by employing the $[2, 3]_8$ memory 1.0 code at low E_b/N_0 are more significant especially for precoded MC-CAFS (note that the AWGN-OFDM curve shown here is worse than that obtained when the $[5, 7]_8$ code is employed).

4.6 Summary

We have looked at precoding for MIMO-OFDM, particularly for the case of partial channel knowledge. We have shown that transmitting in the direction of the transmit correlations at each subcarrier was the optimum direction for average PEP minimization. We have analyzed the BER performance for both the coded and uncoded transmission and for perfect and imperfect knowledge of the transmit correlations. It was shown that the MMSE-BLE does

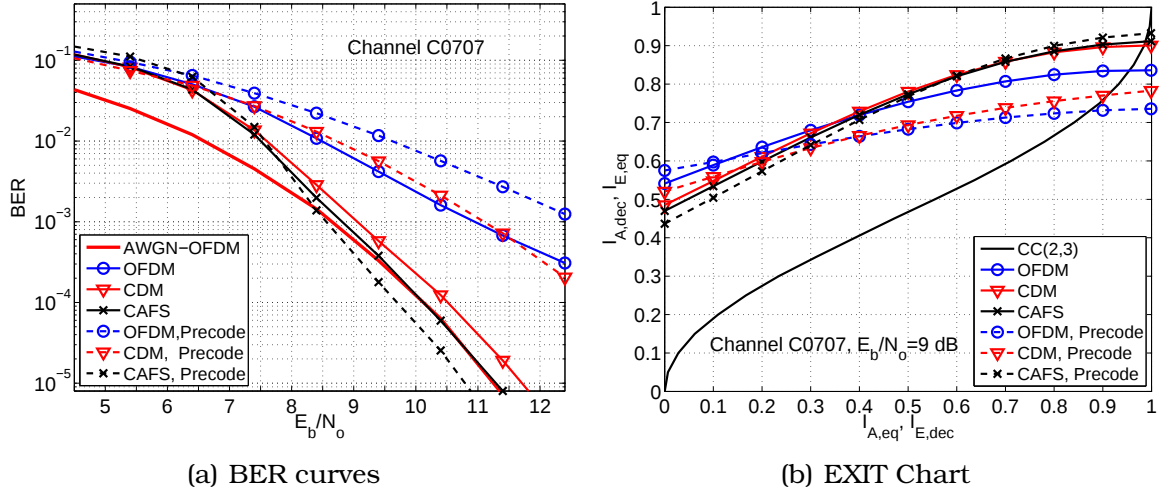


Figure 4.17: RNN: Effect of code memory on the BER of C0707 channel, convolutional code, rate = $1/2$, $m = 1$, $[2, 3]_8$.

not benefit from precoding. As a matter of fact, the BER performance was shown to deteriorate. The MMSE-BDFE, on the other hand, benefits significantly from precoding due to propagation error reduction. The SCE and RNN, for uncoded transmission, were also shown to benefit from precoding for OFDM, MC-CDM and MC-CAFS. In the coded case however, where iterative equalization and decoding was employed, precoding with OFDM and MC-CDM resulted in a worse BER performance than without precoding in most cases due to worsened MFB. Only MC-CAFS spreading was shown to always lead to the lowest BER when the SCE was employed. Similar results were observed with the RNN equalizer for moderate to low interference values. The BER performance was found to be highly dependent on the amount of interference in the channel. Furthermore, we showed that using codes with lower memory improves the BER performance only at low E_b/N_0 values. As the E_b/N_0 increases the performance is however worse than that of higher memory codes.

4.A Appendix to Chapter 4

4.A.1 Antenna Correlations in Frequency Domain

For the sake of clarity, we derive the expression for $\underline{\underline{k}}_{\text{Tave}}$ for a 2×2 MIMO system with $L = 2$. However, the following derivations can be easily extended to any $n_T \times n_R$ MIMO system with any number of uncorrelated channel taps L . Accordingly, the 2×2 correlation matrices at each channel tap, l , with $|\rho_l| \leq 1$

$$\underline{\underline{k}}(l) = \begin{bmatrix} 1 & \rho_l \\ \rho_l^* & 1 \end{bmatrix} = \begin{bmatrix} e_l & \varrho_l \\ \varrho_l^* & e_l \end{bmatrix} \begin{bmatrix} e_l & \varrho_l \\ \varrho_l^* & e_l \end{bmatrix} \quad (4.39)$$

where $e_l \in \mathbb{R}$, $\rho_l \in \mathbb{C}$, $\varrho_l \in \mathbb{C}$, $e_l^2 + |\varrho_l|^2 = 1$ and $2e_l\varrho_l = \rho_l$.

We shall only consider transmit correlations, but the same derivations can be applied to obtain the receive $\underline{\underline{k}}_{\text{Rave}}$. The channel matrices at each tap, l are,

$$\underline{\underline{h}}^c(l) = \begin{bmatrix} h_{11}(l) & h_{12}(l) \\ h_{21}(l) & h_{22}(l) \end{bmatrix} \begin{bmatrix} e_l & \varrho_l \\ \varrho_l^* & e_l \end{bmatrix}, \quad (4.40)$$

Without loss of generality, we shall only consider the first subcarrier frequency, $k = 1$. Thus, we start by calculating the transmit correlation in the frequency domain at frequency $k = 1$ as follows

$$\underline{\underline{k}}_k = \frac{1}{n_R} \mathcal{E}\{\underline{\underline{H}}_k^{cH} \underline{\underline{H}}_k^c\} = \frac{1}{n_R} \mathcal{E}\{\underline{\underline{H}}_1^{cH} \underline{\underline{H}}_1^c\}, \quad (4.41)$$

where, using the discrete Fourier transform, the channel transfer functions at $k = 1$ are given by,

$$\underline{\underline{H}}_1^c = \begin{bmatrix} h_{11}^c(1) + h_{11}^c(2) & h_{12}^c(1) + h_{12}^c(2) \\ h_{21}^c(1) + h_{21}^c(2) & h_{22}^c(1) + h_{22}^c(2) \end{bmatrix} = \begin{bmatrix} H_{11}^c & H_{12}^c \\ H_{21}^c & H_{22}^c \end{bmatrix}. \quad (4.42)$$

Accordingly

$$\underline{\underline{H}}_1^{cH} \underline{\underline{H}}_1^c = \begin{bmatrix} |H_{11}^c|^2 + |H_{21}^c|^2 & H_{11}^{c*} H_{12}^c + H_{21}^{c*} H_{22}^c \\ H_{11}^c H_{12}^{c*} + H_{21}^c H_{22}^{c*} & |H_{12}^c|^2 + |H_{22}^c|^2 \end{bmatrix}, \quad (4.43)$$

Similar to the derivation in Appendix 3.A.1, and assuming equal PDP for the channel taps ($\mathcal{E}\{|h_{ij}(l)|^2\} = 1/L$), we can calculate the expected value of diagonal elements of the above matrix, as follows,

$$\mathcal{E}\{|H_{11}^c|^2\} = \mathcal{E}\{e_1^2 |h_{11}(1)|^2 + |\varrho_1|^2 |h_{12}(1)|^2 + e_2^2 |h_{11}(2)|^2 + |\varrho_2|^2 |h_{12}(2)|^2\} = 1.0, \quad (4.44)$$

and

$$\mathcal{E}\{|H_{11}^c|^2\} = \mathcal{E}\{|H_{12}^c|^2\} = \mathcal{E}\{|H_{21}^c|^2\} = \mathcal{E}\{|H_{22}^c|^2\} = 1.0, \quad (4.45)$$

and the off diagonal elements,

$$\begin{aligned}\mathcal{E}\{H_{11}^{*c}H_{12}^c\} &= \mathcal{E}\{e_1\varrho_1|h_{11}(1)|^2 + e_1\varrho_1|h_{12}(1)|^2 + e_2\varrho_2|h_{11}(2)|^2 + e_2\varrho_2|h_{12}(2)|^2\} \\ &= \frac{1}{2}(\rho_1 + \rho_2) = \frac{1}{L} \sum_{l=1}^L \rho_l,\end{aligned}\quad (4.46)$$

and

$$\mathcal{E}\{H_{11}^{c*}H_{12}^c\} = \mathcal{E}\{H_{21}^{c*}H_{22}^c\} \quad (4.47)$$

Substituting the above two equations in Eqn. 4.41, we get

$$\underline{\underline{k}}_1 = \begin{bmatrix} 1 & \frac{1}{L} \sum_{l=1}^L \rho_l \\ \frac{1}{L} \sum_{l=1}^L \rho_l^* & 1 \end{bmatrix} = \frac{1}{L} \sum_{l=1}^L \underline{\underline{k}}(l). \quad (4.48)$$

The above derivation can also be repeated for any frequency k , however the result does not change, that is

$$\underline{\underline{k}}_k = \underline{\underline{k}}_1 = \underline{\underline{k}}_{Tave} \quad (4.49)$$

Thus, we can still express $\underline{\underline{K}}_T$ as follows,

$$\underline{\underline{K}}_{Tave} = \underline{\underline{k}}_{Tave} \otimes \underline{\underline{I}}_N. \quad (4.50)$$

In case of non-equal PDP ($\mathcal{E}\{|h_{ij}(l)|^2\} = \sigma_l^2$),

$$\underline{\underline{k}}_{Tave} = \begin{bmatrix} 1 & \sum_{l=1}^L \sigma_l^2 \rho_l \\ \sum_{l=1}^L \sigma_l^2 \rho_l^* & 1 \end{bmatrix}, \quad (4.51)$$

where σ_l^2 is the power in the l th MIMO channel tap, and $\sum_{l=1}^L \sigma_l^2 = 1.0$.

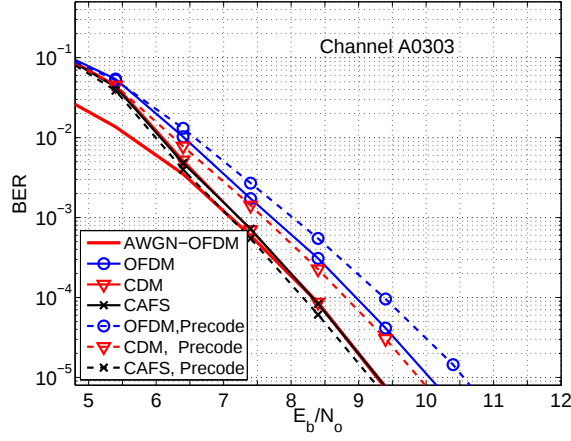
4.A.2 BER curves and EXIT-Charts

BER curves and EXIT-charts for:

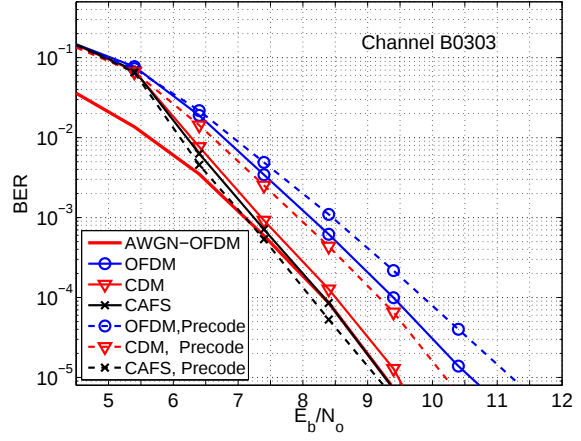
- A0303, B0303, C0303 and D0303
- A0700 and B0700
- A0707 and B0707

in Table 4.2 for RNN and SCE equalizers and convolutional code with rate=1/2, memory 2 and generator polynomial $[7, 5]_8$.

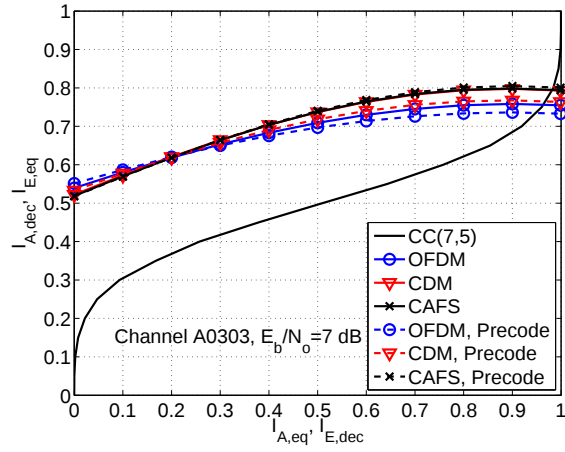
4 Precoding



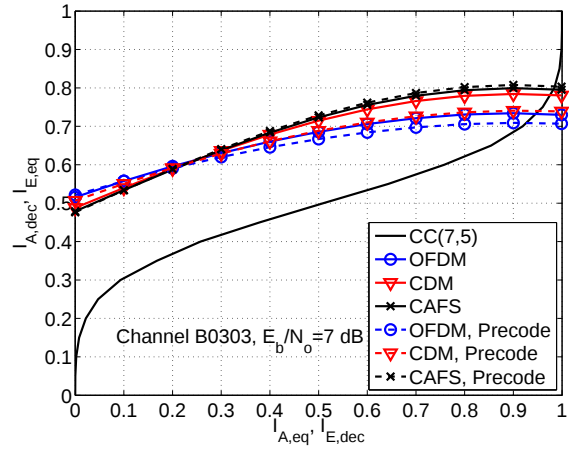
(a) BER curves for Channel A0303



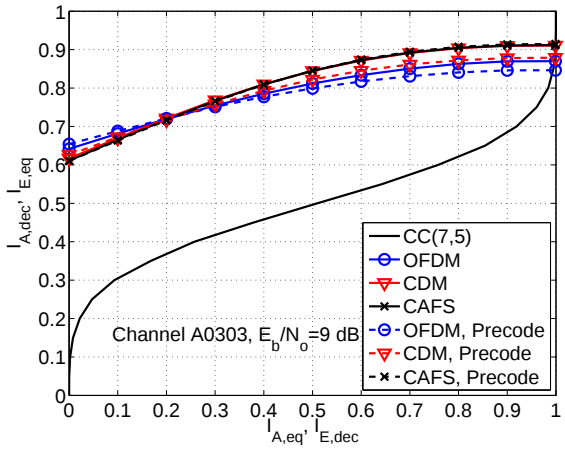
(b) BER curves for Channel B0303



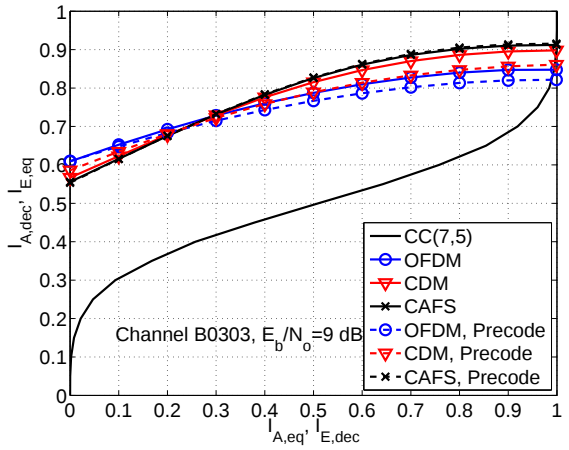
(c) EXIT-Chart: Channel A0303 at 7 dB



(d) EXIT-Chart: Channel B0303 at 7 dB



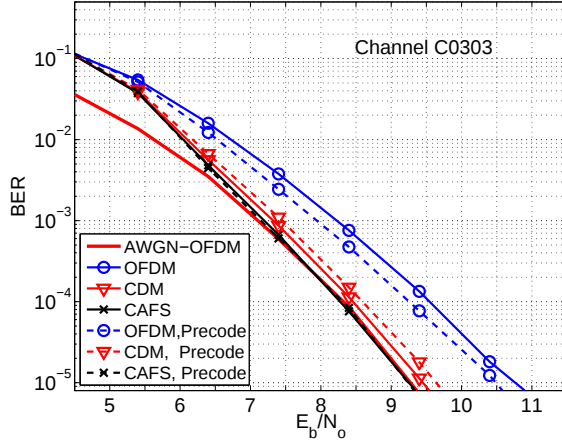
(e) EXIT-Chart: Channel A0303 at 9 dB



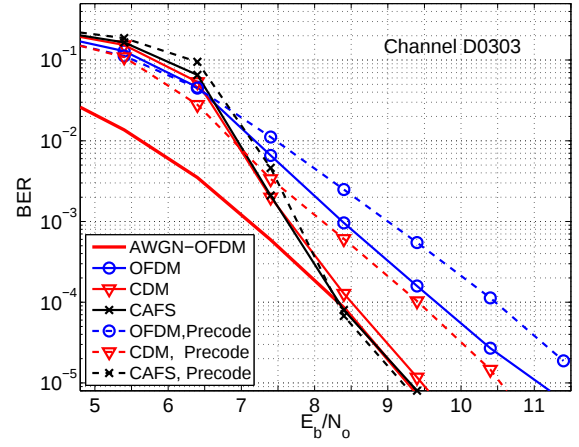
(f) EXIT-Chart: Channel B0303 at 9 dB

Figure 4.18: RNN: BER curves and corresponding EXIT charts for channels A0303 and B0303, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

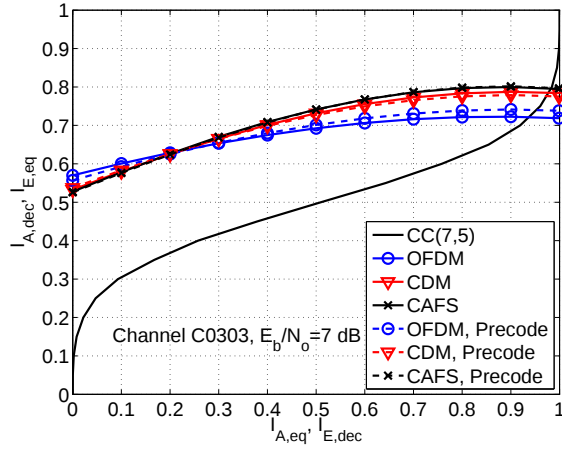
4.A Appendix to Chapter 4



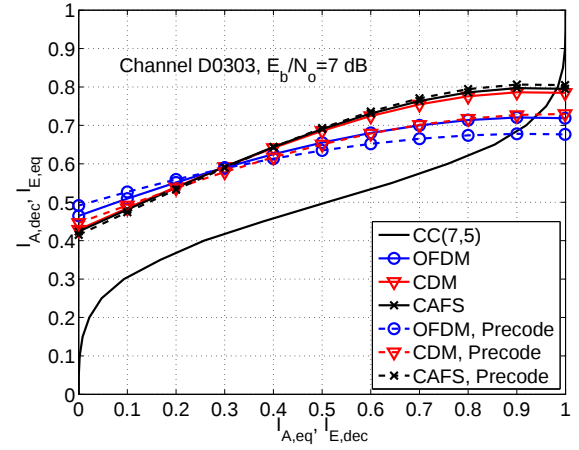
(a) BER curves for Channel C0303



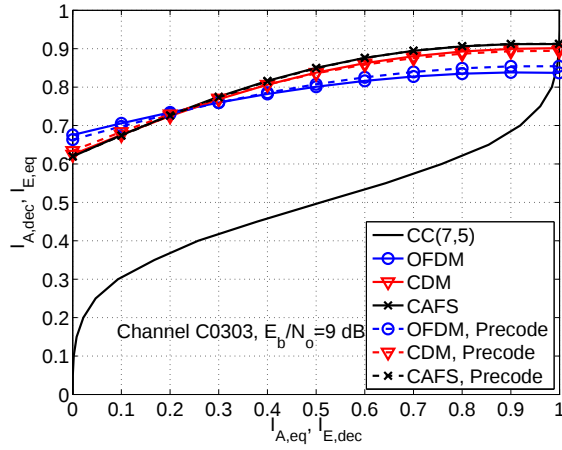
(b) BER curves for Channel D0303



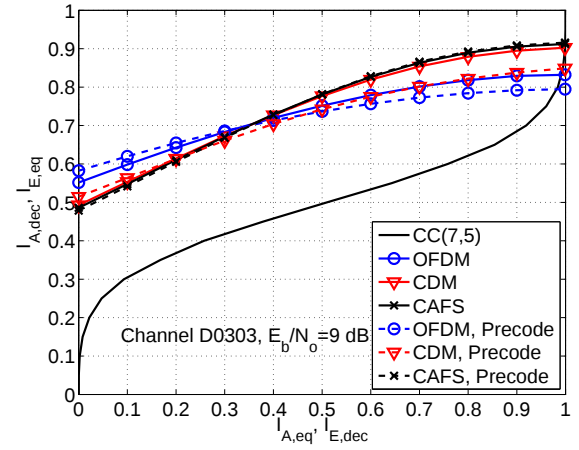
(c) EXIT-Chart: Channel C0303 at 7 dB



(d) EXIT-Chart: Channel D0303 at 7 dB



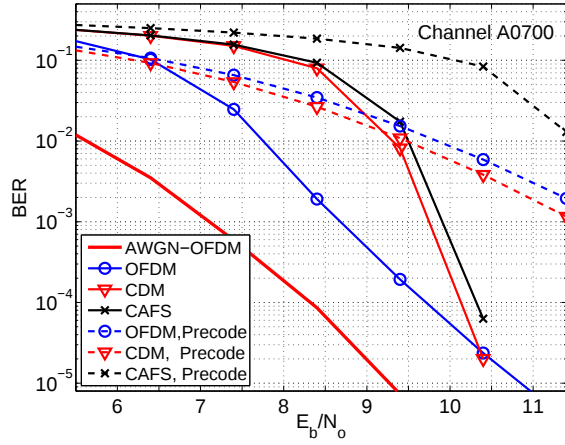
(e) EXIT-Chart: Channel C0303 at 9 dB



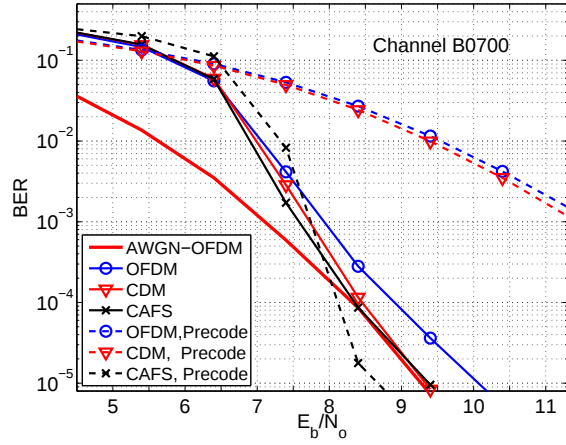
(f) EXIT-Chart: Channel D0303 at 9 dB

Figure 4.19: RNN: BER curves and corresponding EXIT charts for channels C0303 and D0303, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

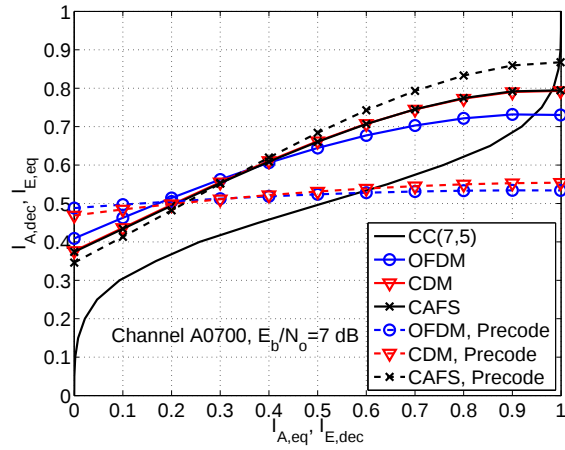
4 Precoding



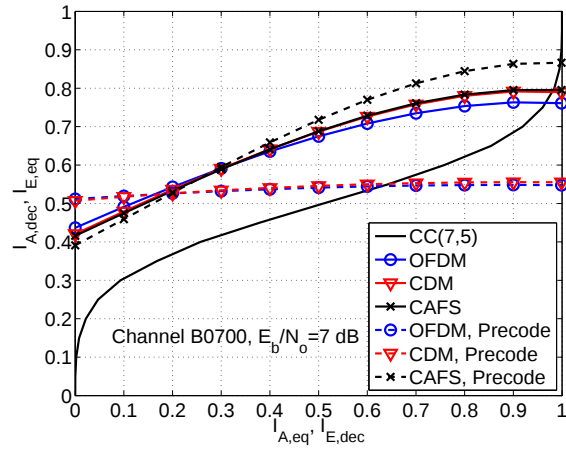
(a) BER curves for Channel A0700



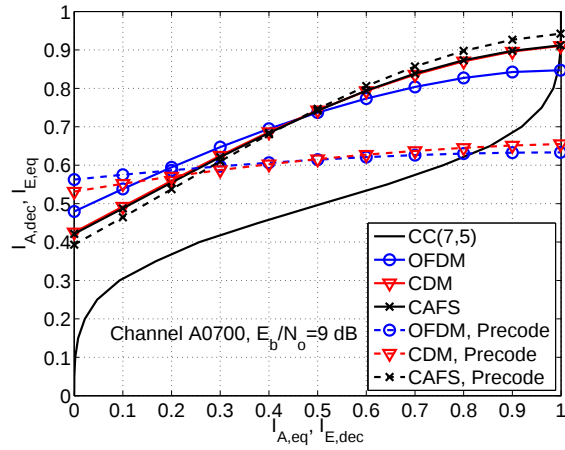
(b) BER curves for Channel B0700



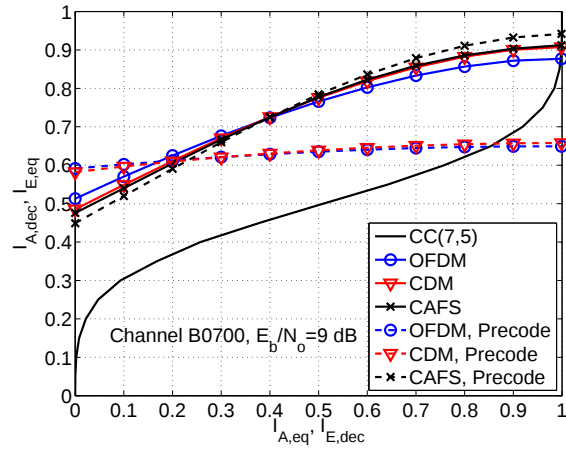
(c) EXIT-Chart: Channel A0700 at 7 dB



(d) EXIT-Chart: Channel B0700 at 7 dB



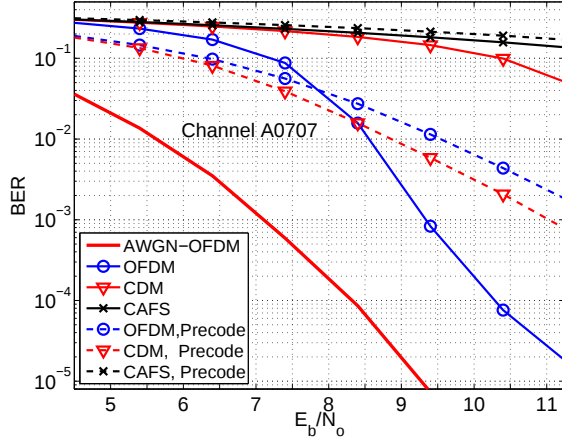
(e) EXIT-Chart: Channel A0700 at 9 dB



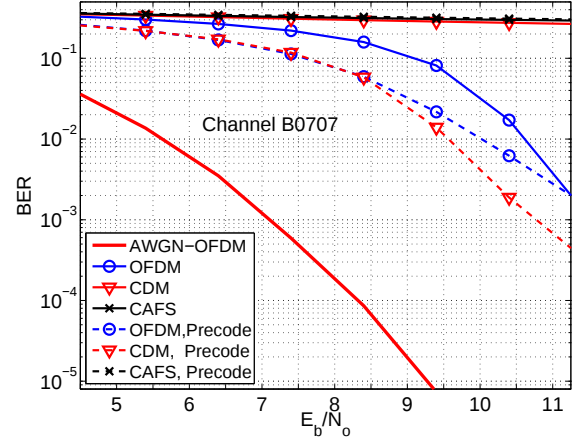
(f) EXIT-Chart: Channel B0700 at 9 dB

Figure 4.20: RNN: BER curves and corresponding EXIT charts for channels A0700 and B0700, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

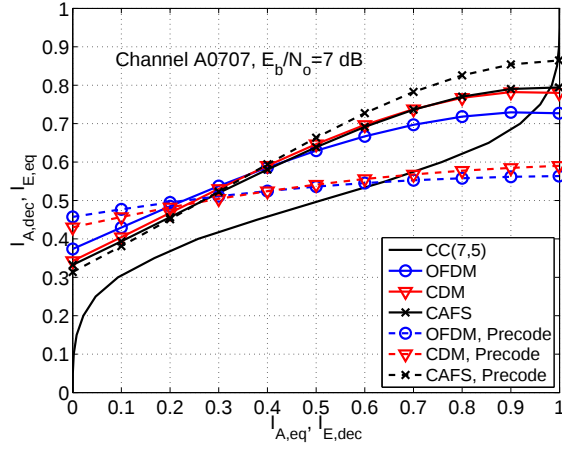
4.A Appendix to Chapter 4



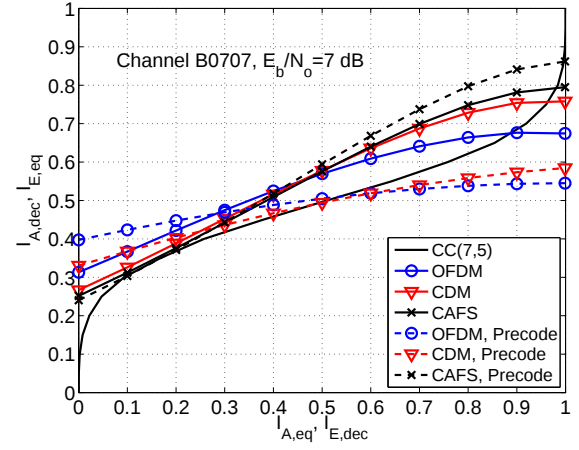
(a) BER curves for Channel A0707



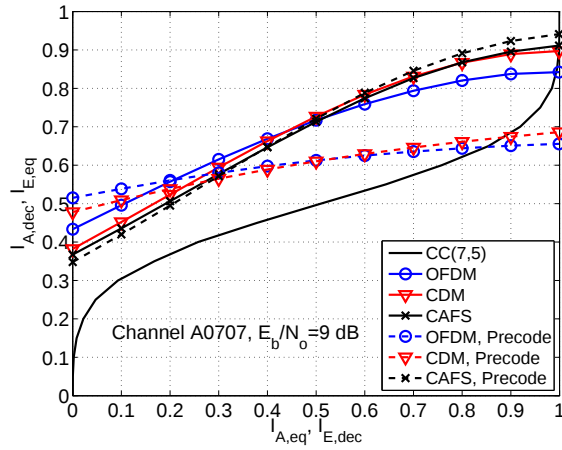
(b) BER curves for Channel B0707



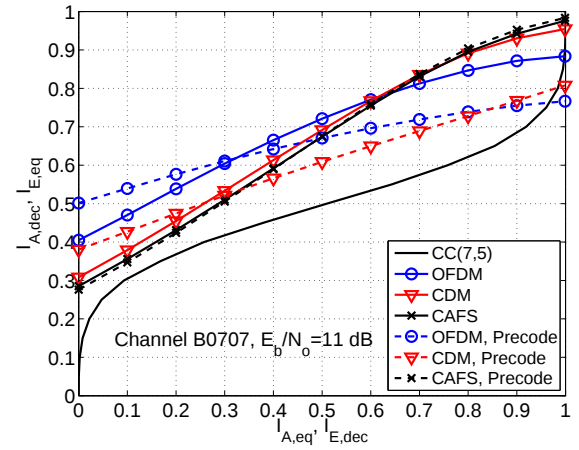
(c) EXIT-Chart: Channel A0707 at 7 dB



(d) EXIT-Chart: Channel B0707 at 7 dB



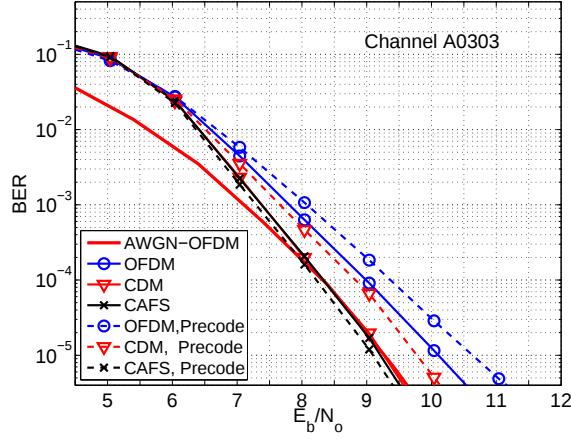
(e) EXIT-Chart: Channel A0707 at 9 dB



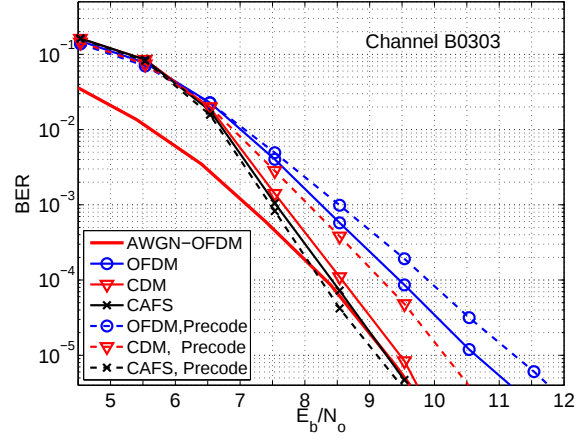
(f) EXIT-Chart: Channel B0707 at 11 dB

Figure 4.21: RNN: BER curves and corresponding EXIT charts for channels A0707 and B0707, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

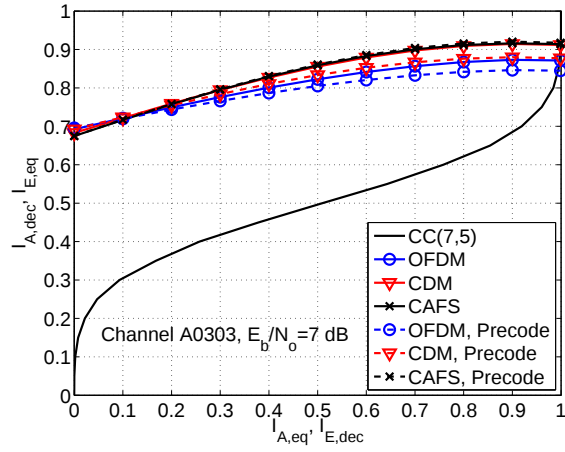
4 Precoding



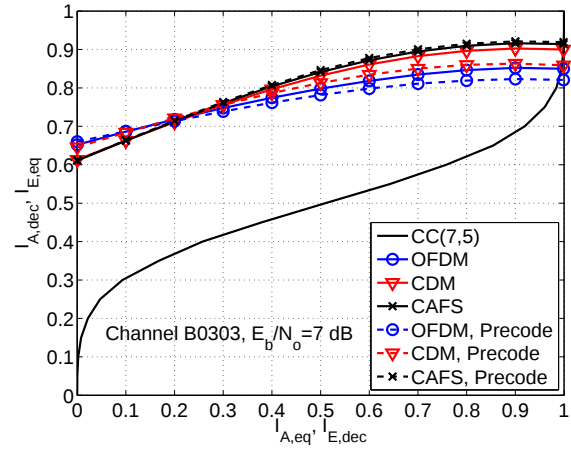
(a) BER curves for Channel A0303



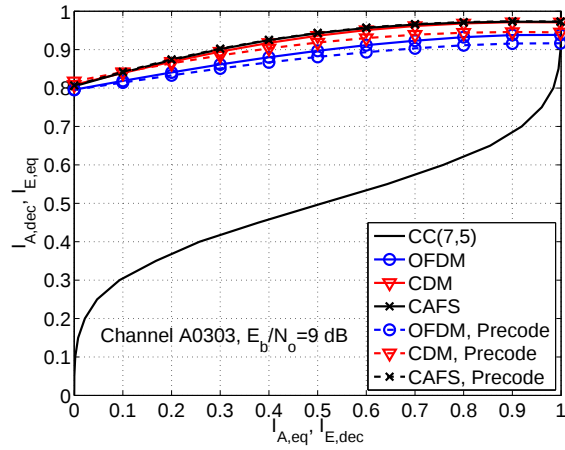
(b) BER curves for Channel B0303



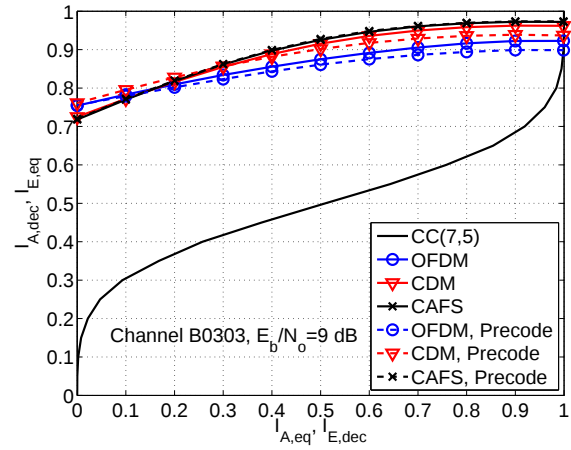
(c) EXIT-Chart: Channel A0303 at 7 dB



(d) EXIT-Chart: Channel B0303 at 7 dB



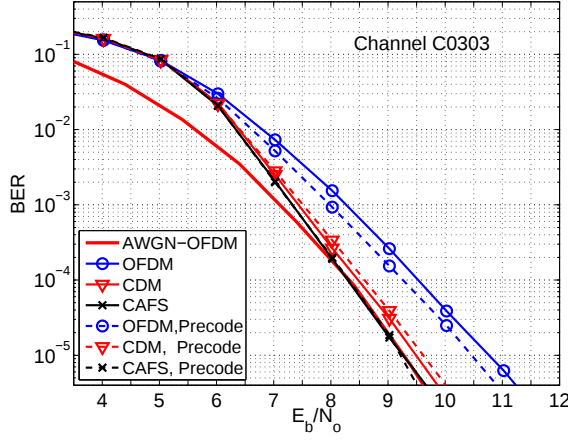
(e) EXIT-Chart: Channel A0303 at 9 dB



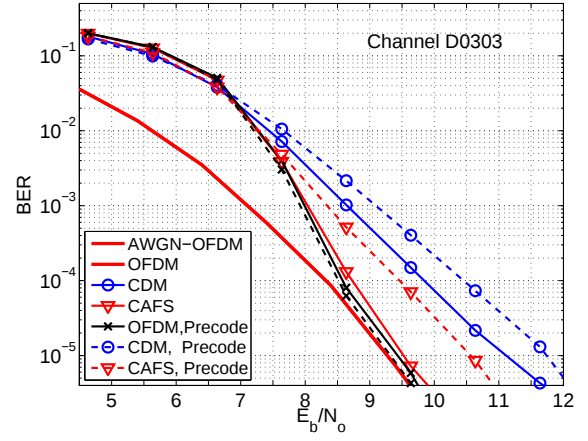
(f) EXIT-Chart: Channel B0303 at 9 dB

Figure 4.22: SCE: BER curves and corresponding EXIT charts for channels A0303 and B0303, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

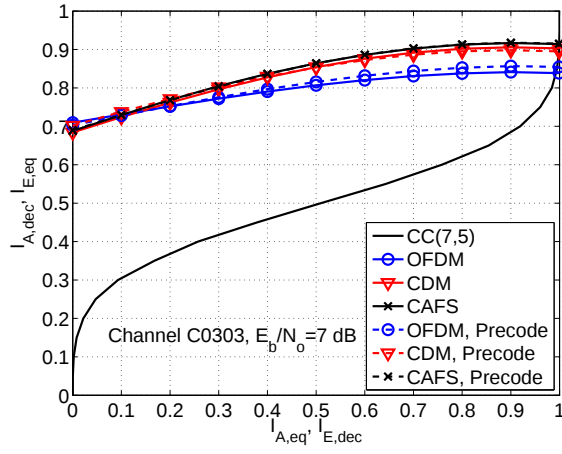
4.A Appendix to Chapter 4



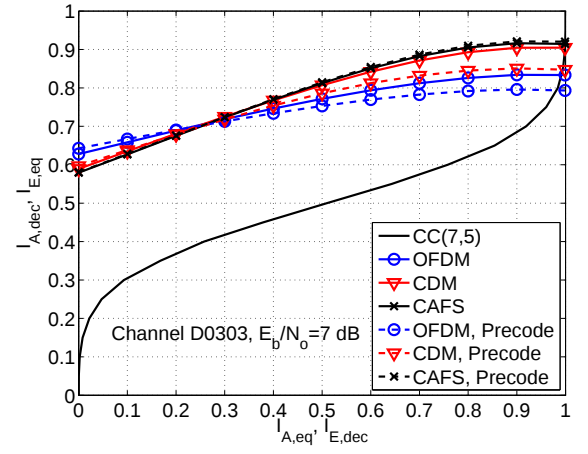
(a) BER curves for Channel C0303



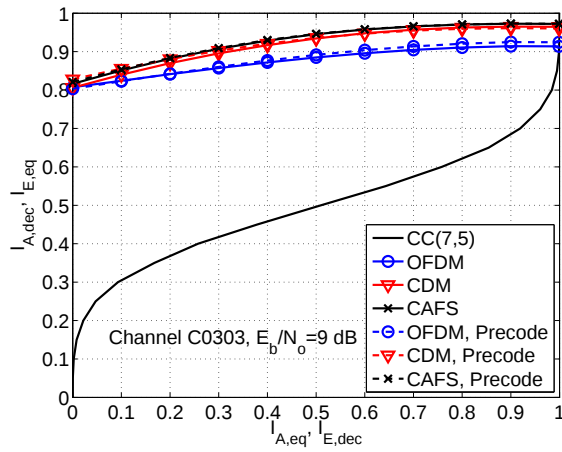
(b) BER curves for Channel D0303



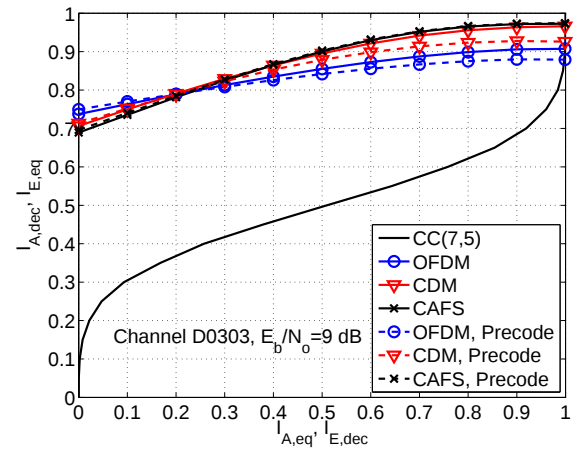
(c) EXIT-Chart: Channel C0303 at 7 dB



(d) EXIT-Chart: Channel D0303 at 7 dB



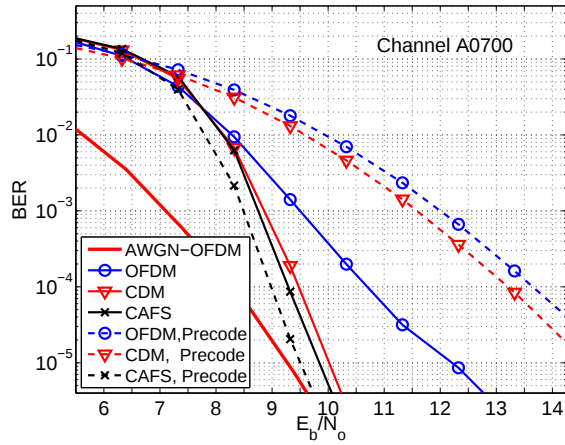
(e) EXIT-Chart: Channel C0303 at 9 dB



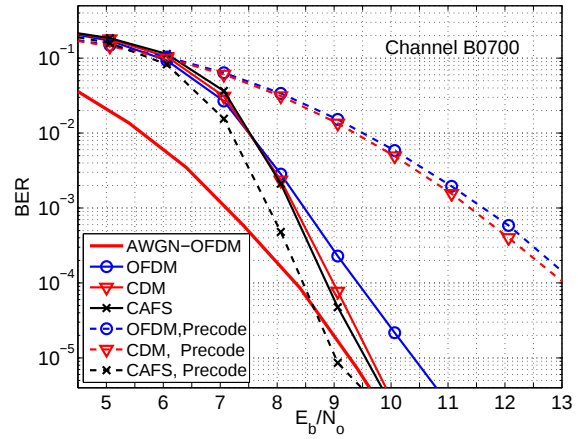
(f) EXIT-Chart: Channel D0303 at 9 dB

Figure 4.23: SCE: BER curves and corresponding EXIT charts for channels C0303 and D0303, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

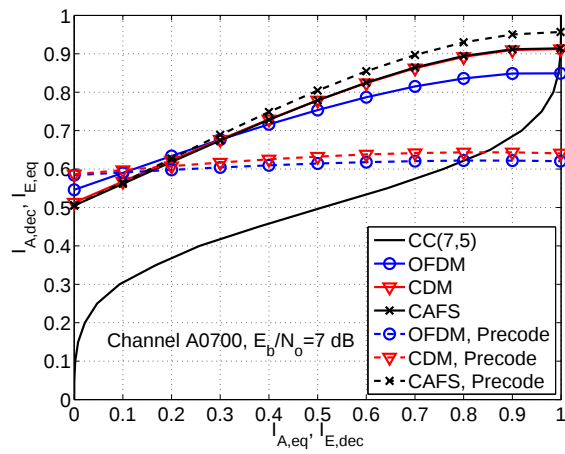
4 Precoding



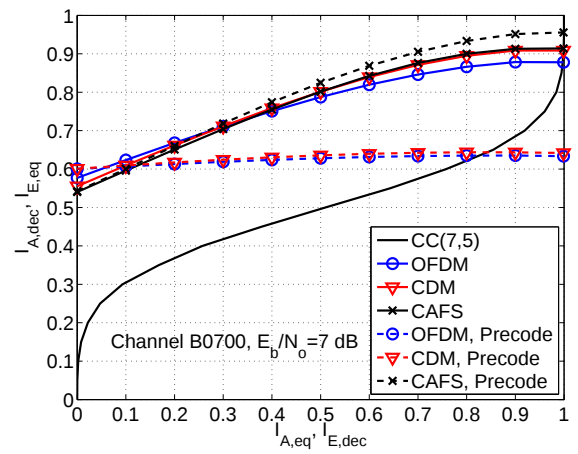
(a) BER curves for Channel A0700



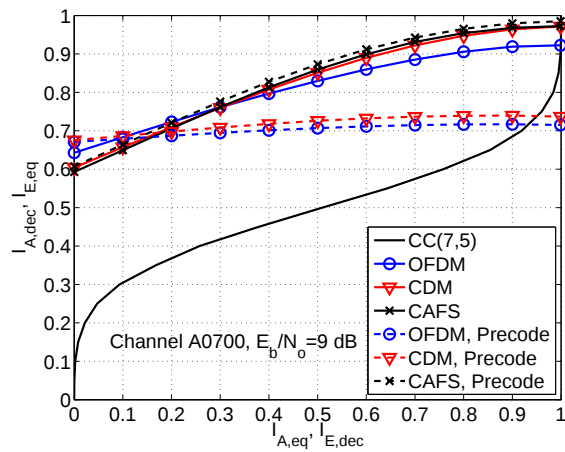
(b) BER curves for Channel B0700



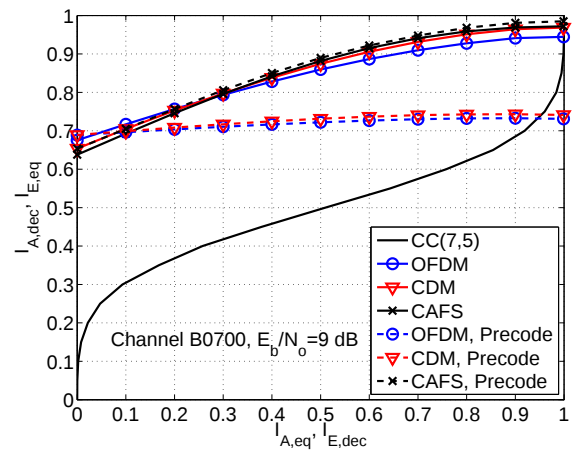
(c) EXIT-Chart: Channel A0700 at 7 dB



(d) EXIT-Chart: Channel B0700 at 7 dB



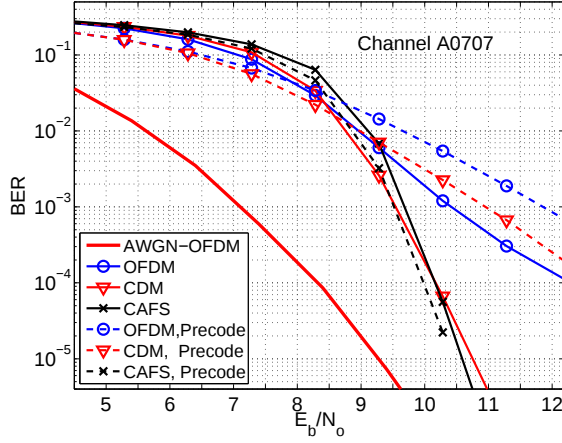
(e) EXIT-Chart: Channel A0700 at 9 dB



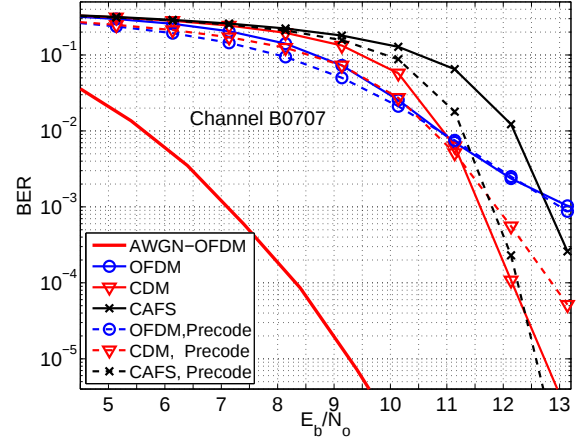
(f) EXIT-Chart: Channel B0700 at 9 dB

Figure 4.24: SCE: BER curves and corresponding EXIT charts for channels A0700 and B0700, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

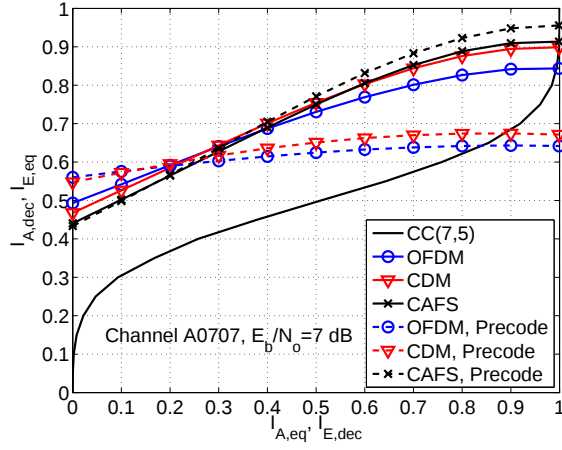
4.A Appendix to Chapter 4



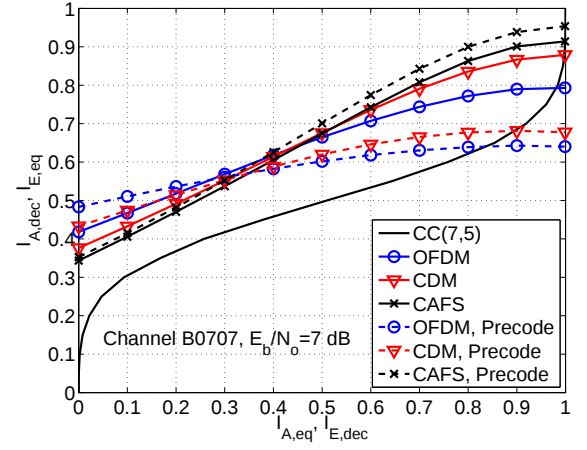
(a) BER curves for Channel A0707



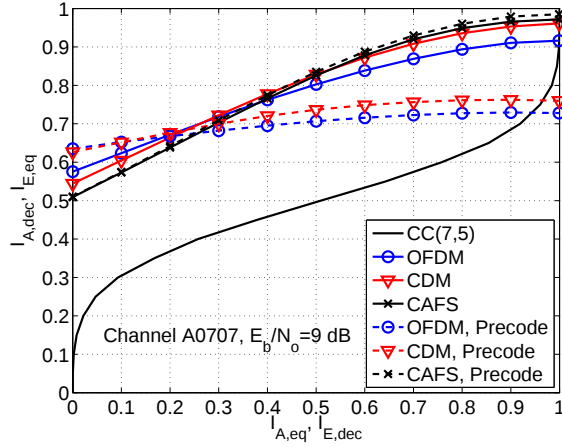
(b) BER curves for Channel B0707



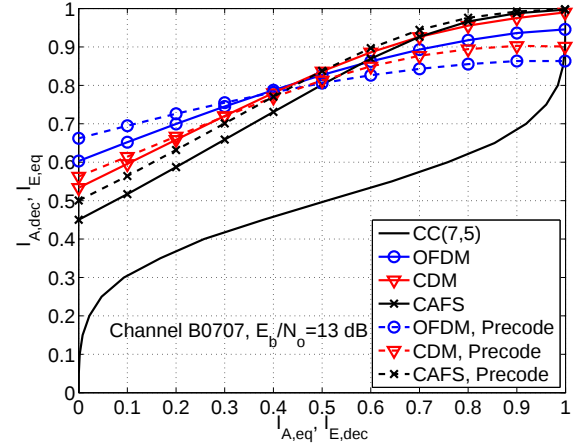
(c) EXIT-Chart: Channel A0707 at 7 dB



(d) EXIT-Chart: Channel B0707 at 7 dB



(e) EXIT-Chart: Channel A0707 at 9 dB



(f) EXIT-Chart: Channel B0707 at 13 dB

Figure 4.25: SCE: BER curves and corresponding EXIT charts for channels A0707 and B0707, convolutional code, rate = $1/2$, $m = 2$, generator polynomial $[7, 5]_8$.

4 *Precoding*

Chapter 5

MIMO Channel Capacity: Theory versus Measurement

In this chapter, we look at the capacity and correlations of measured MIMO channels for a specific outdoor scenario. We first start by giving a theoretical overview of MIMO channel capacity for both flat fading and frequency selective channels in presence of no, partial or full channel knowledge at the transmitter. We then describe the measurement setup and give the corresponding channel capacities and antenna correlations. The calculated capacity of the measured MIMO channels is compared with that of theoretical models.

5.1 MIMO Channel Capacity

In this section, we give the capacity for flat fading MIMO. The expression for the MIMO channel capacity was first introduced in [1]. The author differentiated between deterministic channels (channel state information is present at the transmitter CSIT) and randomly changing channels (Channel unknown

5 MIMO Channel Capacity: Theory versus Measurement

to the transmitter). As mentioned in the previous chapter, although full channel knowledge is not always available at the transmitter, partial knowledge (e. g. fading correlations) is more readily available and can be utilized by the transmitter. In what follows, we thus present the MIMO channel capacity for three cases: no channel knowledge, full channel knowledge and partial channel knowledge at the transmitter. We always assume that the channel is known to the receiver.

5.1.1 No channel knowledge at the transmitter

The capacity of any channel can be calculated either in the frequency or the time domain. We first look at the capacity of a flat fading random channel in the time domain. In this case, the MIMO ergodic (mean) channel capacity is given by [1, 4]

$$C_R = \mathcal{E}\{\log_2 \det(\underline{I}_{n_R} + \frac{P}{N_o n_T} \underline{h} \underline{Q} \underline{h}^H)\} = \mathcal{E}\{\log_2 \det(\underline{I}_{n_R} + \frac{\rho_o}{n_T} \underline{h} \underline{h}^H)\}, \quad (5.1)$$

where $\underline{h}$ is the channel matrix in the time domain and the expectation is taken over all channel realizations, P is the total transmit power, N_o is the noise power, and thus $\rho_o = P/N_o$ is the signal to noise ratio (SNR). The covariance matrix of the transmit signals, $\underline{Q}$, is given by

$$\underline{Q} = \underline{I}_{n_T}. \quad (5.2)$$

That is, the total transmit power is distributed equally over all transmit antennas n_T . It is important to mention here that this is not the Shannon capacity in the true sense, since, as shown next, in presence of full channel knowledge a signal covariance matrix can be chosen that outperforms that in Eqn. 5.2 [22]. Nonetheless, as is customarily done in the literature, we shall still refer to Eqn. 5.1 as the channel capacity. Equation 5.1 can also be written in terms of the channel singular values, λ_i , (i. e. the eigenvalues of $\underline{h} \underline{h}^H$) as follows [1, 4, 3]

$$C_R = \mathcal{E}\{\log_2 \det(\underline{I}_{n_R} + \frac{\rho_o}{n_T} \underline{h} \underline{h}^H)\} = \mathcal{E}\{\sum_{i=1}^r \log_2(1 + \frac{\rho_o}{n_T} \lambda_i)\}, \quad (5.3)$$

where r is the rank of $\underline{h}$.

As mentioned above, the channel capacity can be equivalently calculated in the frequency domain. For a frequency selective channel, it is easier to calculate the capacity in the frequency domain. In this case, we can make use of the discrete Fourier transform (DFT) [79, 91]. The capacity C_R of a

5.1 MIMO Channel Capacity

frequency selective channel can be calculated, for n_R receive antennas and n_T transmit antennas, as follows [79, 91]:

$$C_R = \frac{1}{N} \mathcal{E} \left\{ \sum_{k=1}^N \log_2 \det \left(\underline{I}_{n_R} + \frac{\rho_o}{n_T} \underline{H}_f(k) \underline{H}_f^H(k) \right) \right\}, \quad (5.4)$$

where $\underline{H}_f(k)$ is a matrix of size $n_R \times n_T$ and represents the channel transfer function at frequency k for $k = 1, \dots, N$ frequencies. Similar to the time domain, the capacity can also be calculated from the eigenvalues λ_i of $\underline{H}_f(k) \underline{H}_f^H(k)$ for $k = 1, \dots, N$ as follows [91]

$$C_R = \frac{1}{N} \mathcal{E} \left\{ \sum_{i=1}^{\min(Nn_R, Nn_T)} \log_2 \left(1 + \frac{\rho_o}{n_T} \lambda_i \right) \right\}. \quad (5.5)$$

If we only consider one channel realization, we can write the instantaneous channel capacity as [1, 22]

$$C_R = \log_2 \det \left(\underline{I}_{n_R} + \frac{\rho_o}{n_T} \underline{h} \underline{h}^H \right) = \sum_{i=1}^r \log_2 \left(1 + \frac{\rho_o}{n_T} \lambda_i \right). \quad (5.6)$$

That is, $\underline{h}$ is deterministic.

When random channels are considered, the channel is normalized such that $\mathcal{E} \{ \sum_l |\underline{h}_{ij}(l)|^2 \} = 1$, where $\underline{h}_{ij}(l)$ are the channel impulse responses of the MIMO channel in the time domain between transmit antenna j ($j = 1, \dots, n_T$) and receive antenna i ($i = 1, \dots, n_R$) having $l = 1, \dots, L$ taps.

Asymptotic capacity of uncorrelated MIMO channels At high SNR, the ergodic capacity of uncorrelated MIMO channels grows linearly with $n = \min(n_T, n_R)$ [1, 2, 92, 22]. Figure 5.1(a) shows the capacity of several $n \times n$ MIMO ($n_T = n_R = n$) channels for various SNR. We can see the enormous gains in capacity by increasing the number of both transmit and receive antennas. In contrast, increasing the number of receive antennas (receive diversity only) while keeping n_T constant leads to a logarithmic increase in the capacity [1, 92]. This can be seen in Fig. 5.1(b). At high SNR, the capacity increases by n b/s/Hz for every 3 dB increases in SNR. In addition, the capacity of a MIMO channel for $n_T = n_R = n$ approaches

$$C_R \rightarrow n \log_2(1 + \rho_o)$$

as $n \rightarrow \infty$ since $\frac{1}{n} \underline{h} \underline{h}^H \rightarrow \underline{I}_n$ as $n \rightarrow \infty$ [1, 22]. That is, the capacity increases linearly with n .

5 MIMO Channel Capacity: Theory versus Measurement

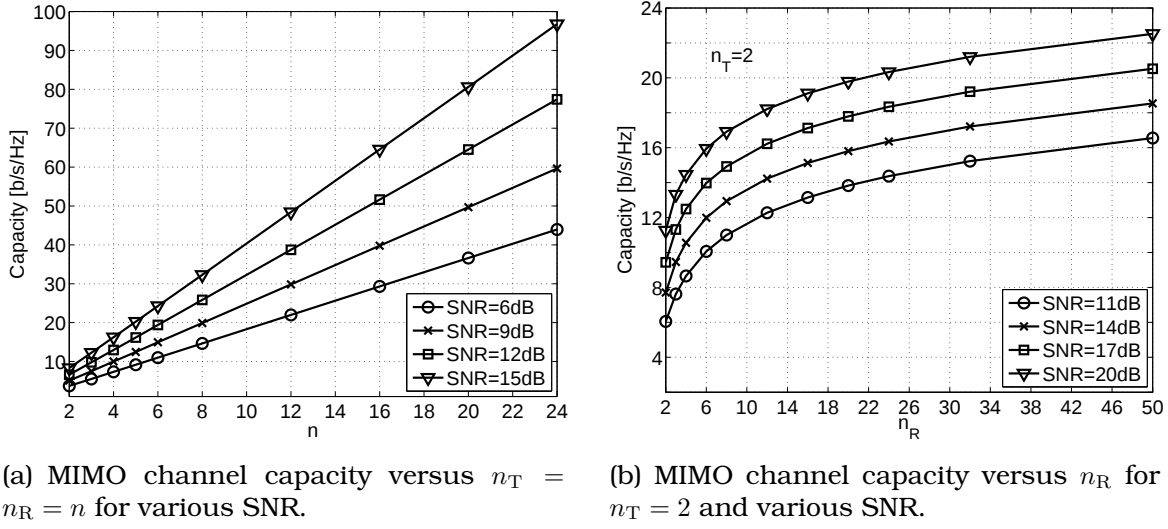


Figure 5.1: MIMO Channel Capacity.

5.1.2 Full channel knowledge at the transmitter

In the previous section, it was shown that if no CSIT is available, the best one can do is to transmit using equal power at all transmit antennas. If full channel knowledge is available at the transmitter, our goal would be to find the optimum $\underline{\underline{Q}}$ maximizing the following equation [1, 4]

$$C = \max_{\underline{\underline{Q}}} \log_2 \det(\underline{\underline{I}}_{n_R} + \frac{\underline{\underline{h}} \underline{\underline{Q}} \underline{\underline{h}}^H}{N_o}) \quad (5.7)$$

subjected to $\text{Tr}(\underline{\underline{Q}}) = n_T$. In this case, the best solution would be to transmit in the direction of the left singular vectors of the channel, $\underline{\underline{V}}_h$, where $\underline{\underline{h}} = \underline{\underline{U}}_h \underline{\underline{\Sigma}} \underline{\underline{V}}_h^H$ and $\underline{\underline{\Sigma}}$ is a diagonal matrix containing the channel singular values, σ_i . At the receiver, the received signal is multiplied by $\underline{\underline{U}}_h^H$. That is, the channel is decomposed into r parallel SISO channels [1, 3, 22]. The capacity can thus be written as the sum of the capacities of those parallel SISO channels as follows,

$$C_R = \sum_{i=1}^r \log_2(1 + \frac{\rho_o \gamma_i}{n_T} \lambda_i), \quad (5.8)$$

where γ_i is the portion of the transmit power allocated to the i th subchannel under the constraint $\sum_{i=1}^r \gamma_i = n_T$ and $\lambda_i = \sigma_i^2$. Thus, the problem now is to find the optimum γ_i to maximize Eqn. 5.8 under the aforementioned constraint. The optimal power allocation is found iteratively through the waterfilling algorithm [53] (see Eqn. 4.7). Thus $\underline{\underline{Q}} = \underline{\underline{V}}_h \underline{\underline{\lambda}}_{opt} \underline{\underline{V}}_h^H$, where $\underline{\underline{\lambda}}_{opt}$ is a

diagonal matrix containing the optimum γ_i values. As mentioned in Sec.4.2 depending on the channel eigenvalues and the available transmit power some of the weak modes might receive zero power. If only one eigenmode gets all of the available transmit power ($\underline{\underline{Q}}$ is rank 1), we denote this transmission strategy beamforming [93].

5.1.3 Partial channel knowledge at the transmitter

In the previous sections, we considered the cases of either full or no channel knowledge at the transmitter. Now, we consider the case of partial channel knowledge being available. To be more specific, we also assume that only the transmit correlation matrix, $\underline{\underline{k}}_T$, is known to the transmitter. In this case, the channel capacity is given by

$$C = \max_{\underline{\underline{Q}}} \mathcal{E} \left\{ \log_2 \det \left(\underline{\underline{I}}_{n_R} + \frac{\underline{\underline{H}} \underline{\underline{Q}} \underline{\underline{H}}^H}{N_o} \right) \right\}$$

subjected to $\text{Tr}(\underline{\underline{Q}}) = n_T$. In this case, the optimum input covariance matrix is given by [94, 80]

$$\underline{\underline{Q}} = \underline{\underline{v}}_T \underline{\underline{\lambda}}_{opt} \underline{\underline{v}}_T^H,$$

where $\underline{\underline{k}}_T = \underline{\underline{v}}_T \underline{\underline{e}}_T \underline{\underline{v}}_T^H$ and, as mentioned in Sec.4.3, the diagonal matrix $\underline{\underline{\lambda}}_{opt}$ is obtained through numerical optimization resembling that of waterfilling, where stronger modes are allocated more power than weaker ones [94, 80]. The authors in [82] considered both transmit and receive correlations and have shown that, although one still needs to transmit in direction of the transmit correlation eigenvectors, the energy distribution at the transmitter is dependent on both the transmit and receive correlations.

Although numerical power allocation strategies are optimum, transmitting using less optimum solutions can still lead to results very close to the optimum ones. In [75], a non optimum power allocation strategy, *stochastic water-filling*, was proposed where waterfilling is done on the *weighted* eigenvalues of $\underline{\underline{k}}_T$. This stochastic strategy performs very close to the optimum power allocation when fading correlations are high.

Similarly, for frequency selective channels, it was shown in [21] that transmitting in the direction of the transmit correlations eigenvectors at each subcarrier maximizes the capacity. Likewise, the power allocation is done through numerical optimization. The authors nonetheless showed that waterfilling on the eigenvalues of the transmit correlations, although suboptimal, leads to almost optimum results.

Figure 5.2 compares the MIMO capacity for correlated fading with that of uncorrelated fading assuming full, partial or no channel knowledge at the

5 MIMO Channel Capacity: Theory versus Measurement

transmitter. For the correlated case, the exponential correlation model is assumed with $\rho = 0.9$. In case of partial CSIT, the stochastic water-filling algorithm was employed. As expected, the presence of antenna correlations causes a loss in capacity, especially if absolutely no channel knowledge is available at the transmitter. Nonetheless, in case of partial CSIT, the channel capacity using this suboptimum stochastic waterfilling method is seen to lie very close to that of full channel knowledge.

5.1.4 Impact of Antenna Correlations on MIMO Capacity

To closely examine the effect of antenna correlations on the MIMO capacity, we consider the MIMO capacity at high SNR assuming $n_T = n_R = n$ and $\underline{k}_T$ and $\underline{k}_R$ to be full rank. The capacity can thus be written as [22]

$$C \approx \log_2 \det\left(\frac{\rho_o}{n_T} \underline{h} \underline{h}^H\right) + \log_2 \det(\underline{k}_T) + \log_2 \det(\underline{k}_R), \quad (5.9)$$

where $\underline{h}$ are the white uncorrelated zero mean complex channel coefficients. Clearly, $\underline{k}_T$ and $\underline{k}_R$ have the same effect on the channel capacity. Since the $\det(\underline{k}) = \prod_i \lambda_i(\underline{k})$ and $\sum_i \lambda_i(\underline{k}) = n$ and making use of the inequality of the arithmetic and geometric means, we have $\prod_i \lambda_i(\underline{k}) \leq 1.0$. That is, $\log_2(\det(\underline{k})) \leq 0$ and is zero only if $\underline{k} = \underline{I}$. Thus, correlations are detrimental to the MIMO capacity leading to loss in capacity as is also shown in Fig. 5.2.

5.1.5 Impact of Line of Sight on MIMO Capacity

The effect of the line of sight (LOS) component on the MIMO channel capacity can be examined by considering Rician fading where the channel is modeled as a sum of a LOS matrix and a Rayleigh fading matrix as follows

$$\underline{h} = \sqrt{\frac{K}{1+K}} \bar{\underline{h}} + \sqrt{\frac{1}{1+K}} \underline{h}_w \quad (5.10)$$

where $\bar{\underline{h}}$ is the fixed component, $\underline{h}_w$ the Rayleigh component and K the Rician factor. K is equal to the ratio of the power in the LOS component to that in the pure fading channel. $K = 0$ represents pure Rayleigh fading, and $K \rightarrow \infty$ represents a non fading channel. The performance of the Rician channel is thus dependent on K as well on the components of the matrix $\bar{\underline{h}}$. Thus, if $K = 0$, the capacity reduces to that of a Rayleigh channel. On the other side, if $K \rightarrow \infty$, then the capacity is dependent on the structure of $\bar{\underline{h}}$ as we show next.

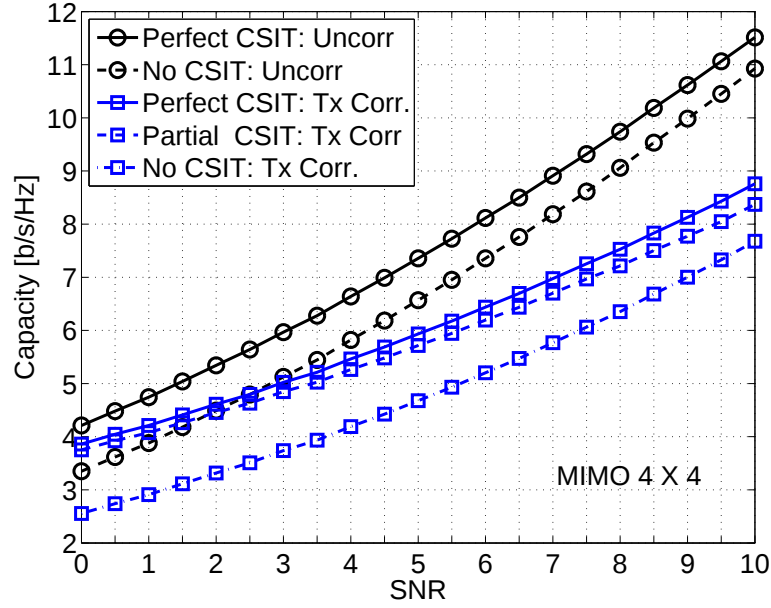


Figure 5.2: MIMO 4x4 Channel Capacity: uncorrelated fading versus correlated fading, exponential correlation model, $\rho = 0.9$.

Maximum Capacity: A channel, that achieves the maximum possible capacity, satisfies the following equation [2, 22],

$$\underline{\underline{h}}^H \underline{\underline{h}} = n_R \underline{\underline{I}}_{n_R}, \quad (5.11)$$

for $n_R \geq n_T$. That is, orthogonal channels maximize the capacity. The eigenvalues are in this case all equal and the capacity can be given by

$$C_{max} = n_T \log_2 \left(1 + \frac{\rho_o}{n_T} n_R \right). \quad (5.12)$$

Thus for orthogonal MIMO channels, reducing the K factor (i. e. Rayleigh component gaining more power) only worsens the capacity. On the other extreme, a LOS matrix $\underline{\underline{h}}$ that is rank deficient –for example $\underline{\underline{h}} = \underline{\underline{1}}$ – would benefit, if the K factor decreases.

In [95], it was shown that in a pure line of sight (LOS) environment (i. e. no scattering, $L = 1$), there exists a range of distances between the transmitter and receiver arrays where the channel matrix is orthogonal. This range contains all distances below and at D_{orth} , which is given by:

$$D_{\text{orth}} = \frac{d_{\text{Tx}} d_{\text{Rx}} n_R}{\lambda}, \quad (5.13)$$

5 MIMO Channel Capacity: Theory versus Measurement

where d_{Tx} is the spacing between the antenna elements in the transmitter, d_{Rx} the spacing between the antenna elements in the receiver and λ is the wavelength. Note that Eqn. 5.13 only considers the orthogonality of the signatures for a pair of adjacent transmit antennas [95]. That is, it considers the orthogonality of the vectors $\underline{h}_k$ and $\underline{h}_{k+1}$, where $\underline{h}_k$ is the k th column of the $n_R \times n_T$ channel matrix $\underline{h}$ in the time domain. At distances around D_{orth} , the channel is not necessarily orthogonal. However, the channel condition number ($\lambda_{max}/\lambda_{min}$) remains in general low [71]. Similar results were also found in [96], where the channel capacity for a simulated 3×3 MIMO LOS channel was found to increase as the transmitter approached the receiver reaching its maximum value at the distance given by Eqn. 5.13. Note, that Eqn. 5.13 also assumes that the waves arriving at the receiver antennas are plane, i. e. large separations between transmitter and receiver. For short distances, this is not necessarily satisfied. The distances D_{orth} obtained from Eqn. 5.13 for typical antenna spacings (e.g. $d_{Tx} = d_{Rx} = 10\lambda$, $n_R = 4$, $D_{orth} \approx 23$ m) are within ranges where the incident waves are not necessarily plane anymore. Thus, Eqn. 5.13, should only be seen as an approximation. Still, we will compare the measurement results with the above model in what follows.

5.2 Measurement Setup

In this section, we briefly describe the measurement setup at both the transmitter and the receiver as shown in Fig. 5.3. The street, where the measurements were done, is a two way street with two lanes for each direction. The lanes are separated with crash barriers and heavy foliage. In addition, heavy foliage was present on either side of the street. The transmitter (Tx) was mounted on a vehicle moving at a constant low speed (around 10 km/h, Doppler shift thus neglected) towards a stationary receiver (Rx), which was mounted on a bridge. The channel measurements were started when the vehicle was about 217 m in front of the bridge and stopped after it passed the bridge by around 62 m (Fig. 5.3). The channel measurements were carried out at a center frequency, f_o , of 5.2 GHz and for a bandwidth, BW, of 120 MHz. The different MIMO channels are not measured in parallel, but successively using a multiplexer at both the transmitter and receiver. Each transmit antenna remains active for multiple test signal periods, while the receiver measures one test signal for one pair of transmit and receive antennas. The multiplexer at the receiver then switches to the next receive antenna and the measurement is repeated [97]. After going through all receive antennas, the multiplexer at the transmitter switches to the next transmit antenna and the whole measurement is then repeated. One snapshot in this case does not define one channel measurement, but all MIMO channel measurements.

5.2 Measurement Setup

For more details about the channel sounder and channel sounder measurements, please refer to [97, 98, 99].

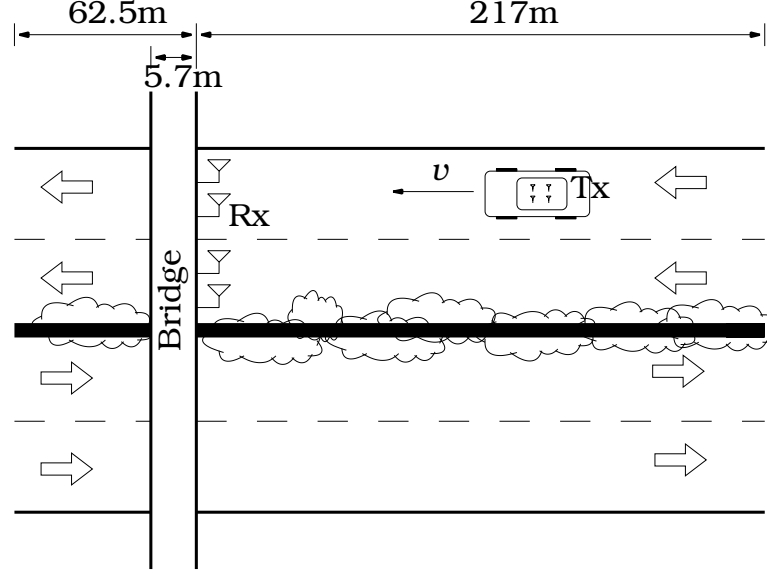


Figure 5.3: Schematic of measurement setup.

The transmitter and receiver array setups are shown in Fig. 5.4. The receive elements were tilted down by 45° . The MIMO channel was measured for three different setups at the receiver and two setups at the transmitter. The setups correspond to different antenna element spacings within either the transmitter or receiver arrays. The total span, W , and separation between the antennas, d (also given as a function of the wavelength, $\lambda = 0.0577$ m), for the three setups at the receiver are given in the following table.

Receiver Setup	W [m]	d [m]
Large (L)	17.5	$2.5 \approx 43\lambda$
Medium (M)	6.16	$0.88 \approx 15\lambda$
Small (S)	1.022	$0.146 \approx 2.5\lambda$

At the transmitter, the distances d_1 and d_2 for the two setups are as follows:

Transmitter Setup	d_1 [m]	d_2 [m]
Large (L)	$0.862 \approx 15\lambda$	$0.944 \approx 16\lambda$
Small (S)	$0.185 \approx 3\lambda$	$0.185 \approx 3\lambda$

All combinations of the transmitter and receiver setups (six in total) were used for channel measurement as well as for the capacity calculations. In all what follows, we shall denote the setups by the receiver setup (L, M or S) followed by the transmitter setup (L or S). For example, LS denotes the results

5 MIMO Channel Capacity: Theory versus Measurement

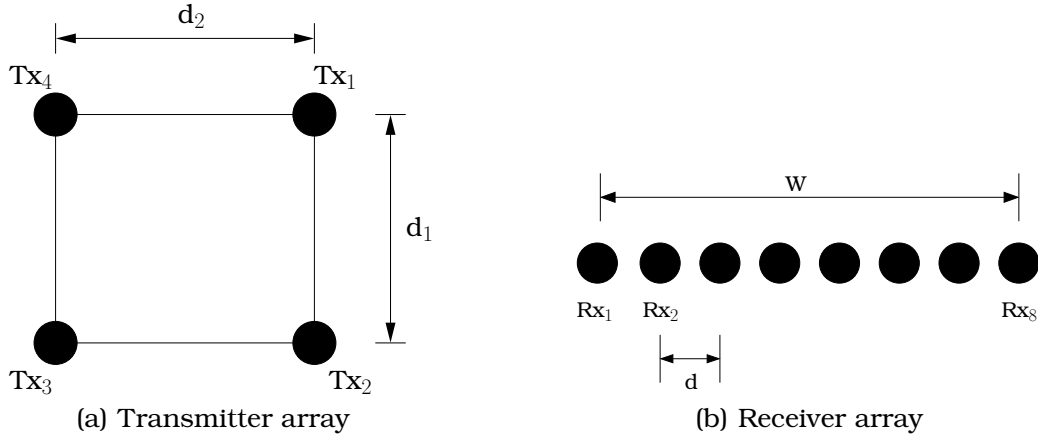


Figure 5.4: Transmitter and receiver array setups.

of the MIMO channel measured using large receive and small transmit array element spacings.

Figure 5.5 shows the impulse response of a typical measured channel (absolute time versus delay) between one pair of transmit and receive antennas. It is clear that the channel has a strong LOS component with the delay spread increasing as the car approaches the bridge. Thus, according to Sec. 5.1.5, large capacities can be expected. In the next section, we compare the actual measured capacities with the maximum achievable capacity expected in such strong LOS channels. Further analysis will be given in Sec. 5.4

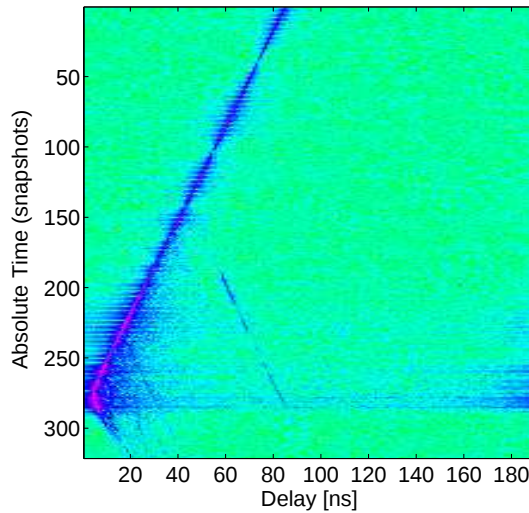


Figure 5.5: Channel impulse response of a typical measured channel.

5.3 Measured Channel Capacity

The MIMO channels at each snapshot were treated as deterministic and the capacity was calculated using Eqn. 5.6. We shall term this capacity instantaneous capacity. The calculations were done in the frequency domain for a bandwidth of 20 MHz and $N = 256$, where N is the number of subcarriers, and were plotted versus the distance between the transmitter and receiver arrays. The capacity was calculated for the above given bandwidth and subcarriers to insure the channel is flat within each subchannel. The receiver (bridge) is located at distance 0. Positive distances represent distances during which the transmitter (vehicle) is approaching the bridge, while negative ones represent those during which the vehicle is moving away from bridge. For the capacity calculations, the eigenvalues of $\underline{\underline{H}} \underline{\underline{H}}^H$ at each snapshot were normalized such that [100, 101]

$$\sum_{i=1}^{\min(Nn_R, Nn_T)} \lambda_i = n_T n_R N. \quad (5.14)$$

We have opted for this normalization instead of normalizing each $\underline{h}_{ij}(l)$ such that $\sum_l |\underline{h}_{ij}(l)|^2 = 1$, since the normalization in Eqn 5.14 maintains the condition number of the measured MIMO channels while at the same time factoring out the pathloss effects (i. e. all channels experience the same SNR). This approach was also employed in [102].

The channel measurements were done using all four transmit and eight receive antennas. Yet, for the capacity calculations either all channels or a subset of them were used. That way we could examine the effect of the number of transmit or receive antennas as well as the element spacing within either the transmitter or the receiver on the capacity. The antennas (see Fig.5.4) corresponding to the different channels used for the capacity calculations are given in the following table. For example, in case of MIMO 2×2 , the channels corresponding to transmit antennas 1 and 3 and receive antennas 1 and 8 were used to calculate the capacity.

MIMO	2×2	2×4	2×8	4×4	4×8
Tx	1,3	1,3	1,3	1-4	1-4
Rx	1,8	1,3,5,7	1-8	1,3,5,7	1-8

Table 5.1: Antennas groups used for capacity calculations.

Figures 5.6 to 5.9 show how the capacity changes with distance to bridge for the Large-Large (LL), Medium-Large (ML), Medium-Small (MS) and Small-

5 MIMO Channel Capacity: Theory versus Measurement

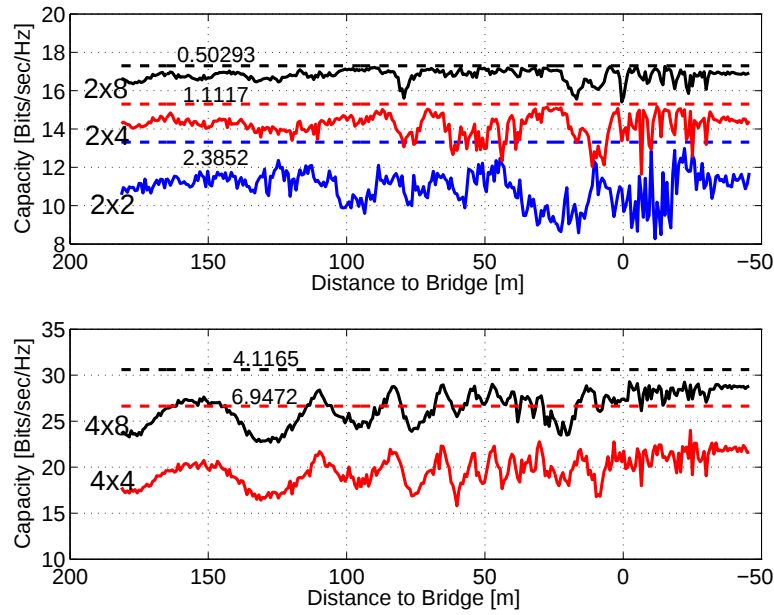


Figure 5.6: Instantaneous capacity versus distance for Large-Large (LL) setup, $\rho_o = 20$ dB.

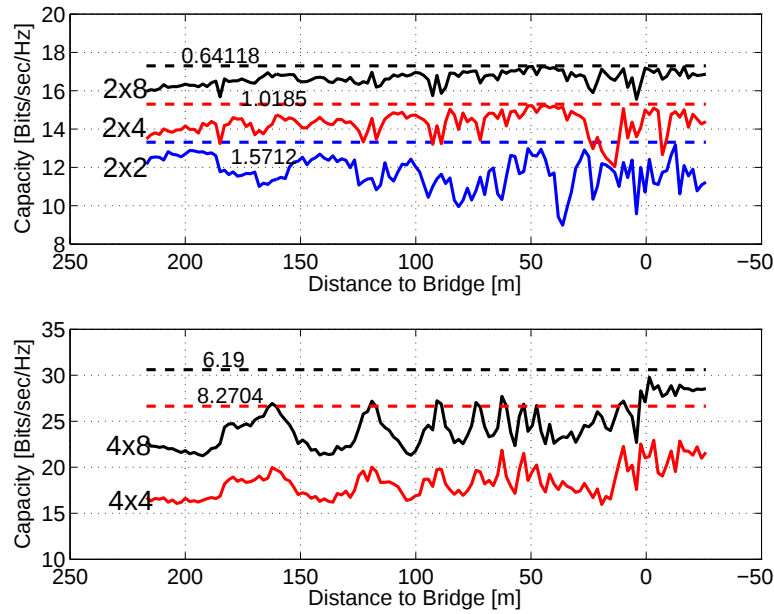


Figure 5.7: Instantaneous capacity versus distance for Medium-Large (ML) setup, $\rho_o = 20$ dB.

5.3 Measured Channel Capacity

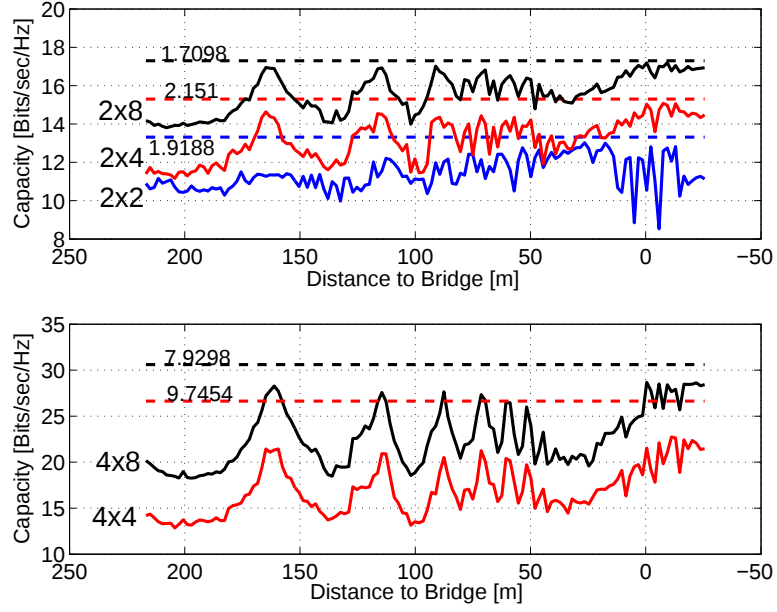


Figure 5.8: Instantaneous capacity versus distance for Medium-Small (MS) setup, $\rho_o = 20$ dB.

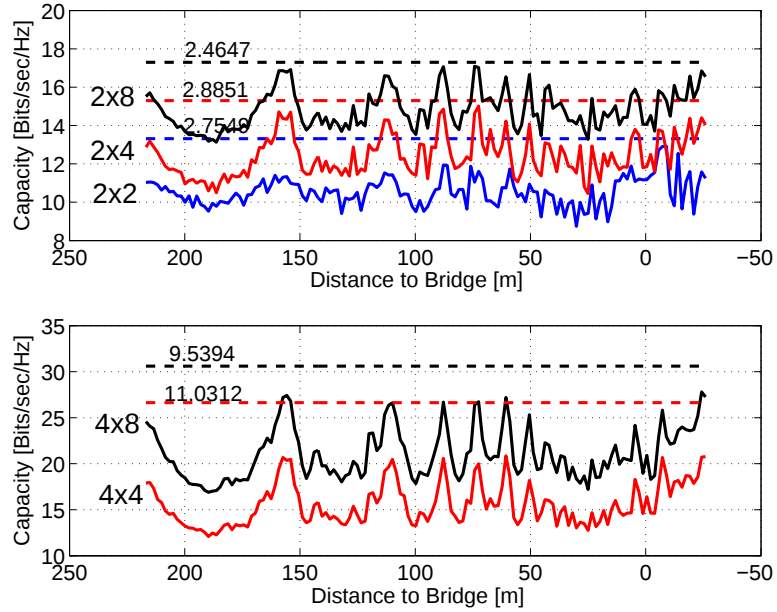


Figure 5.9: Instantaneous Capacity versus distance for Small-Small (SS) setup, $\rho_o = 20$ dB.

5 MIMO Channel Capacity: Theory versus Measurement

Small (SS) array setups and different number of transmit and receive antennas for SNR=20 dB. The dashed horizontal lines represent the maximum capacity that can be achieved for the given $n_R \times n_T$ MIMO system (Eqn. 5.12). The numbers on the lines are the difference between the mean capacity over distance and the maximum one. As expected, the capacity increases as the number of either transmit or receive antennas is increased. Also, as the number of receive antennas increases, the gap between the measured capacity and the maximum one decreases. In addition, the larger the antenna spacing, either at the transmitter or receiver, the higher the calculated capacity.

Furthermore, the calculated capacities are seen to fluctuate as the transmitter approaches the receiver. This is especially true for small antenna spacing within either the transmitter or receiver arrays. Reflections from the street surface that lead to either destructive or constructive interference as the vehicle moves may explain this behavior. To test this theory, the LOS model was extended to a two-path model. The results of the simulations will be shown in Sec. 5.4. The figures also show that the fluctuations seem to be more obvious the smaller the antenna element spacing within either the transmitter or receiver arrays. Channel correlations, which are higher the closer the antennas are to each other, are most probably the reason why the fluctuations are more obvious in for example the SS setup than in the LL setup. In other words, the MIMO channels experience almost the same fading at all antennas the closer the antennas are to each other. This is especially clear in the $2 \times n_R$ capacities, as we have chosen antennas that are furthest from each other (see above table). In the LL setup, the $2 \times n_R$ MIMO channel capacities experience no obvious fluctuations in contrast to the SS setup. Antenna correlations will be discussed in more details in Sec. 5.5.

As mentioned in the previous section, the observed channels exhibit a strong LOS and we thus wish to compare the calculated capacities with the maximum achievable according to Sec. 5.1.5. The following table gives approximate values for D_{orth} according to Eqn. 5.13. The same D_{orth} is obtained for $n_R = 4$ and 8, since the distance between two receive antennas for $n_R = 4$ is twice that for $n_R = 8$.

n_R	LL	ML	MS	SS
2	600 m	213 m	40 m	7 m
4, 8	346 m	122 m	23 m	4 m

Table 5.2: D_{orth} for the different setups.

According to the above table, the LL setup as well as the ML with $n_R = 2$ lie within the orthogonal range through out the whole measurement distance.

For the other setups, the orthogonal range begins either halfway between the transmitter and the receiver, or at distances very close to the receiver. By looking at Figs. 5.6 and 5.7, it is obvious that only the capacities for the 2×8 channels of the LL and ML setups remain very close to the maximum. Despite the strong LOS component, a very slight change in the capacity, as the transmitter approached the receiver, is observed. The capacity, on the average, does not seem to change much with decreasing distance between the transmitter and receiver arrays. In addition, around the distances D_{orth} given in the previous table no significant change in the channel capacity was observed. The results seem to be in contrast to what was expected according to Sec. 5.1.5. However, as mentioned previously, the plane wave assumption is not satisfied for short distances between the transmitter and receiver arrays as is the case in this measurement campaign.

Figures 5.10(a) and 5.10(b) show the mean capacity over distance for all array spacing combinations (i. e. mean of the instantaneous capacity over all snapshots). The solid lines represent the change in the mean capacity with increasing array spacing at the receiver and large array spacing at the transmitter (SL, ML, LL). The dashed one represents the change in the mean capacity for the small array spacing at the transmitter (SS, MS, LS). As expected, the larger the spacing between the antennas within either the transmit or receive arrays, the higher the capacity. The MIMO $2 \times n_R$ capacity does not always confirm to this expectation. This, as mentioned earlier, is probably due to the fact that we have chosen antenna elements that are furthest away from each other, and thus the effect of the antenna spacing is not as strong especially for the 2×2 system. Also note that the large antenna separation at the transmitter is almost equal to the medium one at the receiver. The mean capacities for SL and MS setups are thus, as expected, almost equal (Figs. 5.10(a) and 5.10(b)).

The above results lead to the conclusion that the capacity of the measured MIMO channels is more susceptible to antenna element spacing than to the distance between the transmitter and the receiver. This is however no surprise, since antenna correlations are one of the detrimental factors for MIMO performance.

5.4 Two Path Channel Model

In this section, we model the measured channel between **one** pair of antennas elements using a two-path geometrical channel model with delay difference, $\Delta\tau = \tau_2 - \tau_1$. The delays are given by: $\tau_i = d_i/c$, for $i = 1, 2$, where d_i is the distance traveled by the signal over the i th path and c is the speed of light. We have assumed no doppler shift, i. e. $f_d = 0$. The first path is a line

5 MIMO Channel Capacity: Theory versus Measurement

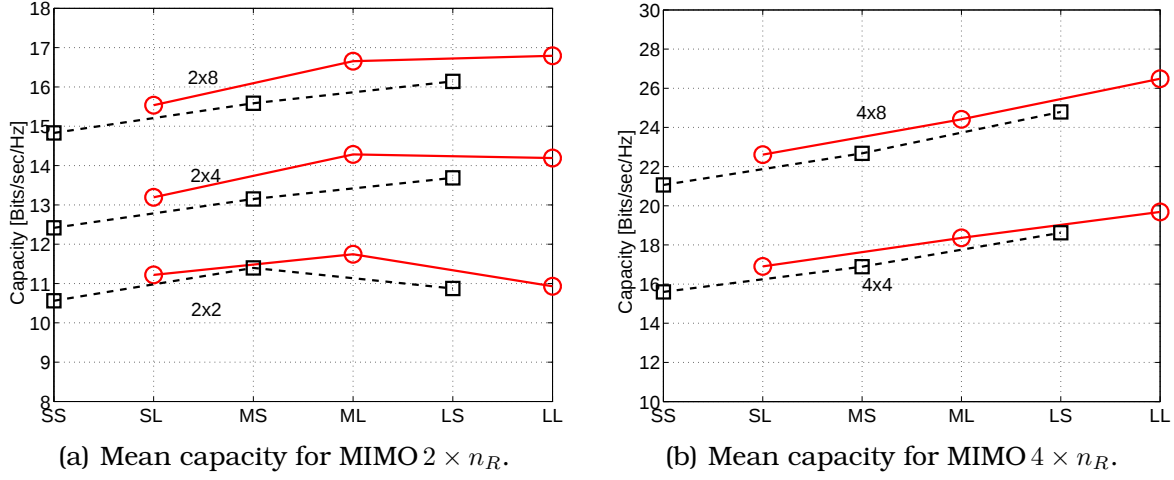


Figure 5.10: Mean capacity for all measured MIMO channels, $\rho_o = 20$ dB.

of sight (LOS) path. The second path is assumed to occur due to reflections from the street surface (see Fig. 5.11). In the low pass domain, this two-path channel model can be described as follows [6],

$$\begin{aligned}
 h_T(\tau) &= a_1 e^{-j2\pi f_o \tau_1} \delta_T(\tau - \tau_1) + a_2 e^{-j2\pi f_o \tau_2} \delta_T(\tau - \tau_2) \\
 &= \sum_{i=1}^2 a_i e^{-j2\pi f_o \tau_i} \delta_T(\tau - \tau_i),
 \end{aligned} \tag{5.15}$$

where f_o is the center frequency of the transmitted signal and a_1 and a_2 are the amplitude factors for the LOS and reflection paths respectively. They are normalized such that $a_1^2 + a_2^2 = 1$. The Dirac function, δ_T , is defined by [6] $\delta_T(\tau) = \delta(\tau) * h_{LP}(\tau)$, where $h_{LP}(\tau)$ is the impulse response of an ideal lowpass filter. Note that Eqn. 5.15 can be easily generalized to any number of paths. As mentioned in Section 5.2, $f_o = 5.2$ GHz and $BW = 120$ MHz. These values were also used for the simulations to calculate the delay difference, $\Delta\tau$. The distances d_i are calculated from a simple geometrical model of the actual measurement scenario (see Fig. 5.11). The obtained delays were then used to calculate the delay difference as well as the channel transfer functions as follows [6],

$$H_T(f, t) = a_1 + a_2 e^{j2\pi(f_d t - f \Delta\tau)}, \tag{5.16}$$

where the doppler frequency $f_d = 0$. At large separations, the delay difference $\Delta\tau$ is small and increases with decreasing distance between the transmitter and receiver. Figure 5.12 shows a spectrogram of a typical measured channel (left) and that of the simulated channel (right) using Eqn. 5.16 for $a_1 = a_2$. As mentioned earlier, the amplitude square of the simulated two path channel was normalized to $a_1^2 + a_2^2 = 1$. The measured channel shown was not

5.4 Two Path Channel Model

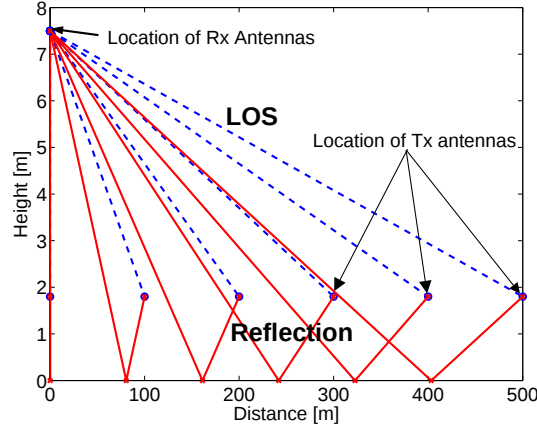


Figure 5.11: Two path Channel Model at different separations between the transmitter and the receiver: Direct path (---LOS) and Reflected path (—Reflection).

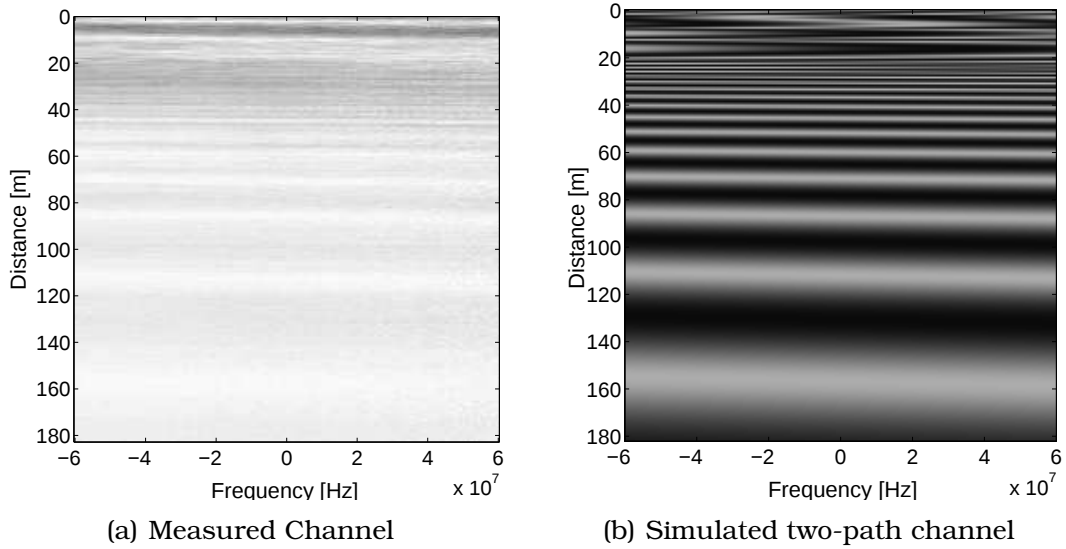


Figure 5.12: Spectrogram of measured and simulated two-path channel ($a_1 = a_2$)

normalized which explains the difference in the gray intensity of the shown spectrograms. Apart from that, both spectrograms are quite comparable, which backs the assumed street reflections. Both exhibit flat fading at large separations between the transmitter and the receiver. Only at very short separations (distances $< 40\text{m}$) do they start to exhibit frequency selective behavior.

5.5 Antenna Correlations

To calculate the spatial correlation, ρ_{ij} , between any pair of channel impulse responses for antenna elements i and j , the following equation was applied [103, 104]

$$\rho_{ij} = \frac{\mathcal{E}\{h_i h_j^*\} - \mathcal{E}\{h_i\}\mathcal{E}\{h_j^*\}}{\sqrt{(\mathcal{E}\{|h_i|^2\} - |\mathcal{E}\{h_i\}|^2)(\mathcal{E}\{|h_j|^2\} - |\mathcal{E}\{h_j\}|^2)}}, \quad (5.17)$$

where the denominator normalized the random variables. In Chapter 2, MIMO channel correlations were defined by the following equations,

$$\underline{k}_T = \frac{1}{n_R} \mathcal{E}\{\underline{h}^H \underline{h}\}, \quad (5.18)$$

$$\underline{k}_R = \frac{1}{n_T} \mathcal{E}\{\underline{h} \underline{h}^H\}. \quad (5.19)$$

The above equations basically correspond to Eqn. 5.17, where however in Eqn. 5.18 the correlations are averaged over all receive antennas and in Eqn. 5.19 they are averaged over all transmit antennas. They also assume the channel to be zero mean and the average channel energy to be one. Thus, before the correlations can be calculated, the channel should be normalized and the mean subtracted. In all what follows, the antenna correlations were calculated using the second tap, the one with the highest amplitude. The channel mean was first calculated for 50 snapshots and subtracted, and then the channel matrices were normalized. Finally, the correlations were calculated by consecutively averaging over 200 snapshots.

Transmit Correlations

Figures 5.13 and 5.14 show the absolute values of the transmit correlations for the LL, SL, LS and SS setups respectively versus the snapshot number. Larger snapshot values correspond to shorter distances between the transmitter and receiver. The average of the correlations for antenna elements lying behind each other for an observer on the bridge (Tx1 and Tx4 as well as Tx2 and Tx3 in Figure 5.4(a)) are compared to the average of the correlations for all other antenna element combinations. In addition to the fact that the smaller the distance between the transmit antenna elements the higher the correlations, three important observations can be made from those figures. First, the antenna correlations are dependent of the geometrical setup of the antennas. Correlations for antennas lying behind each other are in general higher than the correlations for all other antenna element combinations. This can be perceived best for the LL setup. Second, the transmit correlations are affected by the antenna configuration at the receiver. By comparing the correlations for the LL and SL setups in Fig. 5.13,

5.5 Antenna Correlations

we can see that although the setup at the transmitter (L) has not changed, the correlations did change when the receive antenna setup was altered (from L to S). In this case, higher correlations were calculated for the latter setup. Third, the correlations are observed to decrease as the vehicle approaches the bridge. This was to be expected, since the difference between the path delays, τ_{ij} , $i = 1 \cdots n_R$, $j = 1 \cdots n_T$, between the different antenna pairs, increases as the transmitter nears the receiver. Comparable path delays correspond to similar channel impulse responses (see Sec. 5.4).

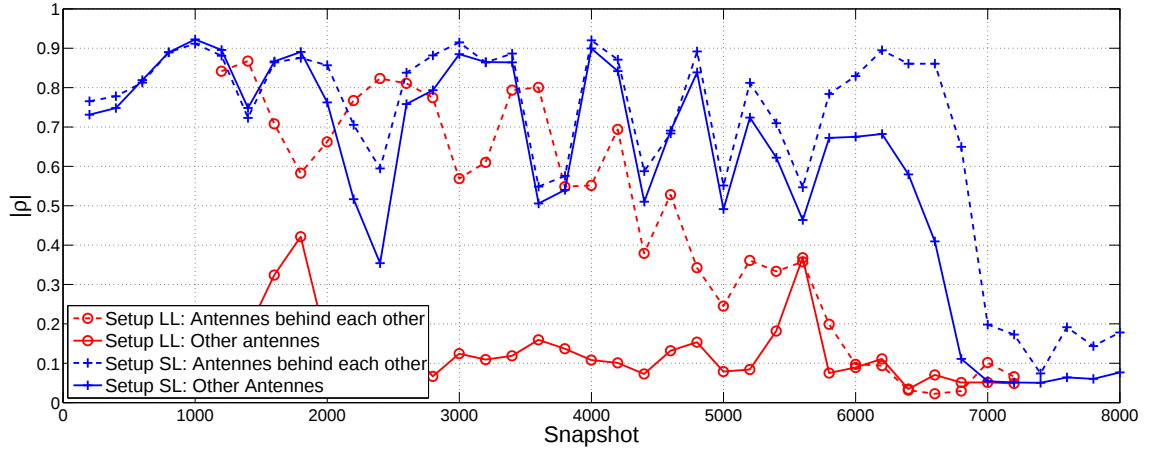


Figure 5.13: Transmit correlations for setups LL and SL.

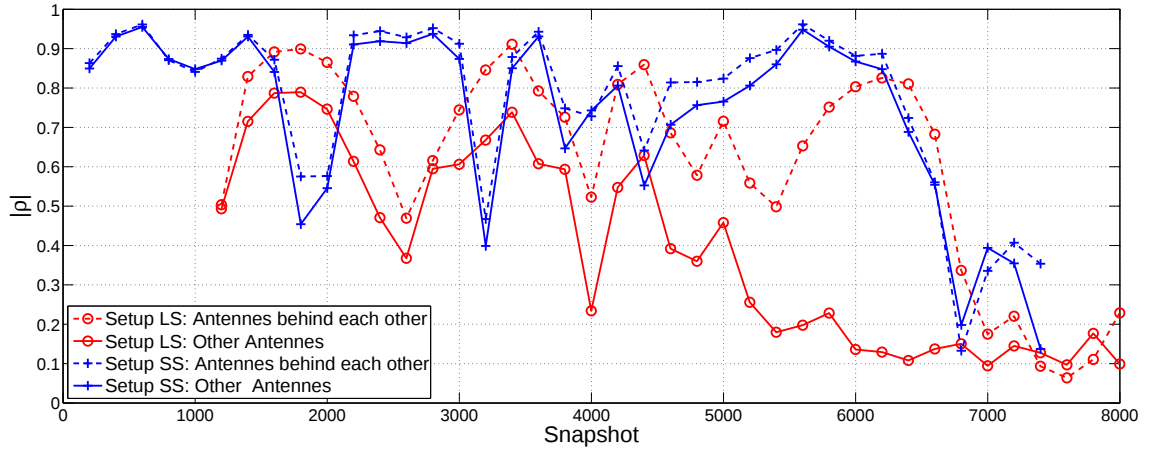


Figure 5.14: Transmit correlations for setups LS and SS.

5 MIMO Channel Capacity: Theory versus Measurement

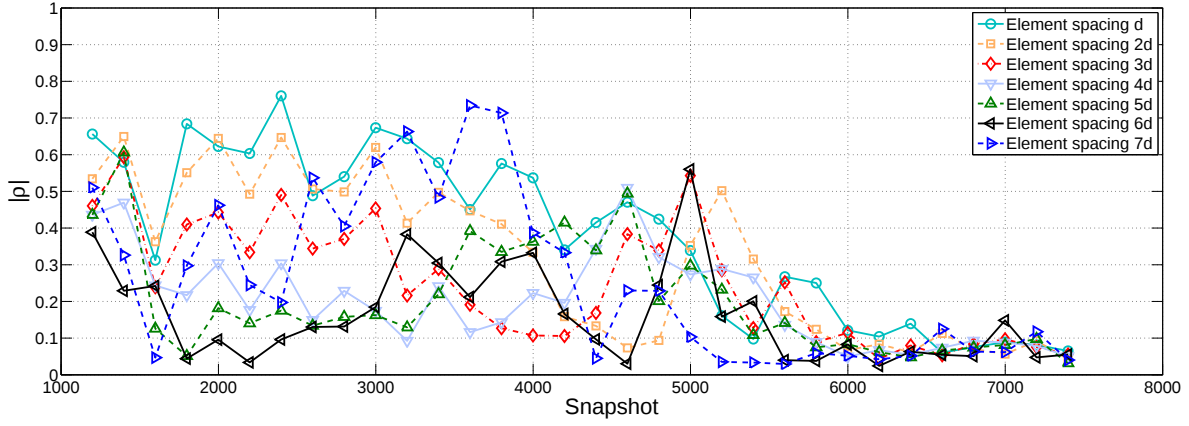


Figure 5.15: Receive correlations for ML Setup.

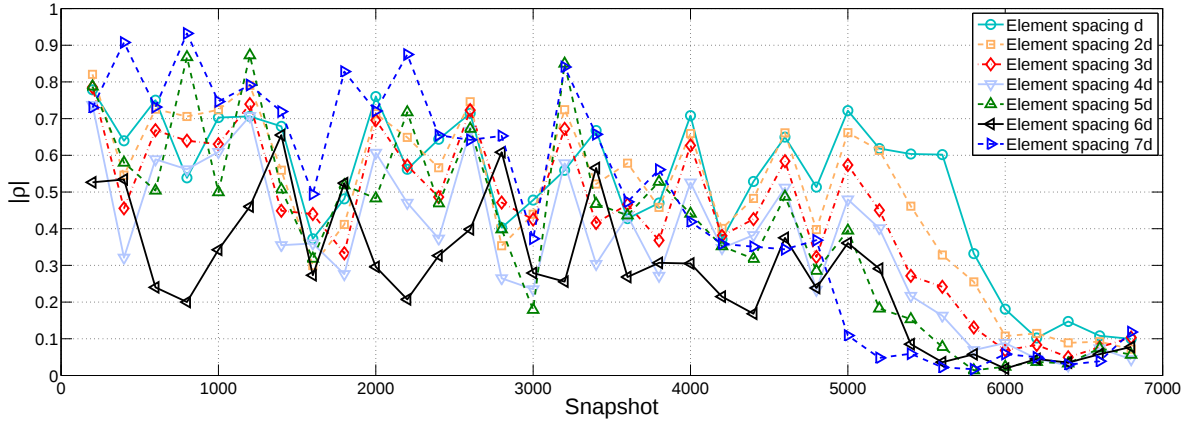


Figure 5.16: Receive correlations for MS Setup.

Receive Correlations

Figures 5.15 and 5.16 show the absolute value of the correlations between antenna elements at the receiver for the ML and MS setups. Shown are the means of the correlations for antenna elements with same spacing, e. g. ρ_{4d} is the mean of ρ_{15} , ρ_{26} , ρ_{37} and ρ_{48} . Similar to transmit correlations, the receive correlations are affected by the transmit array configuration. Thus, the assumption of independence between the transmit and receive correlations used in the Kronecker correlation model is not valid in this scenario. Similar observations were also made in [105]. The receive correlations are also observed to decrease as the transmitter approaches the receiver. In addition, it can be seen from Figs. 5.15 and 5.16 and Table 5.3 that the correlations, in general, decrease as the element spacing increases. However, contrary to what is expected, the correlations for elements with spacing $7d$ are high

5.6 Summary

compared to other correlation values. This behavior was also observed in other setups. As we have seen in transmit correlation results, not only does the element spacing affect the correlation values, but also the geometrical setup of the elements has an effect. By taking a closer look at the correlation between elements that are symmetric to the center of the antenna array (e.g. $\rho_{18}, \rho_{27}, \rho_{36}$ and ρ_{45}), it was found that these correlations on average are higher than the corresponding mean correlations ($\overline{\rho_{5d}}, \overline{\rho_{3d}}, \overline{\rho_{1d}}$). Table 5.4 gives an overview of the absolute values of the correlations averaged over all snapshots. Clearly, correlations between elements that are symmetric to the array center are higher than those between elements at the same distance, but not symmetric to the mean. Again, we see here that the geometrical setup may very well be affecting the correlations.

Setup	$\overline{\rho_{1d}}$ $\overline{\text{Re} + j \text{Im}}$	$\overline{\rho_{2d}}$ $\overline{\text{Re} + j \text{Im}}$	$\overline{\rho_{3d}}$ $\overline{\text{Re} + j \text{Im}}$	$\overline{\rho_{4d}}$ $\overline{\text{Re} + j \text{Im}}$
LL	0,163 + j 0,151	0,117 + j 0,103	0,114 + j 0,110	0,113 + j 0,127
LS	0,254 + j 0,232	0,194 + j 0,214	0,161 + j 0,170	0,130 + j 0,163
ML	0,234 + j 0,258	0,166 + j 0,233	0,172 + j 0,142	0,133 + j 0,125
MS	0,327 + j 0,338	0,274 + j 0,347	0,286 + j 0,273	0,233 + j 0,233
SL	0,400 + j 0,376	0,347 + j 0,368	0,278 + j 0,312	0,229 + j 0,289
SS	0,410 + j 0,375	0,379 + j 0,387	0,324 + j 0,340	0,267 + j 0,296
Setup	$\overline{\rho_{5d}}$ $\overline{\text{Re} + j \text{Im}}$	$\overline{\rho_{6d}}$ $\overline{\text{Re} + j \text{Im}}$	$\overline{\rho_{7d}}$ $\overline{\text{Re} + j \text{Im}}$	
LL	0,124 + j 0,109	0,072 + j 0,098	0,085 + j 0,088	
LS	0,137 + j 0,130	0,112 + j 0,095	0,121 + j 0,094	
ML	0,135 + j 0,119	0,115 + j 0,091	0,177 + j 0,158	
MS	0,250 + j 0,258	0,173 + j 0,193	0,271 + j 0,318	
SL	0,312 + j 0,280	0,238 + j 0,231	0,416 + j 0,307	
SS	0,324 + j 0,309	0,199 + j 0,233	0,412 + j 0,403	

Table 5.3: Receive correlation coefficients: Mean ρ_{Rx} for antenna elements with same separation (1d – 7d) averaged over all snapshots.

5.6 Summary

We looked at the capacity of MIMO channels and gave a theoretical overview for MIMO capacity in presence of no, partial and full channel knowledge. We showed that the optimum solution for capacity maximization for the two latter cases is to transmit independent Gaussian inputs along the eigenvectors of $\underline{k}_T$ or the left singular vectors of $\underline{h}$ respectively. The variances of the Gaussian inputs are given by the power allocation strategy. We then presented

5 MIMO Channel Capacity: Theory versus Measurement

Setup	$ \overline{\rho_{45}} $ / $ \overline{\rho_{1d}} $	$ \overline{\rho_{36}} $ / $ \overline{\rho_{3d}} $	$ \overline{\rho_{27}} $ / $ \overline{\rho_{5d}} $	$ \overline{\rho_{18}} $
LL	0,243 / 0,248	0,192 / 0,175	0,232 / 0,185	0,143
LS	0,448 / 0,382	0,329 / 0,260	0,248 / 0,211	0,174
LU	0,347 / 0,325	0,234 / 0,217	0,197 / 0,157	0,105
ML	0,513 / 0,382	0,335 / 0,249	0,256 / 0,202	0,257
MS	0,675 / 0,518	0,612 / 0,436	0,519 / 0,400	0,462
MU	0,750 / 0,541	0,694 / 0,452	0,546 / 0,407	0,425
SL	0,794 / 0,600	0,682 / 0,468	0,651 / 0,462	0,546
SS	0,861 / 0,607	0,796 / 0,520	0,701 / 0,507	0,642
SU	0,847 / 0,628	0,789 / 0,569	0,707 / 0,558	0,570

Table 5.4: Receive correlations: Correlation values for antenna elements symmetric to the array mean in contrast to those of elements at the same distance, but not symmetric to the mean.

our measurement scenario and gave the channel capacities and correlations of the measured MIMO channels. We found that the measured MIMO channels, as expected, possessed a strong LOS. In addition, the channels exhibited a flat fading behavior throughout most of the measurement distance. The capacity was found to fluctuate as the transmitter approached the receiver. Reflections from the street surface, that periodically destructively and constructively interfered with the LOS path, are suspected to be the reason behind this behavior. A two path channel model was used to examine this speculation. Finally, we presented the transmit and receive correlations for this scenario. We found that the correlations, are not only affected by the distance between the elements within the array, but also by the geometrical setup.

Chapter 6

Summary and Conclusion

In recent years, there has been a continuous demand for reliable wireless transmission at even higher data rates. This trend is very unlikely to change and, as a matter of fact, the demands are expected to increase. Thus, switching to MIMO communication systems is inevitable. In this work, several transmission techniques for MIMO-OFDM were studied. The aim was to improve the system reliability depending on the amount of information present at the transmitter and type of equalizer at the receiver mitigating the interference. We have focused our attention on two cases: no and partial channel state information at the transmitter. Both are of practical importance, since full channel state information is hardly available at the transmitter.

In Chapter 2, a detailed theoretical background for MIMO-OFDM as well as an overview of four block equalizers employed in this work for equalization was presented. Chapter 3 focused on the case of no channel state information at the transmitter. Two criteria, that need to be satisfied by spreading matrices if full diversity is to be achieved, were defined. A family of spreading matrices, MC-CAFS, which were devised based on those criteria was then

6 Summary and Conclusion

presented. In addition, we demonstrated how spreading and antenna correlations affect the interference and the matched filter bound and in turn the BER performance. The main conclusions of this chapter are as follows:

- Spreading improves the matched filter bound, but increases the interference. The higher the diversity offered by the spreading matrix, the higher the interference.
- Spreading with full diversity, although increases the interference to its maximum, leads to the lowest BER for a range of antenna correlations, 0 to some ρ value. The width of this range depends on the equalizer used as well as on the E_b/N_o . The range gets wider as the E_b/N_o increases.
- The matched filter bound is dependent not only on the diversity, but also on the number of correlated diversity branches.
- Antenna correlations increase the interference, which makes it harder for the equalizer to mitigate the interference as correlations increase. In addition, receive correlations worsen the matched filter bound.
- Rotated MC-CAFS can lead to lower BER. This is achieved at no extra cost.

Chapter 4 concentrated on partial channel state information at the transmitter, yet an overview of precoding in presence of full channel state information at the transmitter was also presented. We considered precoding under covariance information (transmit correlations knowledge). We showed that transmitting in the direction of the transmit correlation eigenvectors at each subcarrier was the optimum direction for average pairwise error probability minimization. For the uncoded transmission, precoding was shown to improve the BER for all equalizers except for the MMSE-BLE. Coded transmission with iterative equalization and decoding was also investigated in details. The results can be summarized as follows:

- At high E_b/N_o and for MIMO-OFDM and MC-CDM precoding leads to higher BER since the matched filter bound worsens.
- Precoding along with MC-CAFS and SCE always achieved the lowest BER at high E_b/N_o .
- The BER is highly dependent on the interference in the channel and less on the condition number. The RNN was shown to be especially sensitive to interference and sometimes does not even converge.
- The code memory affects the BER performance. A code with low memory improves the BER at low E_b/N_o compared to a high memory code. However, the reverse is true at high E_b/N_o .

- EXIT charts are a very useful convergence analysis tool that can aid in designing the communication system, e. g. deciding on the code memory and equalizer used.

Thus, with a powerful equalizer such as the SCE and a full diversity spreading matrix as MC-CAFS, precoding can significantly reduce the BER.

Finally, in Chapter 5 a theoretical overview for MIMO channel capacity was given to demonstrate the potential of MIMO transmission. The capacity and the antenna correlations for a measured outdoor scenario with a strong line of sight were calculated and shown in this chapter. The measured capacities were found to increase by increasing the number of transmit or receive antennas, as well as by increasing the antenna element spacing within the transmitter or receiver arrays which agrees with the theoretical predictions. Nonetheless, in contrast to the theories, we found that high antenna correlations are present even for large antenna element spacings. In addition, contrary to the theoretical Kronecker correlation model, the transmit correlations were found to depend on the receiver array configuration and vice versa.

6 *Summary and Conclusion*

Appendix $\mathcal{A}$

Matrix Basics

A.1 Matrix Basics

A.1.1 Inverse, Transpose and Hermitian

- $(\underline{\underline{A}} \underline{\underline{B}})^{-1} = \underline{\underline{B}}^{-1} \underline{\underline{A}}^{-1}$
- $(\underline{\underline{A}} \underline{\underline{B}} \underline{\underline{C}} \dots)^{-1} = \dots \underline{\underline{C}}^{-1} \underline{\underline{B}}^{-1} \underline{\underline{A}}^{-1}$
- $(\underline{\underline{A}}^T)^{-1} = (\underline{\underline{A}}^{-1})^T$
- $(\underline{\underline{A}} + \underline{\underline{B}})^T = \underline{\underline{A}}^T + \underline{\underline{B}}^T$
- $(\underline{\underline{A}} \underline{\underline{B}})^T = \underline{\underline{B}}^T \underline{\underline{A}}^T$
- $(\underline{\underline{A}} \underline{\underline{B}} \underline{\underline{C}} \dots)^T = \dots \underline{\underline{C}}^T \underline{\underline{B}}^T \underline{\underline{A}}^T$
- $(\underline{\underline{A}}^H)^{-1} = (\underline{\underline{A}}^{-1})^H$
- $(\underline{\underline{A}} + \underline{\underline{B}})^H = \underline{\underline{A}}^H + \underline{\underline{B}}^H$

A Matrix Basics

- $(\underline{\underline{A}}\underline{\underline{B}}\underline{\underline{C}}\cdots)^H = \cdots \underline{\underline{C}}^H \underline{\underline{B}}^H \underline{\underline{A}}^H$

A.1.2 Trace, Determinant and Frobenious Norm

- $\text{Tr}(\underline{\underline{A}}) = \sum_i a_{ii}$, where a_{ii} is the i th diagonal element of $\underline{\underline{A}}$
- $\text{Tr}(\underline{\underline{A}}) = \sum_i \lambda_i \quad \lambda_i = \text{eig}(\underline{\underline{A}})$
- $\text{Tr}(\underline{\underline{A}}) = \text{Tr}(\underline{\underline{A}}^T)$
- $\text{Tr}(\underline{\underline{A}}\underline{\underline{B}}) = \text{Tr}(\underline{\underline{B}}\underline{\underline{A}})$
- $\text{Tr}(\underline{\underline{A}} + \underline{\underline{B}}) = \text{Tr}(\underline{\underline{B}} + \underline{\underline{A}})$
- $\text{Tr}(\underline{\underline{A}}\underline{\underline{B}}\underline{\underline{C}}) = \text{Tr}(\underline{\underline{B}}\underline{\underline{C}}\underline{\underline{A}}) = \text{Tr}(\underline{\underline{C}}\underline{\underline{A}}\underline{\underline{B}})$
- $\det(\underline{\underline{A}}\underline{\underline{B}}) = \det(\underline{\underline{A}}) \det(\underline{\underline{B}})$
- $\det(\underline{\underline{A}}^{-1}) = 1/\det(\underline{\underline{A}})$
- $\det(\underline{\underline{A}}) = \prod_i \lambda_i \quad \lambda_i = \text{eig}(\underline{\underline{A}})$
- $\|\underline{\underline{A}}\|_F^2 = \sum_{i=1}^n \sum_{j=1}^n |a_{ij}|^2$
- For an $n \times n$ positive definite matrix $\underline{\underline{A}}$
 - $\det(\underline{\underline{A}}) = \prod_i \lambda_i \leq \prod_{i=1}^n a_{ii}$.
 - $\text{Tr}(\underline{\underline{A}}) = \sum_{i=1}^n \lambda_i$.
 - $\|\underline{\underline{A}}\|_F^2 = \text{Tr}(\underline{\underline{A}}^H \underline{\underline{A}}) = \sum_{i=1}^n \lambda_i^2 \quad \lambda_i = \text{eig}(\underline{\underline{A}}) \quad \text{and} \quad \lambda_i^2 = \text{eig}(\underline{\underline{A}}^H \underline{\underline{A}})$

A.2 Mathematical Definitions

A.2.1 Kronecker Product

Definition

$$\underline{\underline{A}} \otimes \underline{\underline{B}} = \begin{pmatrix} a_{11}\underline{\underline{B}} & \cdots & a_{1n}\underline{\underline{B}} \\ \vdots & \ddots & \\ a_{m1}\underline{\underline{B}} & \cdots & a_{mn}\underline{\underline{B}} \end{pmatrix} \quad (\text{A.1})$$

Properties of Kronecker product

- $(\underline{\underline{A}} \otimes \underline{\underline{B}})^T = \underline{\underline{A}}^T \otimes \underline{\underline{B}}^T$
- $(\underline{\underline{A}} \otimes \underline{\underline{B}})^{-1} = \underline{\underline{A}}^{-1} \otimes \underline{\underline{B}}^{-1}$
- $(\underline{\underline{A}} \otimes \underline{\underline{B}})^H = \underline{\underline{A}}^H \otimes \underline{\underline{B}}^H$
- $(\underline{\underline{A}} \otimes \underline{\underline{B}}) \otimes \underline{\underline{C}} = \underline{\underline{A}} \otimes (\underline{\underline{B}} \otimes \underline{\underline{C}})$
- $(\underline{\underline{A}} + \underline{\underline{B}}) \otimes \underline{\underline{C}} = \underline{\underline{A}} \otimes \underline{\underline{C}} + \underline{\underline{B}} \otimes \underline{\underline{C}}$
- $\underline{\underline{A}} \otimes (\underline{\underline{B}} + \underline{\underline{C}}) = \underline{\underline{A}} \otimes \underline{\underline{B}} + \underline{\underline{A}} \otimes \underline{\underline{C}}$
- $(\underline{\underline{A}} \otimes \underline{\underline{B}})(\underline{\underline{C}} \otimes \underline{\underline{D}}) = (\underline{\underline{A}} \otimes \underline{\underline{C}})(\underline{\underline{B}} \otimes \underline{\underline{D}})$
- $\text{rank}(\underline{\underline{A}} \otimes \underline{\underline{B}}) = \text{rank}(\underline{\underline{A}}) \text{rank}(\underline{\underline{B}})$
- For any $m \times m$ complex matrix $\underline{\underline{A}}$ with eigenvalues $\lambda_i, i = 1 \cdots m$ and an $n \times n$ complex matrix $\underline{\underline{B}}$ with eigenvalues $\mu_j, j = 1 \cdots n$, then
 - the eigenvalues of $\underline{\underline{A}} \otimes \underline{\underline{B}}$ are $\lambda_i \mu_j, i = 1 \cdots m, j = 1 \cdots n$
 - $\det(\underline{\underline{A}} \otimes \underline{\underline{B}}) = \det(\underline{\underline{A}})^{\text{rank}(\underline{\underline{B}})} \det(\underline{\underline{B}})^{\text{rank}(\underline{\underline{A}})}$
 - $\text{Tr}(\underline{\underline{A}} \otimes \underline{\underline{B}}) = \text{Tr}(\underline{\underline{A}}) \text{Tr}(\underline{\underline{B}})$

A.2.2 Hadamard Product

Definition, for any two $m \times n$ matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$

$$\underline{\underline{A}} \circ \underline{\underline{B}} = \begin{pmatrix} a_{11}b_{11} & \cdots & a_{1n}b_{1n} \\ \vdots & \ddots & \\ a_{m1}b_{m1} & \cdots & a_{mn}b_{mn} \end{pmatrix} \quad (\text{A.2})$$

Properties of Hadamard product

- $\underline{\underline{A}} \circ \underline{\underline{B}} = \underline{\underline{B}} \circ \underline{\underline{A}}$
- $\underline{\underline{A}} \circ (\underline{\underline{B}} + \underline{\underline{C}}) = \underline{\underline{A}} \circ \underline{\underline{B}} + \underline{\underline{A}} \circ \underline{\underline{C}}$
- $\underline{\underline{A}} \circ (\lambda \underline{\underline{B}}) = \lambda(\underline{\underline{A}} \circ \underline{\underline{B}})$
- **Theorem** For any two $m \times n$ matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$
 $\text{rank}(\underline{\underline{A}} \circ \underline{\underline{B}}) \leq \text{rank}(\underline{\underline{A}}) \text{rank}(\underline{\underline{B}})$
- **Schur Product Theorem**
 If the square matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are positive semidefinite, then $\underline{\underline{A}} \circ \underline{\underline{B}}$ is also positive semidefinite
- If the square matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are positive semidefinite, then
 - $\det(\underline{\underline{A}} \circ \underline{\underline{B}}) \leq \det(\underline{\underline{A}}) \det(\underline{\underline{B}})$
 - $\lambda_{\min}(\underline{\underline{A}} \circ \underline{\underline{B}}) \geq \lambda_{\min}(\underline{\underline{A}}) \lambda_{\min}(\underline{\underline{B}})$
 - $\lambda_{\max}(\underline{\underline{A}} \circ \underline{\underline{B}}) \leq \lambda_{\max}(\underline{\underline{A}}) \lambda_{\max}(\underline{\underline{B}})$

Appendix $\mathcal{B}$

Mathematical Notations, Symbols and List of Abbreviations

Mathematical Notation

$(\cdot)^*$	Conjugate
$(\cdot)^H$	Conjugate transpose of a matrix $(\cdot)$ (hermitian)
$\det(\cdot)$	Determinate of a matrix $(\cdot)$
$\text{BlkDiag}([\underline{A} \cdots \underline{Z}])$	Forms a block diagonal matrix from a set of matrices
$\text{diag}(\cdot)$	Diagonal of a matrix $(\cdot)$
$\text{Diag}(\cdot)$	Forms a diagonal matrix from a vector
$\mathcal{E}\{x\}$	Expected value a random variable x
$\ \cdot\ _F^2$	Frobenious norm of a matrix $(\cdot)$
$\Im\{\cdot\}$	Imaginary part of a complex number
$\otimes$	Kronecker product
$\log_2$	Log to the base of 2
$\max(\cdot)$	maximum of a set of elements
$\min(\cdot)$	minimum of a set of elements
$\text{rank}(\cdot)$	Rank of a matrix $(\cdot)$

B Mathematical Notations, Symbols and List of Abbreviations

$\Re\{\cdot\}$	Real part of a complex number
Tr	Trace of a matrix $(\cdot)$
$(\cdot)^T$	Transpose of a matrix $(\cdot)$
$\text{var}(x)$	Variance of a random variable x
$\text{vec}(\cdot)$	Stacks the columns of a matrix $(\cdot)$ to form a vector

Symbols

α	Matched filter bound measure
B	MC-CAFS: number of spreading frequencies per antenna
β	Measure of interference
$\beta_{1/2}$	Measure of interference
C_R	Channel Capacity
χ	Condition number
$\underline{e}_T$	Eigenvalues of transmit correlation matrix
f_d	Doppler frequency
f_g	Cut-off frequency of low pass filter
f_o	Center frequency
$\underline{F}$	Fouier matrix
$\underline{\underline{F}}$	Cholesky decompostion of channel correlation matrix
$\Phi(\cdot)$	Correlation measure of a correlaion matrix $(\cdot)$
$g(t)$	Receive signal
$\underline{h}$	SISO channel vector in time domain
$\underline{h}(l)$	MIMO channel matrix in time domain at l th tap
$\underline{H}$	Channel matrix in frequency domain
$\underline{\underline{H}}$	Effective Channel matrix in frequency domain
$I(X, Y)$	Mutual Information between random variables X and Y
$\underline{I}_N$	Identi matrix of size $N \times N$
K	Rician factor
$\underline{k}_T$	Transmit correlation matrix
$\underline{k}_R$	Receive correlation matrix
$\underline{\underline{k}}$	Transmit or Receive correlation matrix
$\underline{L}$	Number of channel taps
L_{cp}	Length of cyclix prefix
$L(q)$	Log-likelihood Ratio of bit q
λ	Wavelength
λ_i	i th eigenvalue of a matrix
λ_{max}	Maximum eigenvalue of a matrix

λ_{min}	Minimum eigenvalue of a matrix
N	Number of OFDM subcarriers
$\underline{n}$	Additive white Gaussian noise vector
$\underline{n}_c$	Additive colored Gaussian noise vector
n_R	Number of receive antennas
n_T	Number of transmit antennas
$\Psi(\cdot)$	Diversity measure of a correlation matrix $(\cdot)$
$p(x)$	Probability density function of random variable x
P	Total transmit power
$P(x=a)$	Probability that $x = a$
$\underline{Q}$	Covariance matrix of a transmit vector
$\underline{\underline{R}}$	Channel correlation matrix
$\underline{\underline{R}}_{K,n}(r)$	K-Symmetric channel
$\underline{\underline{R}}_{KZ,n}(r)$	K-Z-Symmetric channel
ρ	Correlation coefficient
ρ_{Tx}	Transmit correlation coefficient
ρ_{Rx}	Receive correlation coefficient
ρ_l	Correlation coefficient of l th tap
ρ_o	Signal to noise ratio (SNR)
$s(t)$	Transmit signal
σ_l^2	Variance of the l th channel tap
σ_e^2	Variance of the estimation error
T_s	Symbol duration
$\underline{U}$	Spreading matrix
$\underline{\underline{v}}_T$	Eigenvectors of transmit correlation matrix
$\underline{x}$	Transmit symbol vector
$\tilde{\underline{x}}_o$	Received symbol vector

List of Abbreviations

AWGN:	Additive white Gaussian noise
BW	Bandwidth
BW_c	Coherence Bandwidth
BER:	Bit error rate
CIR:	Channel impulse response
CMF:	Channel matched filter
CP:	Cyclic prefix
CSI:	Channel state information
CSIT:	Channel state information at the transmitter
DFT:	Discrete Fourier transform
EXIT:	Extrinsic information transfer chart
IBI:	Inter block interference
IDFT:	Inverse discrete Fourier transform
ISCI:	Inter subcarrier interference
ISI:	Inter symbol interference
LLR:	Log-likelihood Ratio
LOS:	Line of sight
MC-CAFS:	Multi-carrier cyclic antenna frequency spreading
MC-CDM:	Multi-carrier code division multiplexing
MFB:	Matched filter bound
MIMO:	Multiple Input Multiple Output
MMSE-BDFE:	Minimum mean square error block decision feedback equalizer
MMSE-BLE:	Minimum mean square error block linear equalizer
MRC:	Maximum Ratio Combining
OFDM:	Orthogonal frequency division multiplexing
pdf:	Probability density function
PDP:	Power delay profile
PEP:	Pairwise error probability
PSK:	Phase shift keying
RNN:	Recurrent neural networks
SCE:	Soft Cholesky equalizer
SISO:	Single Input Single Output
SNR:	Signal to noise ratio
STBC:	Space Time Block Code

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Index

A

Alamouti Space Time Block
code 44
Antenna Correlations . 16, 96, 148
Capacity 136
Frequency Domain 96, 120
Kronecker Model 17
Measurement Results 148

B

Block Decision Feedback Equal-
izer 24
MMSE-BDFE 24
ZF-BDFE 24
Block Equalizers 22
Block Decision Feedback
Equalizer 24
Block Linear Equalizer 23
Recurrent Neural Network
Equalizer (RNN) 25
Soft Cholesky Equalizer
(SCE) 27
Block Linear Equalizer 23
MMSE-BLE 23
ZF-BLE 23

C

Capacity 131
Antenna Correlations 136
Line of Sight 136
Cholesky Decomposition 24

Coherence Bandwidth 55
Condition Number 19
Correlation Measure 20

D

Discrete Fourier Transform 8
Diversity 42
Diversity Measure 19

E

EXIT Chart 32
Transient Trajectory 34
Exponential Correlation Model 18

H

Hard Decision Equalizers 23

I

Iterative Equalization and Decod-
ing 29
Feedback Equalizers 31

K

K-Symmetric Channel 34
K-Z-Symmetric Channel 35
Kronecker Correlation Model .. 17,
59, 85

L

Log-Likelihood Ratios (L-values) 30

M

Matched Filter Bound 40, 60

Index

Matrix Vector Transmission Model	
MIMO-OFDM	13
Precoding	91
SISO-OFDM	10
Spreading	14
MC-CAFS	49
MC-CDM	48
Measure of Interference	59, 60
Measure of matched filter bound	60
MIMO	4
Channel capacity	131
Channel model	4
MIMO-OFDM	13
Antenna correlations	16
Precoding	93, 96
Spreading Criteria	46
Mutual Information	32

O

OFDM	5
Cyclic prefix	7
MIMO-OFDM	11
Antenna correlations	16
Channel model	12
SISO-OFDM	5
Spreading	14

P

Pairwise Error Probability	45
Precoding	91
Full channel knowledge	93
Power Allocation Strategies	94
Partial channel knowledge .	94
Flat Fading Channels	95
Frequency Selective Channels	96

R

Recurrent Neural Network Equal- izer (RNN)	25
Rotated MC-CAFS	72

S

Soft Cholesky Equalizer (SCE) .	27
Soft Decision Equalizers	25
Space Frequency codes	46
Space Time codes	44
Alamouti	44
Spreading Matrices	
MC-CAFS	49
MC-CDM	48
Rotated MC-CAFS	72

T

Test Channels	34
K-Symmetric Channel	34
K-Z-Symmetric Channel	35
Transient Trajectory	34

