

# Synthesis of Initial Plane-Symmetric Optical Systems using Parabasal Theory



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## Motivation - From Paraxial (PA) to Parabasal (PB)

### Paraxial Theory

- differential region around the reference → **optical axis**
- valid for rotational symmetric systems only
- single Gaussian image plane

rotational symmetric system geometries   double-plane symmetric system geometries   plane symmetric system geometries   symmetry free system geometries

increasing complexity of system geometry

### Parabasal Theory

- differential region around the reference → **base ray (center object-EP)**
- valid for generalized systems
- meridional and sagittal image plane (astigmatism)

**System Reference and Area of Validity**

- base ray: object center → pupil center
- rays that make small angles with the base ray are called parabasal rays

Object   EP   Parabasal Ray Fan

### Derivation of 1st order imaging properties for tilted systems

## Derivation of Parabasal Imaging Framework

### Characteristics in Differential Regime around the Base Ray

- any surface in the vicinity of a specific point is appropriately described by two curvatures → object / entrance pupils are imaged into meridional & sagittal image points / exit pupils

#### Meridional Imaging

#### Sagittal Imaging

### Concept for Parabasal Imaging Framework Derivation

- infinitesimal object point shift → determination of image point shift using Coddingtons eq.

#### Coddingtons Equations

meridional

$$F_m(l_m, l'_m, \theta_m, \theta'_m) = \frac{n' \cos^2 \theta'_m}{l'_m} - \frac{n \cos^2 \theta_m}{l_m} - \frac{n' \cos \theta'_m}{R_m} - \frac{n \cos \theta_m}{R_m} = 0$$

sagittal

$$F_s(l_s, l'_s, \theta_s, \theta'_s) = \frac{n'}{l'_s} - \frac{n}{l_s} - \frac{n' \cos \theta'_s}{R_s} - \frac{n \cos \theta_s}{R_s} = 0$$

⇒ meridional & sagittal image & pupil center position

#### Differential Geometry and Geometric Relations

$O_1$  to  $O_2$  by  $dl, dq \rightarrow d\theta, d\theta' \rightarrow O'_1$  to  $O'_2$  by  $dl', dq'$   
with total derivatives  $dF_m, dF_s \rightarrow \delta'_m, \delta'_s$

⇒ meridional & sagittal image plane tilt

obj. shift along base ray, i.e.  $dq = dq' = 0 \rightarrow M_m, M_s$   
obj. shift perp. to base ray, i.e.  $d\theta = d\theta' = 0 \rightarrow m_m, m_s$

⇒ meridional & sagittal axial ( $M_{m/s}$ ) & lateral ( $m_{m/s}$ ) magnification

### 1st order imaging properties as optimization objective functions

## Real Raytracing Simulation & Final Optimization

### Raytracing Simulation of Initial System

- comparison of 1st order imaging properties derived from parabasal theory (PB) with raytracing (RT) results

Field 1: x = 0 mm, y = 0 mm  
Field 2: x = 0 mm, y = 1 mm  
NA: 0.05

|    | /mm      |          |       |       | /deg     |             |             | /mm   |           |          |          |          |          |          |                 | /deg            |  |
|----|----------|----------|-------|-------|----------|-------------|-------------|-------|-----------|----------|----------|----------|----------|----------|-----------------|-----------------|--|
|    | $l_{2m}$ | $l_{2s}$ | $m_m$ | $m_s$ | $\delta$ | $\delta'_m$ | $\delta'_s$ | $l_1$ | $l_{1-2}$ | $l_{EP}$ | $R_{1m}$ | $R_{1s}$ | $R_{2m}$ | $R_{2s}$ | $\varepsilon_1$ | $\varepsilon_2$ |  |
| PB | -25.0    | -25.0    | 1.0   | 1.0   | 0        | -0.3        | -0.3        | -46.6 | 79.0      | -10.0    | -45.0    | -43.7    | -27.7    | -26.9    | 10.0            | -8.8            |  |
| RT | -25.0    | -25.0    | 0.98  | 0.98  | 0        | -4.69       | 4.16        |       |           |          |          |          |          |          |                 |                 |  |

### Raytracing based Optimization of Initial System

- final optimization of 2 mirror imaging system to achieve imaging properties and minimized aberrations

Field 1: x = 0 mm, y = 0 mm  
Field 2: x = 0 mm, y = 1 mm  
NA: 0.05

|                   | /mm      |          |       |       | /deg     |             |             | /mm   |           |          |          |          |          |          |                 | /deg            |  |
|-------------------|----------|----------|-------|-------|----------|-------------|-------------|-------|-----------|----------|----------|----------|----------|----------|-----------------|-----------------|--|
|                   | $l_{2m}$ | $l_{2s}$ | $m_m$ | $m_s$ | $\delta$ | $\delta'_m$ | $\delta'_s$ | $l_1$ | $l_{1-2}$ | $l_{EP}$ | $R_{1m}$ | $R_{1s}$ | $R_{2m}$ | $R_{2s}$ | $\varepsilon_1$ | $\varepsilon_2$ |  |
| RT <sub>opt</sub> | -25.0    | -25.0    | 1.0   | 1.0   | 0        | 0           | 0           | -54.2 | 88.9      | -10.0    | -66.1    | -65.2    | -25.8    | -20.6    | 13.4            | -16.8           |  |

### Imaging properties & aberration optimized tilted optical system

## Efficient Synthesis of an Initial System

### Example: 2 mirror imaging system folded in y-z-plane

#### Imaging Requirements

- image distances  $l_{2m} = l_{2s} = -25$  mm
- lateral magnification  $m_m = m_s = 1$
- object plane tilt  $\delta = 0^\circ$
- image plane tilt  $\delta'_m = \delta'_s = 0^\circ$

#### Variable System Parameters

- object distance  $l_1$
- mirror distance  $l_{1-2}$
- EP distance  $l_{EP}$
- mirror RoCs  $R_{1m/s}, R_{2m/s}$
- mirror tilts  $\varepsilon_1, \varepsilon_2$

#### Constants

- $n_{1-mirror1} = n_{1-mirror2} = 1$
- $n_{2-mirror1} = n_{2-mirror2} = -1$

### Optimization of System Parameters based on Objective/Merit Functions derived from Coddingtons Equations

| /mm      |          |       |       | /deg     |             |             | /mm   |           |          |          |          |          |          |                 | /deg            |  |
|----------|----------|-------|-------|----------|-------------|-------------|-------|-----------|----------|----------|----------|----------|----------|-----------------|-----------------|--|
| $l_{2m}$ | $l_{2s}$ | $m_m$ | $m_s$ | $\delta$ | $\delta'_m$ | $\delta'_s$ | $l_1$ | $l_{1-2}$ | $l_{EP}$ | $R_{1m}$ | $R_{1s}$ | $R_{2m}$ | $R_{2s}$ | $\varepsilon_1$ | $\varepsilon_2$ |  |
| -25.0    | -25.0    | 1.0   | 1.0   | 0        | -0.3        | -0.3        | -46.6 | 79.0      | -10.0    | -45.0    | -43.7    | -27.7    | -26.9    | 10.0            | -8.8            |  |

### Automatic transfer of initial system parameters to raytracer

## Conclusion

- analytical expressions for 1st order imaging properties of plane-symmetric optical systems
- analytical & optimization based determination of highly tilted, plane-symmetric initial systems
- automatic transfer of initial lens data to commercial raytracing software for final optimization

## Outlook

- derivation of more accurate expression of image plane tilts
- application of imaging framework for initial freeform surface synthesis
- extension of parabasal imaging framework to optical systems without symmetry